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The Nuclear Matter Test of Variational Effective Interactions

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ABSTRACT: *This paper presents the nuclear matter test of seven variational effective interactions derived on the basis of lowest order constrained variational approach (LOCV). Three of these effective interactions, derived from the matrix elements of the Reid93 potential and named N3Y-Fetal, N'3Y-Fetal and N*3Y-Fetal based on nuclear systems A = 16, 40 and 90 respectively, are primarily studied and compared with the first set of four effective interactions, derived from the matrix elements of the Reid68, consisting of the B3Y-Fetal, B'3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal effective interactions based on nuclear systems A = 16, 24, 40 and 90 respectively. The results of this study have revealed that the binding energies* E_{A0} *= -8.6, -12.3 and -7.7 MeV of the cold symmetric nuclear matter (SNM) respectively produced by N3Y-Fetal, N'3Y-Fetal and N*3Y-Fetal effective interactions at the saturation density,* $\rho_0 = 0.17fm^{-3}$ *clearly reflect their mass dependence, but do not satisfy the saturation condition,* E_{A0}*= -16.0±1 MeV, meaning they have failed to reproduce the saturation properties of nuclear matter. Comparing these effective interactions with the B3Y-Fetal, B'3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal effective interactions with the binding energies,* E_{A0} *= -15.2, -14.5, -11.2 and -8.2 MeV respectively for SNM at the saturation density, the findings of this study have established the B3Y-Fetal, out of the seven variational effective interactions presented herein, as the most viable with the highest predictive power, followed by the B'3Y-Fetal effective interaction. Following this, a review of the performance of the B3Y-Fetal effective interaction in nuclear matter and nuclear reactions in its DDM3Yn, BDM3Yn and CDM3Yn density dependences, leading to the presentation of the new B3Y-Fetal-based CDM3Y-K parametrizations and the DDB3Y1-Fetal and BDB3Y1-Fetal curves of equation of state in this work, has so far shown it to be in excellent agreement with the M3Y-Reid and M3Y-Paris effective interactions. These findings indicate the possibility of its successful application in α-nucleus elastic scattering, nuclear fusion, α- and cluster-radioactive decay in future research efforts*

KEYWORDS: nuclear , matter, test of variational, interactions

INTRODUCTION

The discovery of the nucleus by Rutherford in 1911 alongside the discovery of the neutron by

Chadwick in 1932 was a remarkable event that led to a floodgate of active re-searches in nuclear matter and other areas of nuclear physics over the past seven decades.The infinite nuclear matter is an idealized system of bound nucleons with a uniform density that approximates the interior of a heavy nucleus [1]. Since the system is infinite and characterized by strong interactions, complications caused by centre-of-mass motionas well as electromagnetic interactions experienced by finite nuclei are ignored. Becauseof the simple geometry of such an infinite system, the one in which the neutron number N is the same as proton number Z is considered as an important testing ground andsource of invention of new tools to treat the quantitative relationship between the two-body forces and nuclear properties [2, 3]. The main applications of nuclear matter haveso far concerned the nuclear binding energy, equilibrium density and neutron star [4 - 7].

Most information about nuclear matter comes from the semi-empirical formulawhich determines the binding energy of nuclei in terms of a nuclear matter contribution and various corrections for finite nuclei. The well-known formula is the Bethe-Weizsacker formula [8, 9]:

$$E_B(N,Z) = \alpha_v A - \alpha_s A^{\frac{2}{3}} - \alpha_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - \alpha_a \frac{(Z-N)^2}{A} + \alpha_p \frac{\delta}{A^{\frac{3}{4}}} \quad (1)$$

The first term, proportional to A is the volume term which describes the bulk energy of nuclear matter arising from both the kinetic and potential energy associated with the bulk volume of the nuclear system. It is due to the strong nuclear interaction that does not distinguish between neutrons and protons. The second term is the surface term, being due to the strong interaction and a correction to the volume term arising from the factthat nucleons close to the surface have fewer neighbours than the inner ones. The third term accounts for the Coulomb repulsion between protons with α_c as the Coulomb energy parameter. The fourth term is the symmetry energy term whose origin is justified on the basis of Pauli exclusion principle, explaining the experimental evidence that the nuclear force prefers isospin T=O so that nuclei with Z=N are expected to have extra stability and so maximize the binding energy, E_B. The last term is the pairing term, which accounts for the fact that even-even nuclei (i.e nuclei having even Z and even A-Z) are likely to be more stable than even-odd or odd-odd nuclei [1, 3, 10]. The experimentally determined values of the five parameters are: α_v=15.7 MeV, α_s=17.8 MeV, α_c=0.71 MeV, α_a=23.6 MeV and α_p=33.53 MeV. In addition, the pairing parameter, δ is: 1 for even-even, 0 for even-odd and -1 for odd-odd nuclei.

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For infinite nuclear matter, the contributions from the surface term, Coulomb repulsion between protons and the pairing term are ignored, leaving only the volume and symmetry terms in the binding energy formula. Again, for symmetric nuclear matter (SNM), where N=Z, the symmetry term vanishes with only the volume term left in the binding energy. This term is the binding energy per nucleon of infinite nuclear matter, having an empirical value, E_{A0} = -16±1 MeV at a saturation density of 0.17 fm^{-3}. The reproduction of these values together with the compression modulus remains one of the aims of infinite nuclear matter calculations. This has ever been a fundamental and meaningful test for all effective interactions and techniques for many-body problem to pass in order to be used for a successful prediction of nuclear properties of finite nuclei [11]. This is called the nuclear matter test of effective interactions in this work.

All effective interactions that have been used for nuclear matter and nuclear structure calculations within the Hartree-Fock mean-field theory with great success have had this test applied to them. The nuclear matter test usually shows clearly the difference between the effective interactions in terms of their predictive powers. A number of effective interactions that have been found to have good predictive power are the Skyrme, the Gogny, the Love-Franey, the Hamburg and the M3Y interactions [12, 13, 14]. Of all of these effective interactions, the M3Y effective interactions obtained originally by the Michigan State University group from the G-matrix elements of the Paris [15] and Reid [16] nucleon-nucleon (NN) potentials, are known to be the most versatile, working wellin many nuclear models to produce excellent results. Following the success of these G-matrix interactions, Fiase *et al.* [17] derived a similarly motivated potential based on the lowest order constrained variational (LOCV) principle in a work where they investigated the mass dependence of the M3Y-type interactions obtained from the nuclear systems A = 16, 24, 40 and 90, showing their results to be similar to those of Bertsch and collaborators [16] in most reaction channels. These effective interactions are alongside others to be discussed in this paper are collectively called variational effective interactions. Ina bid to determine their viability, the nuclear matter test was applied to this first set of mass-dependent M3Y-type effective interactions in an earlier paper [18] in which they were identified as B3Y-Fetal, B'3Y-Fetal, B^*3Y-Fetal and $B^\dagger$3Y-Fetal effective interactionsbased on nuclear systems A = 16, 24, 40 and 90 respectively. The results of [18]

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have shown the B3Y-Fetal and B′3Y-Fetal effective interactions, based on the nuclear systems A = 16 and 24 respectively, to give correct description of binding energy per nucleon of symmetric nuclear matter (SNM) at the saturation density, $\rho_0 = 0.17 fm^{-3}$, whereas the B*3Y-Fetal and B†3Y-Fetal effective interactions, based on nuclear systems with A =40 and 90 respectively, failed to give correct description of nuclear matter properties at saturation.

The main objective of this paper is to apply the nuclear matter test to a second setof mass-dependent M3Y-type effective interactions derived by Fiase and co-researchers in [19, 20] on the basis of the LOCV method from the matrix elements of the Nijmegen potential, the Reid93 [21, 22] potential. The motivation for the derivation of this second set was the improved phase shift of this high-quality Nijmegen potential, which is believed to have some measurable effects on the members of this set of effective interactions. Itis thought possible in this work that these effects will show up in the predictions of nu- clear matter properties by these effective interactions, which will hopefully be compared with the predictions of the members of the first set of interactions. Since these new mass- dependent M3Y-type effective interactions were derived from nuclear systems with A=16, 40 and 90, we are also intent on evaluating the effect of mass dependence on their pre- dictions of nuclear matter properties in relation to those of the members of the first classin this paper. Additionally, a review of the performances of the B3Y-Fetal and B'3Y-Fetal effective interactions in nuclear matter and nuclear reactions will necessarily be undertaken in this work to determine future research direction for these effective interactions. With this in mind, the organization of this paper is such that Section 2 discusses the functional forms of the variational effective interactions, with Section 3 giving a vivid description of nuclear matter properties, while Section 4 presents and discusses computational results; and Section 5 makes concluding remarks.

The Variational Effective Interactions

The variational effective interactions discussed in this work are in two categories:the first category consists of mass-dependent M3Y-type effective interactions developed based on the *Reid68* [23] nucleon-nucleon (NN) potential within the framework of clus ter expansion technique via the LOCV approach by using two-body correlation functions, whereas the second category is a set of mass-

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dependent M3Y-type effective interactions derived within the framework of LOCV using two-body correlation functions based on the *Reid93*. The details of the calculation were reported in [17, 24] where the matrix elements were shown to be of the form:

$$E_2' = \left\langle \phi \left| \sum_{i>j} [g_2(ij) V_{ij} g_2(ij)] \right| \phi \right\rangle (2)$$

where $\langle\psi|$ represents two-body (harmonic oscillator) wave function and $\boldsymbol{g^{(2)}(ij)}$ are the correlation operators which are meant to take care of the effect of the strong repulsion of the nucleon-nucleon interaction, making the matrix elements finite at short inter-nucleon distances and $\boldsymbol{V_{ij}}$ is the Reid93 [21, 22]. The members of the first set, called the B3Y-Fetal, B′3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal effective interactions, are based on the nuclear systems with A = 16, 24, 40 and 90 respectively. Similarly, the members of the second set, known as N3Y-Fetal, N'3Y-Fetal and N*3Y-Fetal, are based on the nuclear systems A = 16, 40 and 90 respectively. Accordingly, the functional forms of the mass-dependent effective interactions in terms of three Yukawas are [17, 19, 20]:

B3Y-Fetal:

$$v_0^D(s) = \frac{10472.13e^{-4s}}{4s} - \frac{2203.11e^{-2.5s}}{2.5s}$$

$$v_0^{EX}(s) = \frac{499.63e^{-4s}}{4s} - \frac{1347.77e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} (3)$$

B'3Y-Fetal:

$$v_0^D(s) = \frac{6591.54e^{-4s}}{4s} - \frac{1777.55e^{-2.5s}}{2.5s}$$

$$v_0^{EX}(s) = \frac{6158.09e^{-4s}}{4s} - \frac{2253.33e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} (4)$$

B*3Y-Fetal:

$$v_0^D(s) = \frac{7419.23e^{-4s}}{4s} - \frac{1823.98e^{-2.5s}}{2.5s}$$

$$v_0^{EX}(s) = \frac{4754.02e^{-4s}}{4s} - \frac{1984.14e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} (5)$$

B†3Y-Fetal:

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$$v_0^D(s) = \frac{5173.59e^{-4s}}{4s} - \frac{1514.69e^{-2.5s}}{2.5s}$$
$$v_0^{EX}(s) = \frac{4578.66e^{-4s}}{4s} - \frac{1553.56e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} \quad (6)$$

N3Y-Fetal:

$$v_0^D(s) = \frac{4693.63e^{-4s}}{4s} - \frac{1138.92e^{-2.5s}}{2.5s}$$
$$v_0^{EX}(s) = \frac{4666.75e^{-4s}}{4s} - \frac{1928.2e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} \quad (7)$$

N'3Y-Fetal:

$$v_0^D(s) = \frac{10396.28e^{-4s}}{4s} - \frac{2149.44e^{-2.5s}}{2.5s}$$
$$v_0^{EX}(s) = \frac{915.19e^{-4s}}{4s} - \frac{921.55e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} \quad (8)$$

N*3Y-Fetal:

$$v_0^D(s) = \frac{11823.85e^{-4s}}{4s} - \frac{2246.34e^{-2.5s}}{2.5s}$$
$$v_0^{EX}(s) = \frac{3024.25e^{-4s}}{4s} - \frac{551.53e^{-2.5s}}{2.5s} - \frac{7.8474e^{-0.7072s}}{0.7072s} \quad (9)$$

The interaction strengths used for constructing the B3Y-Fetal, B′3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal have been taken from [17]. Here, the superscripts ′,* and † associated withthe letter B were said in [18] to indicate that each of the M3Y-type effective interactions was derived by [17] in Botswana, but different from the others on the basis of massdependence. In the same vein, the superscripts ′ and * associated with the letter N areto indicate that each of the M3Y-type effective interactions in the second set, tested in Nigeria, is different from the others on the basis of mass dependence. In actual sense, they were derived by [19, 20] in Botswana [19] and Nigeria [20].

For correct reproduction of the saturation properties of NM at the saturation densityρ_0 = $0.17 fm^{-3}$ in the present Hartree-Fock (HF) study, each of the M3Y-type effective interactions is used in the density-dependent form:

$$v_0^{D(EX)}(\rho, s) = H_0(\rho) v^{D(EX)}(s), \quad (10)$$

where $H_0(\rho)$, the density dependent factor, has the following explicit forms shown in Equations (11 - 13) [25 - 27]:

$$H_0(\rho) = C(1 + \alpha e^{-\beta\rho}), \quad (11)$$

for the DDM3Yn (n = 1) interaction,

$$H_0(\rho) = C(1 - \alpha\rho^{\beta}) \qquad (12)$$

for the BDM3Yn (n = 0, 1, 2, 3) interaction and

$$H_0(\rho) = C(1 + \alpha e^{-\beta\rho} - \gamma\rho) \qquad (13)$$

for the CDM3Yn (n = 1, 2, 3, 4, 5, 6) interaction

Here, the parameters *C, α*, *β* and *γ* of the density dependences are such that they repro- duce the saturation properties of nuclear matter at density $\rho_0 = 0.17 fm^{-3}$ with a binding energy, E_{A0} = -16 MeV within HF calculations. All of these density dependences have been severally used within mean-field approximation for nuclear matter, nuclear structure and nuclear reaction studies with excellent results [18, 28 - 32]. These interactions (DDM3Yn, BDM3Yn and CDM3Yn) are originally based on the M3Y interaction obtained from the G-matrix elements of the Paris [15] as well as Reid NN potential [16]. They have under-gone several stages of development and improvement over the years with the DDM3Yn interactions as the first to be used [25] followed by the BDM3Yn interactions; whereas the CDM3Yn interactions came into use as the latest, hybrid versions of the first two. One of our M3Y-type effective interactions, the B3Y-Fetal, has been used in all the density dependences for nuclear matter and nuclear reaction calculations with results shown to be in excellent agreement with those of M3Y-Paris and M3Y-Reid effective interactions [33, 34]. The reason for the inclusion of all the density dependences in this study is to help us undertake a review of the performance of the B3Y-Fetal in relation to the other M3Y-type effective interactions under consideration. In Ref. [33], the B3Y-Fetal effective interaction was used in its DDM3Y1, BDM3Y0, BDM3Y1, BDM3Y2 and BDM3Y3 versions for calculations involving symmetric nuclear matter (NM), obtaining the incompressibilities K_0 = 176, 196, 235, 351 and 467 MeV respectively, which have been observed to be in good agreement with those of the M3Y-Reid and M3Y-Paris effective interactions. Furthermore, using the CDM3Y-K approach, a recent study [34] of nuclear matter equation of state with the B3Y-Fetal effective interaction has evolved a new set of useful density-dependent interactions called CDB3Y1-, CDB3Y2-, CDB3Y3-, CDB3Y4-, CDB3Y5-, CDB3Y6-Fetal inter- actions with corresponding incompressibilities K_0 = 188, 204, 217, 228, 241 and 252 MeV respectively, in excellent agreement with those of the M3Y-Paris and M3Y-Reid effective interactions.

The Nuclear Matter Test

Past studies on nuclear matter, nuclear structure, neutron star and nuclear reactions have revealed nuclear matter as the first-level test all effective interactions and techniques for many-body problem must pass to qualify for successful study of the properties of finite nuclei. Symmetric nuclear matter, in which the number of protons equals the number of neutrons [1, 35] is usually the reasonable, starting point. This is theoretically represented by a uniform Fermi gas containing equal protons and neutrons per unit volume [36], characterized by a saturation density ρ_0 = 0.166± 0.018 fm^{-3} and a binding energy per particle (E_{A0}) of -16±1 MeV. Therefore, the full essence of the nuclear matter test in this work is the reproduction of the binding energy per nucleon of symmetric nuclear matter (SNM) at a saturation density of 0.17 fm^{-3} together with the compression modulus. In other words, this test may be said to be a study of saturation of properties of SNM: binding energy per particle or equation of state (EOS), nuclear pressure or saturation condition [6, 25] and compression modulus or incompressibility, which is of special interest in Nuclear Physics because it characterizes the stiffness of nuclear EOS. The well-known standard is that the theoretical equations of state which predict higher K_0 values of about 300 MeV are often called "stiff", whereas those which predict smaller K_0 values of about 200 MeV are said to be "soft" [25, 30]. Results from the experimental data of giant monopole resonances (GMR) have shown that K_0 = 240±20 MeV [37, 38]. Theoretical calculations based on non-relativistic and relativistic mean field models have predicted the ranges 220 - 235 MeV and 250 - 270 MeV respectively for the incompressibility, K_0 of symmetric nuclear matter [30]. Herein, each of the mass-dependent M3Y-type effective interactions, N3Y-Fetal, N'3Y-Fetal and N*3Y-Fetal effective interactions, is subjected to this nuclear matter test to determine its viability in relation to the members of this set as well as the members of the other set. Essentially, correct reproduction of the EOS of SNM by any of the effective interactions will move it to the next stage of the nuclear matter test, which is the computation of incompressibility.

Within the HF mean-field approximation, a good description of the nuclear matter properties

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begins with the evaluation of nuclear matter energy per nucleon (EOS). At the saturation density, ρ_0, the binding energy per nucleon, the saturation condition and the incompressibility of the SNM, respectively, are [6]:

$$E_{A0} = E_0^K + \frac{\rho_0}{2} H_0(\rho_0)(J_0^D + J_0^{EX}), \quad (14)$$

$$0 = 2E_0^K + \frac{1}{2}[\rho_0 H_0'(\rho_0) + H_0(\rho_0)](J_0^D + J_0^{EX}) - H_0(\rho_0) J_{01}^{EX} \quad (15)$$

and

$$K_0 = 9\rho^2 \frac{\delta^2 E_A}{\delta \rho^2} |\rho = \rho_0$$
$$-2E_0^K + 9\rho_0^2 \left[\frac{\rho_0}{2} H_0''(\rho_0) + H_0'(\rho_0)\right] (J_0^D + J_0^{EX})$$
$$-9\rho_0^2 \left[2H_0'(\rho_0) + \frac{5}{3\rho_0} H_0(\rho_0)\right] J_{01}^{EX} + 9\rho_0 H_0(\rho_0) J_{02}^{EX} \quad (16)$$

Here, $E_0^K = \frac{3\hbar^2 k_{F0}^2}{10m}$

$$J_0^D = \int v_0^D(s) d^3 r$$
$$J_0^{EX} = \int v_0^{EX}(s)[\hat{j}_1(k_{F0}s)]^2 d^3 s$$
$$J_{01}^{EX} = \int v_0^{EX}(s) \hat{j}_1(k_{F0}s) j_2(k_{F0}s) d^3 s$$
$$J_{02}^{EX} = \int v_{00}^{EX}(s)[j_2(k_{F0}s)]^2 + j_1(k_{F0}s) j_3(k_{F0}s) d^3 s \quad (17)$$

where, m is the bare nucleon mass, $H_0(\rho)$ is the density dependence of the isoscalar component of the density dependent interactions, J_0^D and , J_0^{EX} are the volume integrals of the direct and exchange parts of the interactions, and $\hat{j}1(x) = 3j_1(x)/x$, with $j_n(x)$ as the nth-order spherical Bessel function. $H'_0(\rho_0)$ and $H''_0(\rho_0)$ in Equations (15) and (16) de- note the first and second derivatives of the density dependence of the isoscalar part of NN interactions with respect to the nuclear density, ρ_0. The integrals $J_0^D, J_0^{EX}, J_{01}^{EX}$ and J_{02}^{EX} depend only on the effective interaction. Consequently, the saturation conditions are further simplified by substituting the explicit forms of the isoscalar density dependence, $H_0(\rho_0)$ given by Equations (11) - (13) and their derivatives into their respective expressions in Equations (14) - (17) to reproduce the desired saturation properties of SNM.

This nuclear matter test has been applied to the first set of the mass-dependent M3Y-type effective interactions in [18] with the B3Y-Fetal as a standard for comparison. The first step taken in the viability test [18] was to use each of the effective interac-tions to determine the binding energy per nucleon of SNM at the saturation density, $\rho_0 = 0.17 fm^{-3}$, with results that showed the B3Y-Fetal, B′3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal effective interactions, based on nuclear systems A = 16, 24, 40 and 90 respectively.

Table 1: Binding Energies per Nucleon of B3Y-Fetal, B′3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal Effective Interactions at Saturation Density.

Effective Interaction	Mass Number (A)	E_{A0} [MeV]
B3Y-Fetal	16	−15.2
B’3Y-Fetal	24	−14.5
B*3Y-Fetal	40	−11.1
B†3Y-Fetal	90	−8.2

to have predicted the binding energies, E_{A0} = -15.2, -14.5, -11.2 and -8.2 MeV respectively at the saturation density. The results are reproduced in Table 1 for the purpose of reference and further discussion. Clearly, the binding energies of -15.2 and -14.5 MeV obtained with the B3Y-Fetal and B’3Y-Fetal respectively are well within the acceptable range of -16.0±1 MeV, with the B3Y-Fetal as a stronger effective interaction than the B’3Y-Fetal and the most viable of all the effective interactions tested in [18]. On the other hand, the B*3Y-Fetal and B†3Y-Fetal effective interactions with the binding energies of -11.1 and -8.2 MeV, respectively, have failed to reproduce the saturation of nuclear matter and are unsuitable for nuclear matter calculations. It is also noteworthy that the trend, shown in Table 1, in which the binding energy per nucleon decreases continuously with increasing mass number, A depicts the effect of mass dependence on the effective interactions.

Following this convincing result, further nuclear matter calculations were carried out with the B3Y-Fetal and B’3Y-Fetal effective interactions in their DDM3Yn and BDM3Yn versions in which the DDB’3Y1-, BDB’3Y0-, BDB’3Y1-, BDB’3Y2- and BDB’3Y3-Fetal effective interactions produced the incompressibilities K_0 = 165, 183, 219, 327 and 434 MeV [18], respectively, in comparison with the higher values of their B3Y-Fetal-based counter- parts. Again, these results are reproduced in Table 2 and Figure 1 (B3Y-Fetal) and Figure2 (B'3Y-Fetal) for further explanation.

Table 2: Parameters of Density Dependence and Nuclear IncompressibilitiesObtained with the B3Y-Fetal and B'3Y-Fetal at Saturation Density.

Density Dependent	Version	C	α	β	K[MeV]
DDB3Y1-Fetal		0.2986	3.1757	2.9605	176
DDB'3Y1-Fetal		0.3483	2.4634	2.9605	165
BDB3Y0-Fetal		1.3045	1.0810	2/3	196
BDB'3Y0-Fetal		1.3047	1.0990	2/3	183
BDB3Y1-Fetal		1.1603	1.4626	1	235
BDB'3Y1-Fetal		1.1676	1.4534	1	219
BDB3Y2-Fetal		1.0160	4.9169	2	351
BDB'3Y2-Fetal		1.0417	4.6098	2	327
BDB3Y3-Fetal		0.9680	20.250	3	467
BDB'3Y3-Fetal		0.9938	23.491	3	434

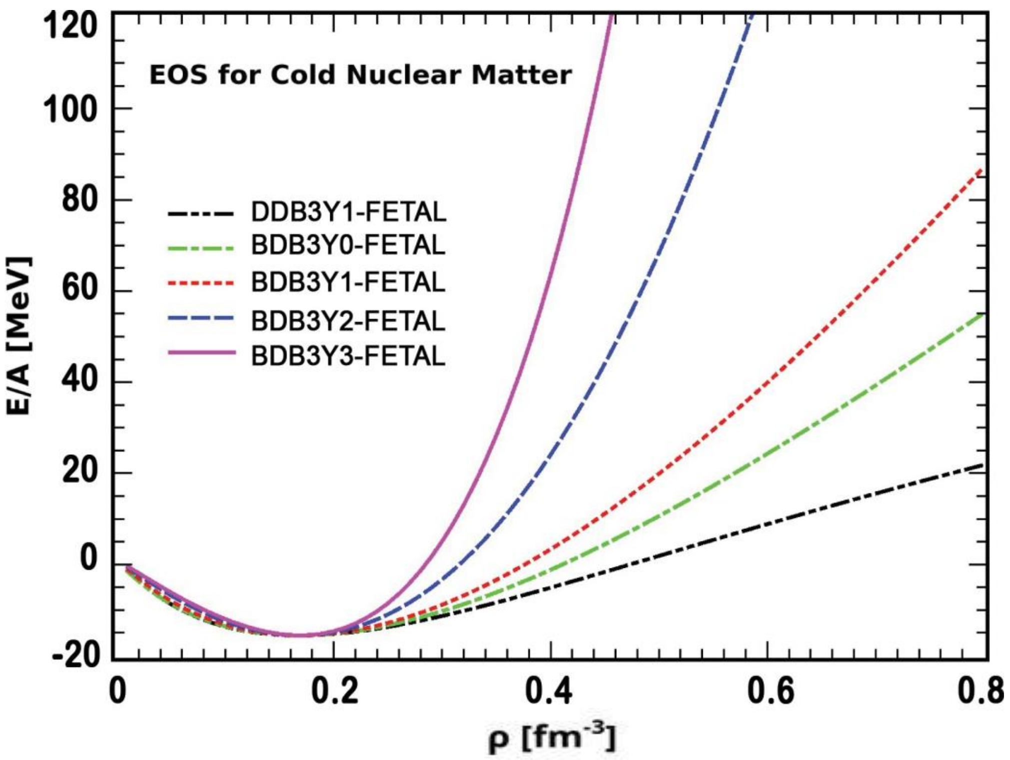

Figure 1: Equations of State of Cold NM Calculated with DDB3Y1- BDB3Y0-, BDB3Y1-, BDB3Y2- and BDB3Y3-Fetal Interactions.

It is interesting to see different equations of state, obtained with DDB3Y1-, BDB3Y0-, BDB3Y1-, BDB3Y2- and BDB3Y3-Fetal effective interactions in Figure 1, pass through the same point at the saturation point. This is largely due to the fact that the B3Y-Fetal accurately reproduced the binding energy per nucleon (E_{A0} = -15.2 MeV) of nuclear matter within the acceptable range E_{A0} = -16±1 MeV. On the contrary, Figure 2 shows that the curves of DDB'3Y1-, BDB'3Y0-, BDB'3Y1-, BDB'3Y2- and BDB'3Y3-Fetal Interactions are not perfectly in alignment at the saturation point, possibly due to the approximation of a binding energy of -14.5 MeV produced by B'3Y-Fetal effective interaction to -15.0 MeV. By this approximation, the B'3Y-Fetal effective interaction has narrowly qualified for use for good description of saturation properties of nuclear matter. But, generally, the curves of B3Y-Fetal-based DDM3Yn and BDM3Yn versions and their B'3Y-Fetal-based counterparts are identical, showing similar differences with increasing nuclear density. This is a good evidence for the viability of the B'3Y-Fetal effective interaction. In addition, the incompressibility K_0 = 219 MeV produced by the BDB'3Y1-Fetal interaction falls within

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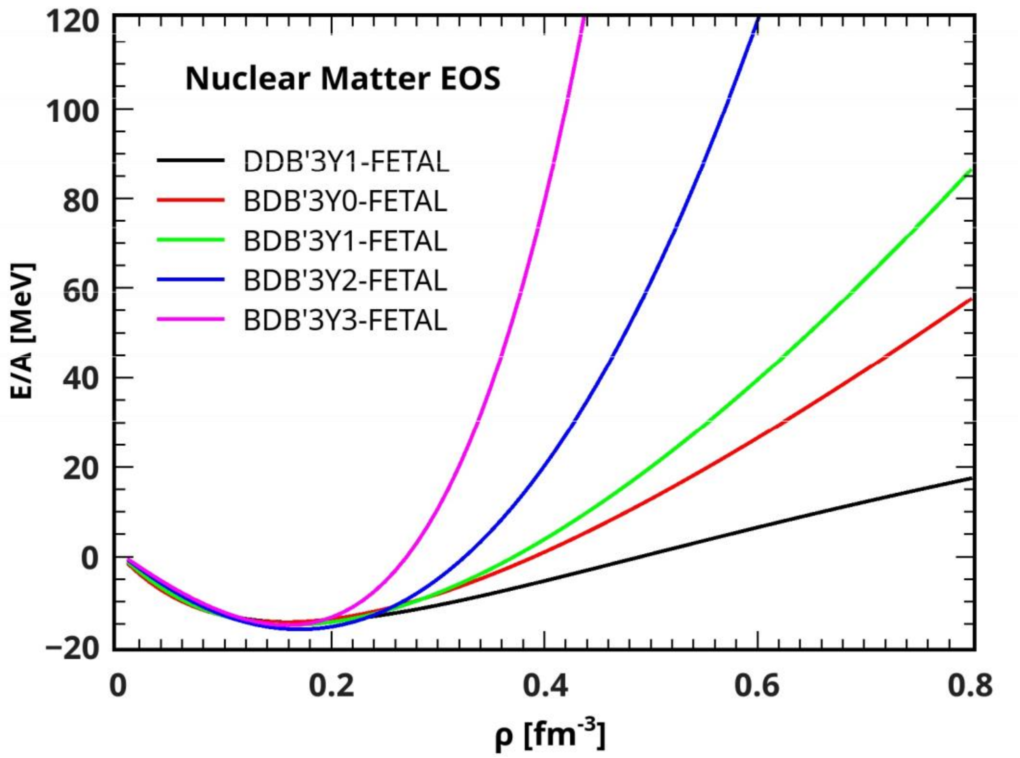

Figure 2: Equations of State of Cold NM Calculated with DDB’3Y1- BDB’3Y0-, BDB’3Y1-, BDB’3Y2- and BDB’3Y3-Fetal Interactions.

the standard incompressibility range K_0 = 240±20 MeV [37, 38]. Also, the B’3Y-Fetal effective interaction was applied to nuclear reaction in [41] with results that compared favourably well with those obtained with the B3Y-Fetal and M3Y-Reid effective interac- tions. These facts have firmly established this effective interaction as the second most viable mass-dependent effective interaction derived by Fiase *et al.* [17]. Results of preliminary nuclear reaction calculations [39] have shown the real folded potential derived from the B’3Y-Fetal effective interaction to have an attractive direct component in a manner similar to the M3Y-Reid-based real folded potential. This interesting feature, believed to be a manifestation of the effect of mass dependence on the variational effective inter- actions, will hopefully be investigated through a systematic study of elastic scattering of some nuclear systems with the B’3Y-Fetal in comparison with the B3Y-

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Fetal and M3Y-Reideffective interactions in subsequent papers.

RESULTS AND DISCUSSION

The aim of this study has been the application of the nuclear matter test to a num-ber of variational effective interactions derived by Fiase *et al.* [19] based on the matrix elements of the *Reid93* nucleon-nucleon potential. The results obtained in this work are presented in Table 3 and discussed accordingly. In discussing these results, the perfor- mances of these effective interactions will be compared with those of their counterpartsin Table 1, paving the way for the review of some of the work done with the B3Y-Fetal interaction.

Table 3: Binding Energies per Nucleon of N3Y-Fetal, N′3Y-Fetal and N*3Y-FetalEffective Interactions at Saturation Density.

Effective Interaction	Mass Number (A)	E_{A0}[MeV]
N3Y-Fetal	16	−8.6
N’3Y-Fetal	40	−12.3
N*3Y-Fetal	90	−7.7

In a bid to determine the viability of the new variational effective interactions, each of them has been used to determine the binding energy per nucleon of SNM atthe saturation density, $\rho_0 = 0.17 fm^{-3}$ in this work. The computational results in Table3 reveal the N3Y-Fetal, N′3Y-Fetal and N*3Y-Fetal effective interactions, based on the nuclear systems A = 16, 40 and 90 respectively, to have predicted the binding energies,E_{A0} = -8.6, -12.3 and -7.7 MeV respectively at the saturation density, $\rho_0 = 0.17 fm^{-3}$. The effect of mass dependence is evidently revealed in the fact that the binding energies produced by the effective interactions are different, but the missing information is the manner in which the mass difference affects the binding energies. Compared with B3Y- Fetal, B’3Y-Fetal, B*3Y-Fetal and B†3Y-Fetal effective interactions in Table 1, in which the binding energy per nucleon (E_{A0}) is seen to decrease continuously with increasing mass number, A, no specific order of variation of binding energy with nuclear mass canbe predicted in Table 3. The binding energy is seen to increase from -8.6 MeV for N3Y-Fetal to -12.0 MeV for the N’3Y-Fetal and then decrease to -7.7 MeV for the N*3Y-Fetal interaction. Also, the binding energies of -8.6, -12.3 and -7.7 MeV obtained with N3Y- Fetal, N′3Y-Fetal and N*3Y-Fetal effective interactions respectively, are completely outside of the acceptable range E_{A0} = -16±1 MeV, meaning that these effective interactions have failed to reproduce the saturation of nuclear matter. The direct implication of this finding in a larger context is that this second set of variational effective interactions is not viable enough to be used for any form of study on finite nuclei. In comparison, the first set of four variational effective interactions, shown in Table 1, has been found in [18] to have produced two viable effective interactions, out of which the B3Y-Fetal, followed by the B’3Y-Fetal, is the most viable. Therefore, the results of the present calculation have evidently shown that only two (B3Y-Fetal and B’3Y-Fetal) of the seven effective interactions derived by Fiase and collaborators [17, 19] are known to be viable. As a proof of the viability of these effective interactions, a brief review of the work done with them was given in Sections 2 and 3; and the performance of the B3Y-Fetal effective interaction is further reviewed shortly in this work.

The B3Y-Fetal effective interaction was the first of all the variational effective in- teractions to be used in its DDM3Yn and BDM3Yn versions for nuclear matter and nuclear reaction studies in [32, 33, 39, 40] with results that were in impressive agreement with those of the M3Y-Reid and M3Y-Paris effective interactions. In all, the DDM3Yn and BDM3Yn have produced the DDB3Y1-, BDB3Y0-, BDB3Y1-, BDB3Y2- and BDB3Y3-Fetal effective interactions corresponding to the incompressibilities K_0 = 176, 196, 235, 351 and 467 MeV [33] respectively at the saturation density, $\rho_0 = 0.17 fm^{-3}$. These incompressibilities, shown in Figure 1, were found to be in excellent agreement with the incompressibilities K_0 = 171 (DDM3Y1-Reid), 191 (BDM3Y0-Reid), 232 (BDM3Y1-Reid), 353 (BDM3Y2-Reid) and 475 (BDM3Y3-Reid) obtained with the M3Y-Reid as well as the incompressibilities K_0 = 176 (DDM3Y1-Paris), 221 (BDM3Y0-Paris), 270 (BDM3Y1-Paris), 418 (BDM3Y2-Paris) and 566 (BDM3Y3-Paris) produced by the M3Y-Paris effective inter- action [27, 33]. Motivated by this impressive performance of the B3Y-Fetal effective interaction, the DDB3Y1-, BDB3Y1-, BDB3Y2- and BDB3Y3-Fetal effective interactions were used for nuclear reaction study considered as the second level of viability test in [18, 39] alongside the DDB3Y1-, BDB3Y1-, BDB3Y2- and BDB3Y3-Reid effective interactions. The findings from that work showed the DDB3Y1-Fetal and BDB3Y1-Fetal as well as DDB3Y1- Reid and BDB3Y1-Reid real folded potentials to give the best description of the elastic data of $^{12}C +^{12} C$ and $^{16}O +^{16} O$ systems, whereas the BDB3Y2-Fetal and BDB3Y3-Fetal gave a very poor description of elastic data in the same way the BDB3Y2-Reid and BDB3Y3-Reid potentials did. This led to the conclusion that the cold nuclear matter possibly has an underlying soft equation of state [42, 43]. The implication of this is that the DDM3Y1 and BDM3Y1 versions of the three density-dependent effective interactions used in the calculation have, altogether, given for SNM at equilibrium incompressibility $K_0 \simeq 171 - 270$ MeV from which B3Y-Fetal predicts $K_0 \simeq 176 - 235$ MeV; M3Y-Reid gives $K_0 \simeq 171 - 232$ MeV and M3Y-Paris predicts $K_0 \simeq 176 - 270$ MeV. But, in agreement with the standard incompressibility range $K_0 = 240 \pm 20$ MeV [37, 38] established based on results from the experimental data of giant monopole resonances (GMR), only the BDB3Y1-Fetal (235 MeV), BDM3Y1-Reid (232 MeV) and BDM3Y0-Paris (221 MeV) can be said to have acceptably reproduced the saturation properties of cold symmetric nuclear matter (SNM). For the M3Y-Paris effective interaction, the incompressibility, K_0 = 270 MeV produced by the BDM3Y1-Paris density-dependent interaction has been considered to be the upper limit of incompressibility (soft EOS) of the cold SNM according to the predictions of theoretical calculations based on relativistic mean field models, which have predicted the range K_0 = 250 - 270 MeV [30]; therefore, it is now scarcely used for nuclear matterand related studies. Consequently, to develop more realistic density-dependent versions of the M3Y effective interaction for consistent use in systematic mean-field studies of NM, finite nuclei and nuclear reaction, Khoa *et al.* [26, 29] performed a systematic HF studyof NM using the M3Y-Paris-based CDM3Yn interactions, producing the incompressibilities K_0 = 188, 204, 217, 228, 241 and 252 MeV corresponding to CDM3Y1-, CDM3Y2-, CDM3Y3-, CDM3Y4-, CDM3Y5- and CDM3Y6-Paris respectively [29]. Among these inter- actions, the CDM3Y3, CDM3Y4, CDM3Y5 and CDM3Y6 versions of M3Y-Paris have been variously used for nuclear matter [44], nuclear reaction [7] and astrophysical studies [45, 46]. Also, using the CDM3Y-K approach [6], the incompressibilities K_0 = 188, 204, 217, 228, 241 and 252 MeV, corresponding to CDM3Y2, CDM3Y3, CDM3Y4, CDM3Y5 and CDM3Y6, were obtained with the M3Y-Reid in its CDM3Y density-dependent version. Similarly, in a recent nuclear matter study [34] with the B3Y-Fetal effective interaction,the equivalents of CDM3Y1-, CDM3Y2-, CDM3Y3-, CDM3Y4-, CDM3Y5- and CDM3Y6-Paris interactions, corresponding to the incompressibilities K_0 188, 204, 217, 228, 241 and 252 MeV, were derived and called CDB3Y1-, CDB3Y2-, CDB3Y3-, CDB3Y4-, CDB3Y5- and CDB3Y6-Fetal interactions respectively, using the CDM3Y-K approach [6], in excellent agreement with those of M3Y-Reid and M3Y-Paris effective interactions. The new CDM3Y-K parametrizations of the B3Y-Fetal effective interaction are presented in Figure 3 together with the DDM3Y1-Fetal and BDB3Y1-Fetal interactions. In the CDM3Y-K approach [6], where incompressibility is a common parameter, the incompressibilities K_0 = 188, 204, 217, 228, 241 and 252 MeV are the same for M3Y-Paris, M3Y-Reid and B3Y-Fetal effective interactions, but each of them has a completely different set of parameters of density dependence C_0, α, β and γ corresponding to each incompressibility value. Therefore, forthe B3Y-Fetal, as explained in [34], the same set of parameters, CDB3Y-K1, CDB3Y-K2, CDB3Y-K3, CDB3Y-K4, CDB3Y-K5 and CDB3Y-K6 in the CDM3Y-Kn approach [6] produced the incompressibilities K_0 = 188, 204, 217, 228, 241 and 252 MeV, respectively, as the set of parameters CDB3Y1, CDB3Y2, CDB3Y3, CDB3Y4, CDB3Y5 and CDB3Y6 in the CDM3Yn approach. For this reason, the CDM3Yn approach has been adopted in [34] to classify the new CDM3Y-K parametrizations of the B3Y-Fetal effective interaction as shown in Figure 3

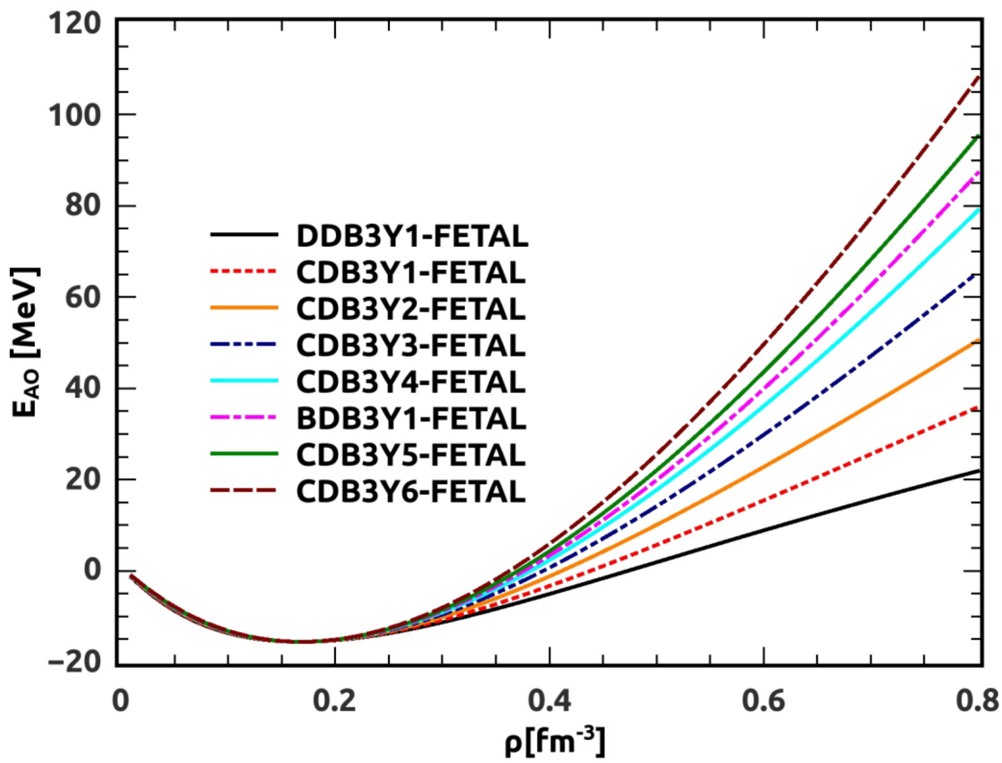

Figure 3: Equations of State of Cold NM Calculated with DDB3Y1- CDB3Y1-, CDB3Y2-, CDB3Y3- and CDB3Y4-, BDB3Y1- CDB3Y5-and CDB3Y6-Fetal Interactions.

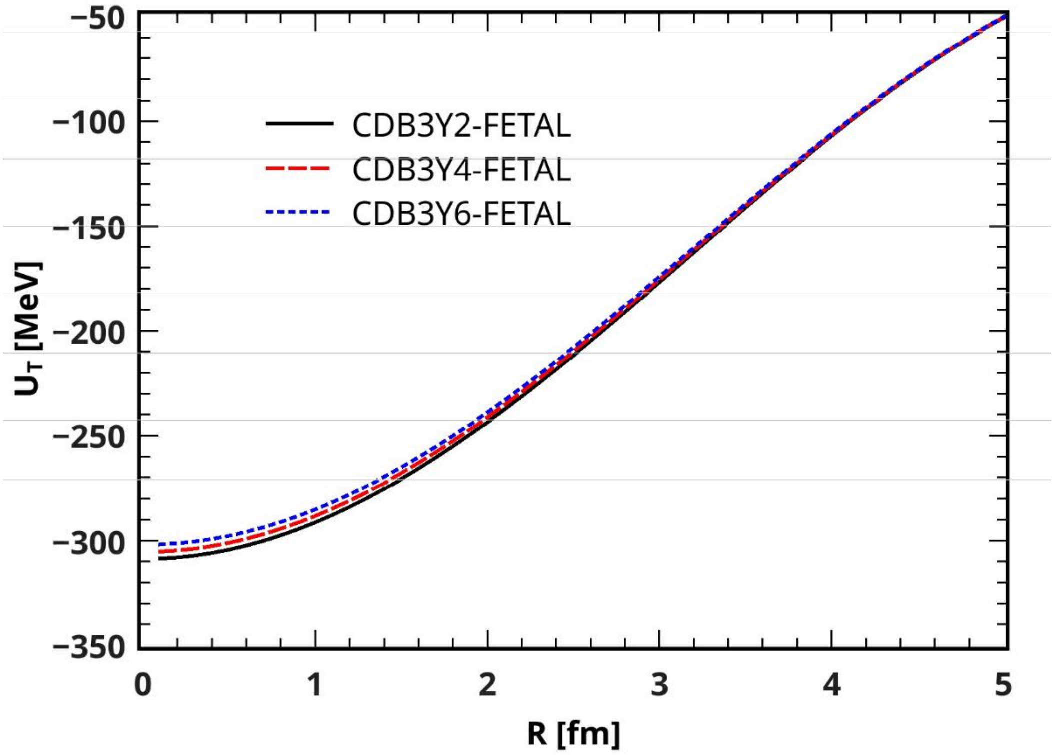

Figure 4: The Real Folded Potentials for the ^{16}O + ^{16}O System Obtained with CDB3Y2-FETAL, CDB3Y4-FETAL and CDB3Y6-FETAL Interactions at the Incident Energy of 350 MeV.

To determine the prospects of the new B3Y-based CDM3Y-K interactions, the CDB3Y6-
Fetal was used along with those of the M3Y-Reid and M3Y-Paris effective interactions within the framework of optical model to construct a nucleus-nucleus optical potential involving ^{16}O +^{16} O nuclear system at the incident energies of 150, 250, 350 and 450 MeV with results that were in good agreement with those of the other effective interactions [34]. To buttress the findings of [34] in this work, the total real folded potentials obtained with the CDB3Y2-Fetal (K=204 MeV), CDB3Y4-Fetal(K=228 MeV) and CDB3Y6- Fetal(K=252 MeV) interactions at the incident energy of 350 MeV are presented in Figure 4, where the magnitudes of these potentials at smaller inter-nuclear distances are shown to be -308.7, -305.3 and -301.9 MeV respectively. Here, the usual trend of decreasing magnitude with increasing incompressibility [28, 42], alongside the good shapes of the optical potentials, confirms the viability of each of these B3Y-Fetal-based interactions in

nuclear reactions.

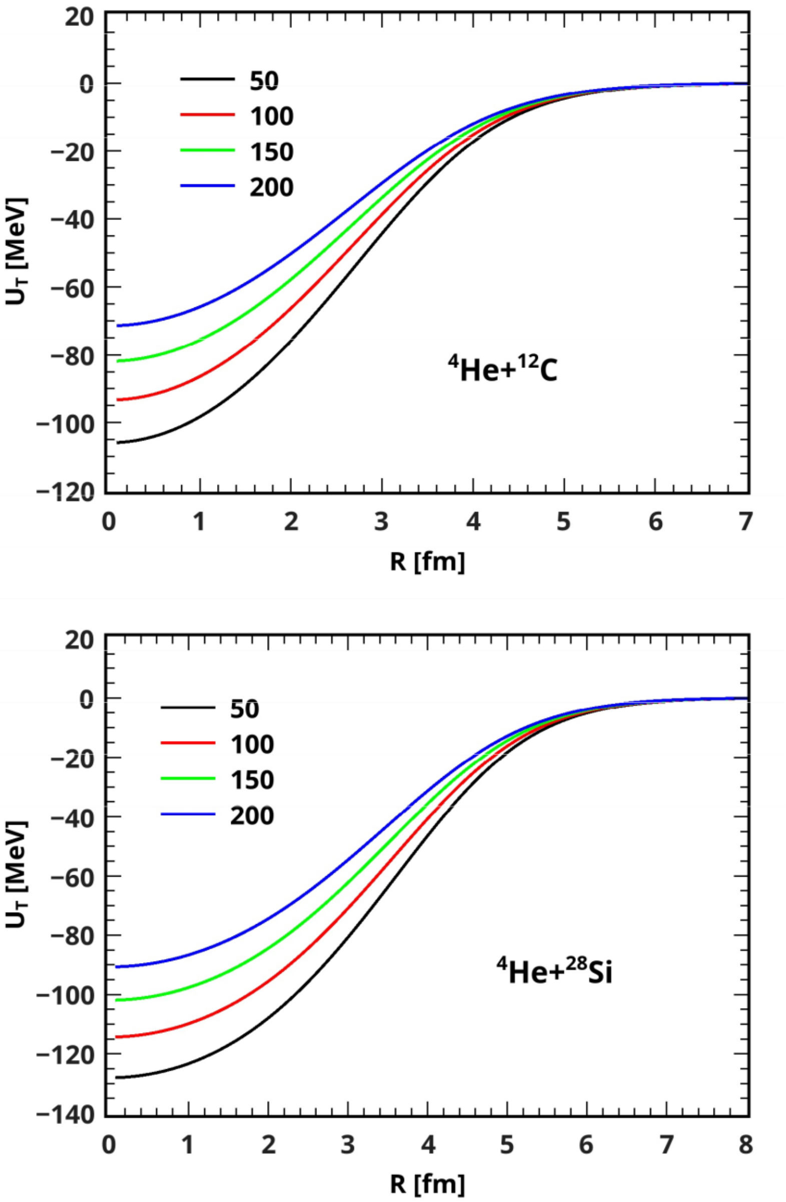

Figure 5: The Total Folded CDM3Y4-Fetal Potentials for the ^{4}He + ^{12}C (Upper) and ^{4}He + ^{28}Si (Lower) Systems at Incident Energies from 50 to 200 MeV.

In addition, the CDB3Y4-Fetal is used in the present study within the framework of double folding model to construct an α-nucleus optical potential for the study of the nuclear reactions involving ^{4}He +^{12} C and ^{4}He +^{28} Si at the incident energies of 50, 100, 150 and 200 MeV. The

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results are shown in performance plots displayed in Figure 5. The total real folded potentials for the ^{4}He + ^{12}C and ^{4}He + ^{28}Si systems are seen to occupy the upper and lower regions, respectively, of Figure 5. These curves are observed to be in good agreement with those of the CDB3Y6-Fetal and CDM3Y6-Reid, discussed in [34], in terms of magnitude, shape and trend. These results have shown the largest values (magnitudes) of the CDB3Y4-Fetal folded potential for the ^{4}He + ^{12}C at smaller inter-nuclear distances to be -108.7, -95.9, -84.1 and -73.5 MeV at the incident energies of 50, 100, 150 and 200 MeV respectively; whereas the largest values of CDB3Y4-Fetal potential for the ^{4}He + ^{28}Si are -131.5, -117.5, -104.8 and -93.3 MeV respectively at the same incident energies. Here, the characteristic trend [28, 42] in which magnitude decreases with increasing incident energy is good evidence that the CDB3Y4-Fetal interaction will likely do well in α-nucleus scattering of many nuclear systems.

Finally, the results obtained in this work show the B3Y-Fetal effective interaction, followed by the B'3Y-Fetal effective interaction, as the most viable of the seven variational effective interactions derived by the authors in Refs. [17, 19]. Consequently, a review of the performance of the B3Y-Fetal undertaken herein indicates that this new effective interaction has done very well in nuclear matter [33, 34, 42] and nucleus-nucleus scattering [42, 43] studies. The new CDM3Y-K parametrizations of the B3Y-Fetal effective interaction have also been observed to have very bright prospects in α-nucleus reactions. Certainly, the findings of this study serve as a strong and broad premise for the application of the B3Y-Fetal effective interaction to the optical model analysis of refractive α-nucleus elastic scattering on targets ranging from ^{12}C to ^{208}Pb in subsequent research efforts. Additionally, the B3Y-Fetal effective interaction will hopefully be applied to several nuclear phenomena (such as fusion dynamics and radioactive decays) that are highly sensitive to the choice of the effective NN potential. It will be compared with the results obtained from the M3Y and its analogous R3Y potential, which is microscopically derived from the relativistic mean-field (RMF) Lagrangian [47]. This step is crucial as it improves the pre- diction and understanding of fusion cross-section and the excitation of various reactions of astrophysical relevance [48], induced reactions on heavy targets [48], and the synthesis of superheavy elements [50]. Moreover, it would be interesting to examine the effect of the B3Y-fetal interaction on the properties (such as binding energies, quadrupole deformation, Q-values, rms charge, and matter radii) of alpha and clusteremitting parent nucleiin both ground and intrinsic-excited states, which are known [51, 52] to be susceptible to the NN interaction employed.

CONCLUSION

We have studied the new variational effective interactions called N3Y-Fetal, N'3Y- Fetal and N*3Y-Fetal effective interactions derived on the basis of the lowest order constrained variational approach [19] from the matrix elements of *Reid93* potential, using the nuclear matter test. This second set has also been compared with the first set of four variational effective interactions known as the B3Y-Fetal, B'3Y-Fetal, B*3Y-Fetal and $B^{\dagger}$3Y-Fetal effective interactions derived by Fiase and collaborators [17] from the matrix elements of the *Reid68* potential. This study has evidently revealed that the different binding energies E_{A0} = -8.6, -12.3 and -7.7 MeV of the cold SNM respectively produced by N3Y-Fetal, N'3Y-Fetal and N*3Y-Fetal effective interactions at the saturation density,$\rho_0 = 0.17fm^{-3}$ clearly reflect the effect of mass dependence, but do not satisy the condition, E_{A0}= 16.0±1 MeV. Thus, these variational effective interactions, failing to reproduce the nuclear matter saturation properties, are shown not to be viable in this work for any meaningful study of nuclear matter and finite nuclei. However, comparing these members of the second set with the members of the first set of variational effective interactions,this study upholds the findings of [18] in which the B3Y-Fetal, followed by the B'3Y-Fetal, was found to be the most viable effective interaction. On the whole, the findings of this study have established the B3Y-Fetal, out of the seven variational effective interactions presented in this work, as the most viable with the highest predictive power, followedby the B'3Y-Fetal effective interaction. Following this conclusion, we have undertakena review of the performance of the B3Y-Fetal effective interaction in nuclear matter inits DDM3Yn, BDM3Yn and CDM3Yn density dependences, leading to the presentation of the new B3Y-Fetal-based CDM3Y-K parametrizations [33] together with the DDB3Y1-Fetal and BDB3Y1-Fetal curves of equation of state in Figure 3. Particularly, the new CDB3Y2-, CDB3Y4- and CDB3Y6-Fetal interactions have been used within the framework of double folding model to construct real folded potentials involving the $^{16}O + ^{16}O$ reaction at an incident energy of 350 MeV with reasonably good results. In the final analysis, the performance of the B3Y-Fetal effective interaction in nuclear matter (symmetric and asymmetric) and nuclear reactions (nucleus-nucleus scattering: $^{16}O + ^{16}O$ and $^{12}C + ^{12}C$) has shown it to be in excellent agreement with the M3Y-Reid and M3Y-Paris effective interactions. This serves as a credible basis for its application in α-nucleus elastic scattering, nuclear fusion, α- and cluster-radioactive decay studies in future research efforts.

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