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DYNAMIC EFFECTS OF VISCOUS DAMPING ON ISOTROPIC RECTANGULAR PLATES RESTING ON PASTERNAK FOUNDATION, SUBJECTED TO MOVING LOADS

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Abstract : *The model governing the vibration problem of damped isotropic rectangular plate resting on Pasternak foundation is a fourth order partial differential equation, which was solved by separating the variables using series, which reduces the equation to a second order differential equation, and it was solved by employing the central difference scheme of the finite deference method. The dynamic effect of viscous damping was investigated. Apart from the fact that the results obtained compares well with some standard results, it was found that the presence of viscous damping on isotropic plate on Pasternak foundation reduces the possibility of resonance and also stabilizes the system.*

Keywords: Pasternak foundation, viscous damping, partially distributed load, Isotropic rectangular plate.

INTRODUCTION

Many structures can be modeled as rectangular plates, like Railway and Highway Bridges, Road Pavements e.t.c. because of the safety and maintenance of these structures; many researchers have worked and are still working on the dynamic response of plates subjected to moving loads. Some of the early works on this very old and ever expanding field of research includes the work in [8], which discussed the differential equation relating to the breaking of railway Bridges. In [2], [10],[9] and [6] interesting results on the vibration of railway Bridges under moving loads are also reported. In [4] it was concluded that the natural frequency of rectangular plates traversed by moving concentrated forces is greater than that of plates subjected to moving concentrated masses. More recently, studies have been carried out on plates resting on elastic foundations. Such studies worthy of note include that of [3]

On the dynamic response to moving concentrated masses of elastic plates on non-Winkler elastic foundation. Also in [4] we have study on the dynamic response of plates on Pasternak foundation to distributed moving loads, and it was found that the presence of foundation moduli reduces the deflection of the plate and that the area of the distribution of the load has significant effect on the displacement amplitude. In most of the works the effect of damping on the system was neglected. To properly understand the control and dynamic response of vibrating structures to moving loads, it is important to carry out objective analyses of the effect of damping on such structures. In most of the early works the effect of damping is either completely neglected or poorly discussed. It was recently in [5] that a proper analysis of effect of viscous damping on the response of rectangular plate resting on elastic Winkler foundation was carried out. Studies in [1] also investigated the dynamic response of damped Orthotropic Plate on elastic foundation to dynamic moving Loads. In [5], it was reported that the deflection profile of the plate depends on the magnitude of the damping coefficient. The results in [5] also agree in some way with that of [1]. Pasternak foundation is a more advanced model than Winkler, so it becomes important to extend the works in [5] and [1] to isotropic rectangular plates resting on Pasternak foundation.

MATHEMATICAL MODEL GOVERNING THE PROBLEM

The equation governing the problem is given as:

$$D\Delta^4 W(x,y,t) + M_1 W_{,tt}(x,y,t) + 2M\gamma W_{,t}(x,y,t) + KW(x,y,t) - G\Delta^2 W(x,y,t) = P(x,y,t)$$

(1)

Where $P(x,y,t)$ is the applied moving load given as:

$$P(x,y,t) = \frac{1}{r}\left(Mg - M\frac{d^2W}{dt^2}\right)\left[H\left(x - vt + \frac{r}{2}\right) - H\left(x - vt - \frac{r}{2}\right)\right]\delta(y - y_1)$$

(2)

$W(x,y,t)$ =deflection of the plate
$H(x) =$ Heaviside step function
$\delta(x) =$ Dirac delta function
K =Foundation
$G =$ Shear modulus of Foundation
$\Delta^4 =$ Biharmonic Operator
$v =$ Velocity
$g =$ Acceleration due to gravity
$\gamma =$ Coefficient of viscous damping
$M_1 =$ Mass density per unit area
$t =$ Time

$$D = \frac{Eh^3}{12(1-v)}$$

Where: E =Young's modulus, ν =Poisson's ratio, h =thickness of plate, r =Length of load
The above model was developed under the following assumptions:
-There is no deformation in the middle plane of the plate, the plane remain neutral during bending
-The small strain in the body is still governed by Hooke's law
-The load is a distributed time load
-The plate is resting on Pasternak foundation

SOLUTION PROCEDURE

The developed model is solved by method of separation of variable.
Let;

$$W(x,y,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}(t) W_n(x) W_m(y) \qquad (3)$$

We now put (3) into RHS of (1), to have;

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$D\left[\sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}(t)W_n^{iv}(x)W_m(y) + 2\sum_{m=1}^{M}\sum_{n=1}^{N} A_{mn}(t)W_n^{ii}(x)W_m^{ii}(y) +\right.$
m=1Mn=1NAmn(t)Wn(x)Wmiv(y)+M1m=1Mn=1NAmn(t)Wn(x)Wm(y)+2M1γm=1 Mn=1NAmn(t)Wn(x)Wm(y)+Km=1Mn=1NAmn(t)Wn(x)Wm(y)− Gm=1Mn=1NAmn(t)Wnii(x)Wm(y)+m=1Mn=1NAmn(t)Wn(x)Wmii(y)=Px,y,t
(4)

The equation of motion of the vibrating plate resting on elastic foundation is given as

$$D\Delta^4 W - G\Delta^2 W + KW + \omega^2 M_1 W = 0 \qquad (5)$$

Where $W = W(x, y, t)$

Substituting (3) into (5),

$$D\left[W_n^{iv}(x)W_m(y) + 2W_n^{ii}(x)W_m^{ii}(y) + W_n(x)W_m^{iv}(y)\right] - G\left[W_n^{ii}(x)W_m(y) + A_{mn}(t)W_n(x)W_m^{ii}(y)\right] + KW_n(x)W_m(y) + \omega^2 M_1 W_n(x)W_m(y) = 0 \qquad (6)$$

Let $\lambda_{mn} = \omega^2 M_1$, such that

$$\lambda_{mn}W_n(x)W_m(y) = D\left[W_n^{iv}(x)W_m(y) + 2W_n^{ii}(x)W_m^{ii}(y) + W_n(x)W_m^{iv}(y)\right] - G\left[W_n^{ii}(x)W_m(y) + A_{mn}(t)W_n(x)W_m^{ii}(y)\right] + KW_n(x)W_m(y) \qquad (7)$$

Putting (7) into (4) we have that

$\sum_{m=1}^{M}\sum_{n=1}^{N}\left[A_{mn}(t)\lambda_{mn}M_1W_n(x)W_m(y) + M_1\ddot{A}_{mn}(t)W_n(x)W_m(y) +\right.$
2M1γAmn(t)Wn(x)Wm(y)=P(x,y,t) (8)

From (2)

$$P(x,y,t) = \frac{1}{r}\left(Mg - M\frac{d^2W}{dt^2}\right)\left[H\left(x - vt + \frac{r}{2}\right) - H\left(x - vt - \frac{r}{2}\right)\right]\delta(y - y_1)$$

Where,

$$\frac{d^2W}{dt^2} = \frac{\partial^2 W}{\partial t^2} + 2v\frac{\partial^2 W}{\partial x \partial t} + v^2\frac{\partial^2 W}{\partial x^2}$$

LHS of (1) now becomes,

$P(x,y,t) = \frac{1}{r}\left(Mg - M\left[\frac{\partial^2 W}{\partial t^2} + 2v\frac{\partial^2 W}{\partial x \partial t} + v^2\frac{\partial^2 W}{\partial x^2}\right]\right)\left[H\left(x - vt + \frac{r}{2}\right) - H\left(x - vt - \frac{r}{2}\right)\right]\delta(y -$
y1

Equation (1) reduces to

$\sum_{m=1}^{M}\sum_{n=1}^{N}\left[A_{mn}(t)\lambda_{mn}M_1W_n(x)W_m(y) + M_1\ddot{A}_{mn}(t)W_n(x)W_m(y) +\right.$
2M1γAmn(t)Wn(x)Wm(y)=1rMg− M∂2W∂t2+2v∂2W∂x∂t+v2∂2W∂x2Hx− vt+r2− Hx− vt− r2δy− y1 (9)

By substituting (3) into the LHS of (9), integrating both sides along the edges of the plate and applying the orthogonality of $W_n(x)$ *and* $W_m(y)$ (9) becomes

$\ddot{A}_{mn}(t) + 2\gamma\dot{A}_{mn}(t) + \lambda_{mn}A_{mn}(t) =$
$\frac{1}{\theta r M_1}\left[MgW_j(y_1)\int_{B_1}^{B_2} W_i(x)dx -\right.$
Mm=1Mn=1NAmntWny1Wmy1B1B2WnxWixdx+2vAmntWmy1Wjy1B1B2W'n'(x)Wixdx+v2AmntWmy1Wjy1B1B2W'n''(x)Wixdx (10)

Where$B_1 = vt - \frac{r}{2}$ and $B_1 = vt + \frac{r}{2}$

Equation (10) is a coupled differential equation to be solved for some specific boundary condition

Simply Supported Plates

Although equation (10) can be solved for various classical end supports, we focus only on simply supported isotropic rectangular plates.

For simply supported rectangular plate with dimension$(a \times b)$, with the edges condition given as:

$$W(0,y,t) = W(a,y,t) = \frac{\partial^2 W(0,y,t)}{\partial x^2} = \frac{\partial^2 W(a,y,t)}{\partial x^2} = 0$$

$$W(x,0,t) = W(x,b,t) = \frac{\partial^2 W(x,0,t)}{\partial x^2} = 0$$

And the initial conditions are

$$W(x,y,0) = \frac{\partial W(x,y,0)}{\partial x} = 0$$

The normalized deflection curve has obtained in [7] to be:

$$W_n(x)W_m(y) = \frac{2}{\sqrt{ab}} sin\frac{n\pi x}{a} sin\frac{m\pi y}{b} \qquad (11)$$

To obtain the Eigen values, we put (11) into (7), to obtain;

$$\lambda_{mn} = \pi^2 S(D\pi^2 S + G) + K \qquad (12)$$

Where $\qquad S = \left[\frac{n^2}{a^2} + \frac{m^2}{b^2}\right]$

The exact governing for simply supported isotropic rectangular plate can be obtained by substituting (11) into (10)

Where;

$q = n + i, c = n - i$

$$\ddot{A}_{mn}(t) + 2\gamma \dot{A}_{mn}(t) + \lambda_{mn} A_{mn}(t) = \frac{1}{\theta r M_1}\left[\frac{4Mga}{\pi i\sqrt{ab}} sin\frac{i\pi y_1}{b} sin\frac{i\pi vt}{a} sin\frac{i\pi r}{2a} - \right.$$

Mm=1Mn=1N4absinmπy1bsiniπy1bAmntaπccoscπvtasincπr2a− aπqcosqπvtasinqπr2a+8nπva2bAmntsinmπy1bsinπy1baqπsinqπvtasinqπra− acπsincvπtasincπr2a− 4π2n2v2a3bAmntsinmπy1bsinjπy1baπccoscπvtasincπr2a− aπqcosqπvtasinqπr2a (13)

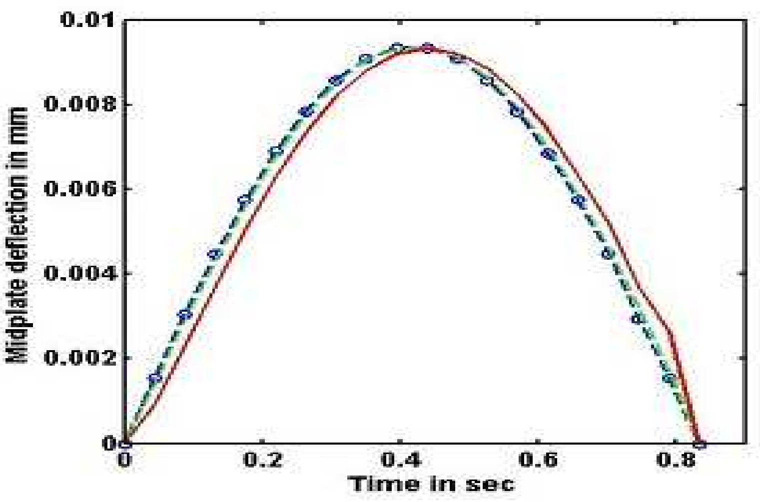
$\ddot{A}_{mn}(t) + 2\gamma \dot{A}_{mn}(t) + \lambda_{mn} A_{mn}(t) = \frac{1}{\theta r M_1}\left[\frac{4Mga}{\pi i\sqrt{ab}} sin\frac{i\pi y_1}{b} sin\frac{i\pi vt}{a} sin\frac{i\pi r}{2a} -\right.$
Mm=1Mn=1N4absinmπy1bsiniπy1bAmntr2− a2nπcos2nπvtasinnπra+8nπva2b Amntsinmπy1bsinπy1ba2nπsin2nπvtasinnπra− 4π2n2v2a3bAmntsinmπy1bsinj πy1br2− a2nπcos2nπvtasinnπra (14)

Equation (13) is for when $n \neq i$ while (14) is for when $n = i$

RESULTS AND DISCUSION

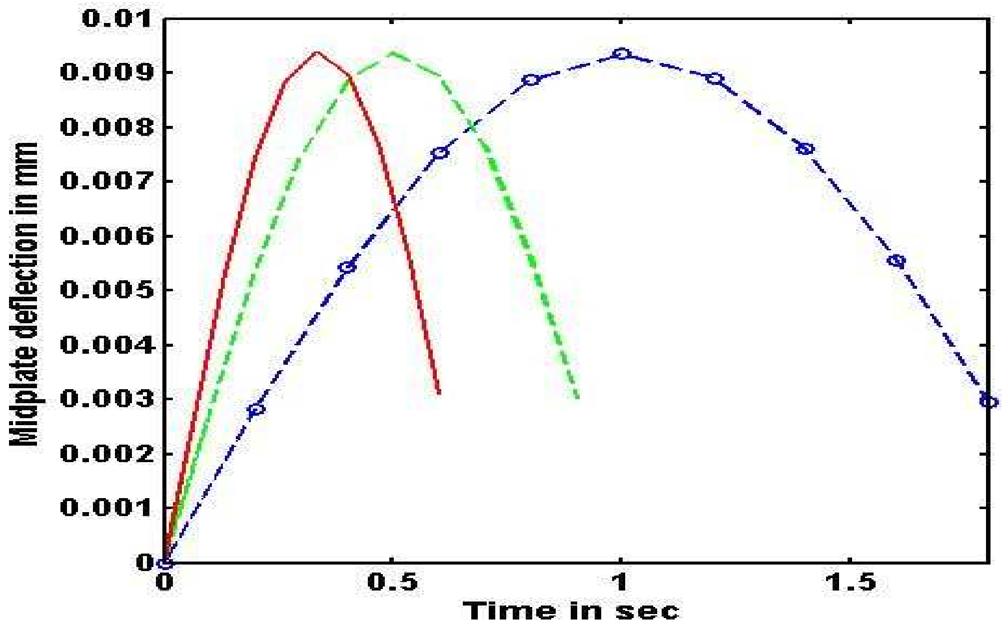
The above coupled differential equations are solved using numerical scheme, the method employed was the central difference approach. The resulting Tri-diagonal matrix was solved using MATLAB. The results obtain are shown graphically below, the dynamic effects of viscous damping on rectangular plates on Pasternak foundation was considered by varying the percentage of viscous damping, and the effect of increase in velocity on a damped system on an elastic foundation is also shown below. In figure 1,it was observed that if the damping ratio is increased the amplitude is reduced. In figure 2, we have deflection for various velocities, velocity depends on damping and higher velocity causes higher mid plate deflection. Figure 3 is for different values of the foundation moduli, and we see that increase in foundation moduli reduces deflection and there by stabilizing the system. Hence damping can be used to prevent build up of amplitude. For comparison sake, the following values are assumed for the corresponding parameters; h=0.25m, E=21090000N/m^2, γ =0, 50, 100, G=4, K=20, M=100kg, g=9.8m/s^2, r=0.5, 1, 1.5, a=10m, b=5m, y_1=2.5m, ν=0.2, v=5m/s, 10m/s and 15m/s.

Figure 1



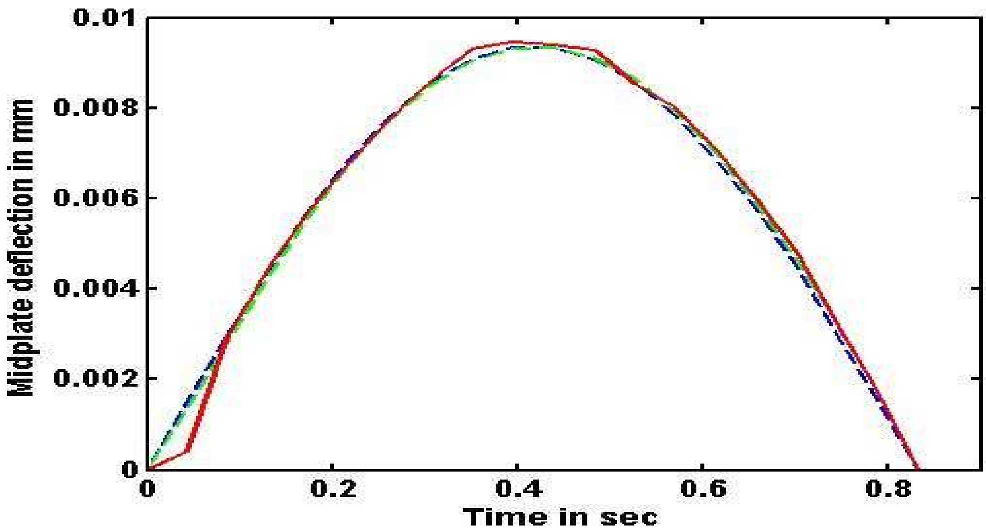
Deflection/time at various damping ratio –o---, γ =0,--, γ =50,-, γ =100

Figure 2



Deflection/time at various velocities –o---- v=5m/s, --, v=10m/s, -,v=15m/s

Figure 3



Deflection/time at various values of G and K. ---------G=0, k=0, - - -, G=4,k=20

CONCLUSION

The equation governing the vibration problem of damped isotropic rectangular plate resting on Pasternak foundation was solved by reducing the fourth order partial differential equation to a coupled second order differential equation. Simply supported boundary condition was investigated, the effects of viscous damping on the system was also studied. The presence of elastic foundation stabilizes the system, and the possibilities of resonance are greatly reduced.

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