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Mathematical Modelling of the Impact of Acoustic Waves on an Incompressible Fluid Flow through a Parallel Channel

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Abstract: *This study investigates the impact of acoustic waves on fluid flow through a channel, and the channel is considered to be a pipe carrying incompressible fluid. The research focused on analysing the impact of an acoustic wave on the fluid flowing through the channel that could change the flow patterns and assessing how effective acoustic wave stimulation can be at disrupting the fluid flow. We modified the Navier-Stokes equation and incorporated the acoustic terms into the system and scaled it to be dimensionless. The scaled mathematical models were solved analytically, and we adopted Wolfram Mathematica, version 12, to perform the numerical computation and investigate the influence of acoustic wave propagation on fluid flow behaviour through the channel. Results: The simulated results reveal that the introduction of acoustic waves induces oscillatory velocity profiles and flow rates, indicating dynamic coupling between acoustic forcing and fluid motion. These oscillations are strongly dependent on acoustic wave parameters such as frequency and amplitude as well as pore geometry and fluid properties.The results suggest that acoustic fields can be strategically tuned to enhance fluid mobility, overcome capillary barriers, and promote redistribution of phases in porous networks.*
Keywords: acoustic, wave, parallel channel, incompressible fluid, mathematical modeling, fluid and haemodynamics research group (FHRG)

INTRODUCTION

Fluid flow in confined spaces, such as channels, is very important in various engineering and scientific fields. These systems are fundamental to technologies like microfluidic devices, heat transfer, chemical reactors, and biomedical instruments. A detailed understanding of how fluids behave in these constrained environments is essential for improving system efficiency,

optimising performance, and supporting the development of innovative technologies. Acoustic wave pressure fluctuations that move through fluids have gained attention as a non-invasive and effective method for influencing fluid flow. When these waves interact with flowing fluids, they can produce complex effects such as acoustic streaming, enhanced mixing, vibration-induced motion and changes in flow resistance. These phenomena are applied in a variety of settings, including ultrasonic cleaning, medical imaging and therapy, chemical processing, and sound-based flow control (Patel et al., 2023). Parallel channel configurations where multiple channels are aligned are common in many practical systems due to their compact design and ability to handle high flow rates. Understanding how acoustic waves affect fluid flow in these channels is essential for designing more effective and efficient devices that rely on acoustic-fluid interactions (Lee & Kim, 2024).

Mathematical modelling offers a powerful tool to study and predict the complex interaction between acoustic waves and fluid motion in such systems. By using equations based on the principles of fluid mechanics and acoustics, researchers can simulate flow patterns, pressure changes, and the influence of acoustic forces under various operating conditions. These models help uncover the underlying physics, reduce the need for costly experimental work, and support the optimisation of design parameters. The interaction between acoustic fields and the fluid motion can trigger mechanical responses through several mechanisms (Chem et al., 2019), such as acoustic streaming, where steady fluid motion is driven by the energy loss of high-frequency waves; pressure gradients, which can redistribute fluid within the channel system; changes in surface tension and contact angle, affecting how easily fluids move or adhere to surfaces; and oscillation of droplets or bubbles, which can cause fluid migration, coalescence, or separation.

In addition, Hajibeygi (2011) proposed an iterative multiscale finite volume (i-MSFV) method for simulating multiphase flow in fractured porous media. His approach incorporates a hierarchical treatment of fractures, modelling them as lower-dimensional entities that interact dynamically with the surrounding matrix. In a similar context, Bajaj (2009) developed an unstructured finite volume simulator aimed at modelling two-phase flow through fractured media. Her method captures the detailed geometry of fractures using an unstructured mesh, allowing for high-resolution simulations of multiphase transport processes. Radišić (2020) contributed to the field by constructing and analysing a two-phase, two-component flow model using a fully implicit finite volume method. Her work emphasises the mathematical rigour of the global pressure formulation and demonstrates convergence and stability through theoretical proofs. The model is designed to simulate compressible, immiscible multiphase flow in heterogeneous domains and is validated using benchmark problems in one, two, and three dimensions. More recently, Abbasi (2025) introduced Physics-Informed Neural Networks (PINNs) as a novel data-driven alternative to traditional simulation techniques. The work focuses on both forward and inverse modelling of multiphase flow, particularly in the context of spontaneous imbibition and fractured reservoir systems. Liu *et al*. (2024) introduced the Multigrid Neural Operator (MgNO), a novel approach inspired by classical multigrid methods, to

tackle these problems efficiently. Their model is specifically designed to incorporate heterogeneous permeability and porosity, which are critical factors in realistic subsurface environments. Sheng *et al*. (2023) investigated how the microstructure of porous foams influences acoustic impedance and absorption characteristics. Their findings indicate that increasing pore size and connectivity enhances sound absorption by promoting viscous and thermal dissipation within the pore. Lee and Kim (2024) investigated how acoustic waves scatter in porous rock formations, finding that both pore shape and fluid saturation significantly affect wave speed and attenuation. These factors are crucial for applications such as seismic imaging and subsurface exploration. The effects of acoustic waves on porous media also have important practical applications in environmental science and engineering. Patel *et al.* (2023) showed that acoustic stimulation can enhance oil recovery by promoting fluid movement within porous reservoirs. Additionally, Nguyen *et al*. (2024) demonstrated that acoustic waves can enhance pollutant removal efficiency in porous biofilters by increasing mass transfer rates. Advances in numerical modelling have greatly improved the ability to predict acoustic behaviour within porous structures. Wang et al. (2023) developed a multiphysics model that couples acoustic wave propagation with fluid flow and solid deformation in porous media. Their simulations revealed how acoustic pressure gradients drive fluid movement inside pores, which in turn affects the material's effective stiffness and damping properties. Furthermore, the study by Oliveira *et al*. (2024) incorporates detailed pore-scale geometries to more accurately simulate sound absorption. These advanced models contribute to optimising porous materials for noise reduction in architectural and automotive applications. Oliveira *et al*. (2024) incorporate detailed pore-scale geometries to more accurately simulate sound absorption. These advanced models contribute to optimising porous materials for noise reduction in architectural and automotive applications.

Ramos et al. (2020) utilised high-performance computing to solve complex acoustic problems, approximating acoustic fields with computer-based models and simulations. Li *et al.* (2022) developed a parallel algorithm for direct numerical simulation of acoustic wave propagation, validating their solver for underwater acoustic waves. Muller *et al*. (2018) investigated acoustic streaming effects caused by surface acoustic waves in microchannel flow, employing computational approaches to model acoustic effects in three dimensions. Their study highlights the importance of considering acoustic streaming effects in microfluidic applications. Temkins's (2001) work on elements of acoustics provides a detailed derivation of the mathematical form of acoustic wave propagation in viscous fluids. Morris's (2017) work on flow acoustic interactions provides a comprehensive overview of theoretical models for flow acoustic simulations. These models are important for understanding complex interactions between fluid flow and acoustic waves. Although much progress has been made in modelling acoustic effects in single-channel systems, the behaviour of fluids under acoustic influence in parallel channels remains underexplored (Sheng et al., 2023). Challenges such as wave interference, coupling between channels, and fluid-structure interactions make this a complex problem that requires detailed modelling. This study aims to address this gap by formulating a robust mathematical model that captures the effects of acoustic waves on fluid flow in parallel channels.

Mathematical Formulation

In this section, we shall derive a system of mathematical models that represent the effect of acoustic waves on the momentum of the fluid flow and we also considered the impact of acoustic wave on the flow by invoking the wave equation. In addition, we shall consider the following realistic assumptions. The fluid is considered incompressible with a constant density, the acoustic wave effect is considered particularly, and the flow is assumed to be 2D and flows fully developed steadily.

Governing Model

In view of above mentioned assumptions and consideration, and following Bunonyo et al. (2021), the models governing the flow is stated mathematically as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\sum F-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3.2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\sum F-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3.3}$$

2.3 Acoustic Wave Equation

The acoustic wave equation can be recall mathematically as:

$$y=A_a sin\left(kx-\omega t+\varphi\right) \tag{3.4}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} u=0, \quad \text{at } y=0 \\ u=0, \quad \text{at } y=R \end{aligned}\right\} \tag{3.5}$$

Since the flow is steady, equations (3.1)-(3.4) are reduced to:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho F_a-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3.6}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\rho F_b-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3.7}$$

Simplifying equations (3.6)-(3.7), we have:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=F_a-\frac{1}{\rho}\frac{\partial P}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3.9}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=F_b-\frac{1}{\rho}\frac{\partial P}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3.10}$$

$$y=A_a sin\left(kx-\omega t+\varphi\right) \tag{3.11}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} u=0, \quad \text{at } y=0 \\ u=0, \quad \text{at } y=R \end{array}\right\} \tag{3.12}$$

Non-dimensional variables

Here we introduce the dimensionless parameters as follows

$$\left.\begin{array}{l} u^*=\frac{u}{U_\infty}, v^*=\frac{v}{U_\infty}, y^*=\frac{y}{R_0}, x^*=\frac{x}{R_0} \\ P^*=\frac{P}{\rho U_\infty^2}, \delta=\frac{R_0 m A_a}{U_\infty^2}, \frac{1}{Re}=\frac{\upsilon}{U_\infty R_0} \end{array}\right\} \tag{3.13}$$

Applying equation (3.13) into equation (3.9), we have:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{R_0}{U_\infty^2}F_a-\frac{\partial P^*}{\partial x^*}+\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{3.14}$$

Recalling that the acoustic wave force is:

$$F_a=m\frac{d^2 y}{dt^2} \tag{3.15}$$

Substituting equation (3.15) into equation (3.14), we have:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{R_0 m}{U_\infty^2}\frac{d^2 y}{dt^2}-\frac{\partial P^*}{\partial x^*}+\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{3.16}$$

Differentiating equation (3.11) to the order in equation (3.16), we have:

$$\frac{d^2 y}{dt^2}=-A_a\omega^2 sin\left(kx-\omega t+\varphi\right) \tag{3.17}$$

Substituting equation (3.17) into equation (3.16), we have:

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{R_0 m A_a\omega^2}{U_\infty^2}sin\left(kx-\omega t+\varphi\right)-\frac{\partial P^*}{\partial x^*}+\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{3.18}$$

Simplifying equation (3.18), we have:

$$\frac{\partial P^*}{\partial x^*}=-\delta\omega^2 sin\left(\frac{2\pi}{\lambda}x-\omega t+\varphi\right)+\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right) \tag{3.19}$$

Simplifying equation (3.19), we have:

$$\frac{d^2u}{dy^2} = P_0Re + \delta\omega^2 Resin\left(\frac{2\pi}{\lambda}x - \omega t + \varphi\right) \tag{3.20}$$

Simplifying equation (3.20), we have:

$$\frac{d^2u}{dy^2} = \chi_0 \tag{3.21}$$

where $\chi_0 = Re\left(P_0 + \delta\omega^2 sin\left(\frac{2\pi}{\lambda}x - \omega t + \varphi\right)\right)$

Integrating equation (3.21) twice, we obtained:

$$u(y) = \frac{\chi_0 y^2}{2} + c_1 y + c_2 \tag{3.22}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} u = 0, \quad \text{at } y = 0 \\ u = 0, \quad \text{at } y = h \end{aligned}\right\} \tag{3.23}$$

Simplifying equation (3.22) using equation (3.23), we have:

$$u(y) = \frac{\chi_0}{2}\left(y^2 - hy\right) \tag{3.24}$$

RESULTS

This section deals with the simulation and presentation of results. Here, the result we shall be talking about is the effect of the oscillatory frequency, Remold's number, phase angle, magnitude of oscillatory amplitude, and dimensionless period time on the fluid velocity profile, while other contributing factors are kept constant. The results are present as follows.

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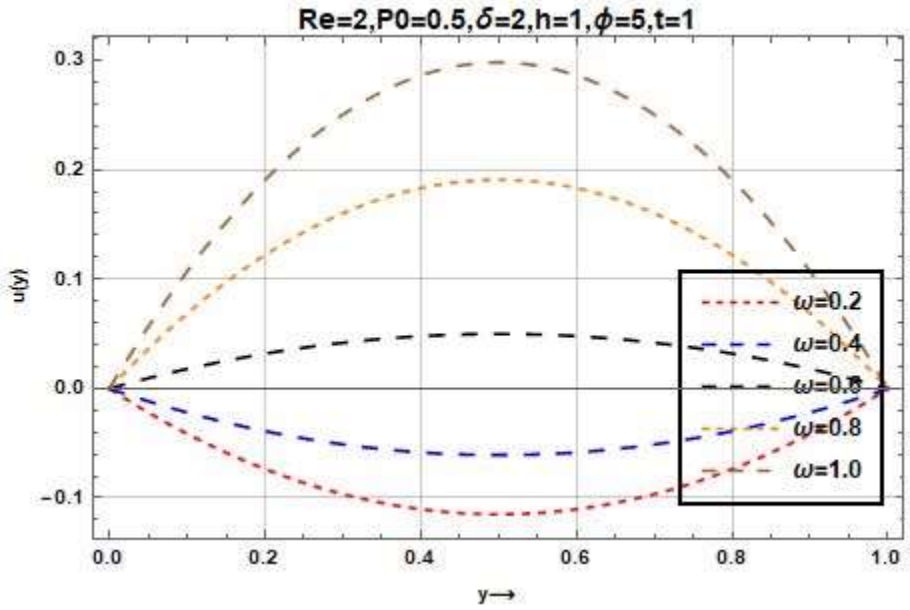

Figure 1: The impact of the variation of acoustic frequency on the fluid velocity

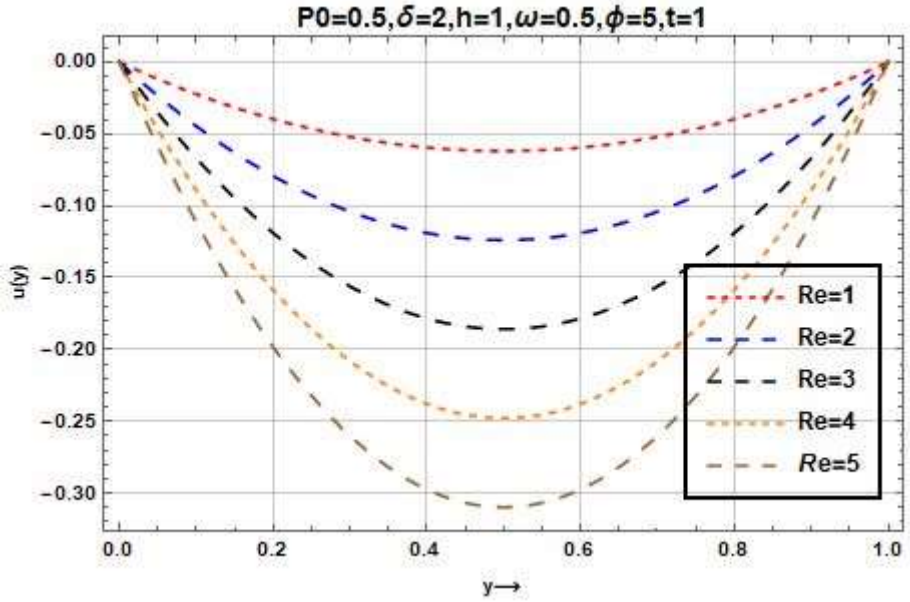

Figure 2: The impact of the variation Reynolds number on the fluid velocity

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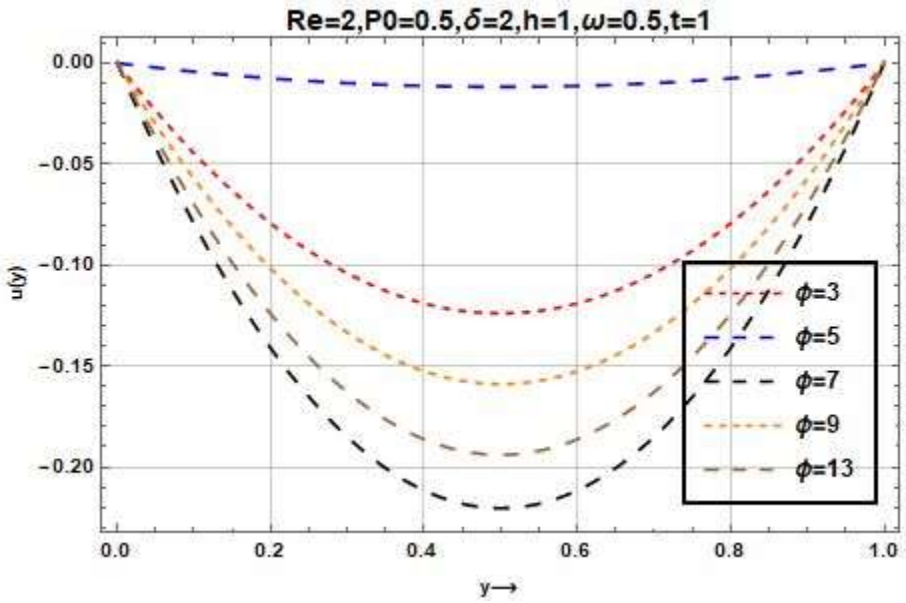

Figure 3 The impact of the variation of wave front on the fluid velocity

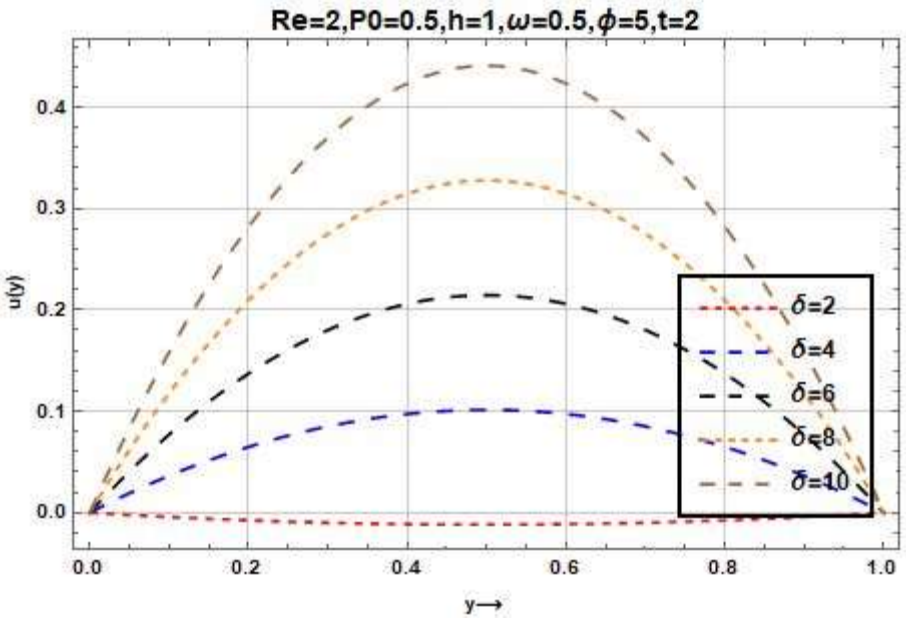

Figure 4: The impact of the variation acoustic amplitude on the fluid velocity

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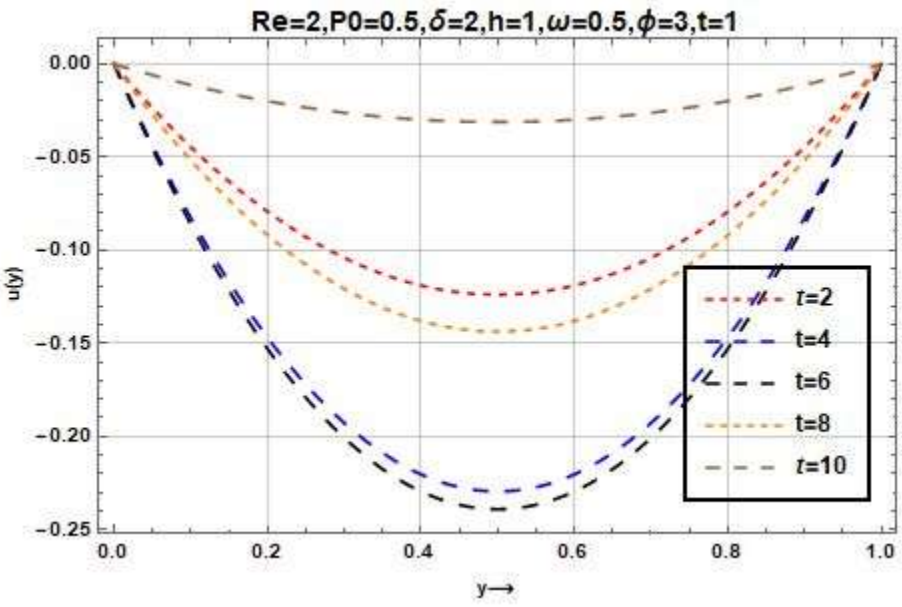

Figure 5: The impact of the variation of dimensionless time on fluid velocity

DISCUSSION OF RESULTS

Figure1 illustrates the parabolic velocity profiles under oscillatory modulation for varying acoustic frequency**.** The x-axis represents the spatial domain across the channel, while the y-axis shows how the velocity profile changes with different acoustic frequency . As ω increases from 0.2 to 1.0, we observe a stronger negative shift in the velocity curves, indicating that higher acoustic angular frequency lead to more pronounced flow velocities. This suggests that acoustic waves are effectively enhancing or disturbing the flow more as increases ω dynamically modulating the velocity field.

The results clearly demonstrate that acoustic energy boosts fluid mobility, consistent with the no-slip boundary condition described in the methodology. Additionally, the velocity profile becomes steeper near the walls as ω increases, meaning the velocity gradient and therefore the shear stress is also increasing. These higher shear stresses induced by stronger acoustic forcing can be particularly useful in practical applications such as enhanced oil recovery or environmental remediation, where dislodging trapped phases is essential.

Conclusions; This graph visually demonstrates the core finding of the research.The deeper thevelocity profile, the stronger the acoustic wave's impact, implying a greater potential to overcome capillary forces, mobilize trapped phases, and redistribute fluids.

Figure 2; Figure 2 show that, The shape of the velocity profile remain the same, as in figure 1 but the depth of the curve increases as Reynolds number increases. As the velocity profile depth increases, the nature of the flow changes significantly. At a profile value of **0.5**, the flow remains very shallow with low peak velocity, where viscous forces dominate and the movement is sluggish. Increasing the profile to **1** results in a slightlydeeper curve, leading to a modest increase in flow velocity. At a profile value of **2**, the curve represents a moderate depth, producing a balanced flow where both viscous and inertial effects are present in reasonable proportion. When the profile deepens further to **4**, the velocity becomes more pronounced, and inertial forces begin to dominate, resulting in a significant acceleration of the flow.Conclusion; Varying the Reynolds number directly controls the magnitude of velocity inducedby acoustic waves in a porous medium. The model clearly shows that, Higher Re leads to greater acoustic-induced fluid motion, while lower Re dampens the response due to viscosity. This insight is key when designing acoustic-enhanced systems for oil recovery, groundwatertreatment, or microfluidic devices.

Figure 3 show the impact of varying the phase angle ϕ in the velocity profile, particularly focusing on its role in modulating fluid flow within porous media under the influence of oscillatory acoustic waves. The velocity field includes a sinusoidal term, where ϕ is the phase angle. This sine component governs both the timing and direction of fluid particle oscillation. By adjusting ϕ we effectively shift the wave in time, thereby altering the phase at which specific flow behaviors such as forward flow, reverse flow, or zero net motion occur at any given instant t. This phase shift manifests as a temporal displacement of the oscillatory pattern. As ϕ increases, the sine wave undergoes a leftward shift along the time axis, which in turn modifies the instantaneous velocity field experienced at different spatial locations. Consequently, the amplitude and sign (direction) of the velocity vector at a given time are strongly influenced by the value of ϕ. Understanding this phase dependence is critical, as it informs the design and control of acoustic stimulation techniques in porous structures such as enhanced oil recovery or biomedical ultrasound applications where precise manipulation of fluid dynamics is required.

Figure 4 show the velocity profiles $u(y)$ displayed in the plot are parabolic and symmetric about the channel centerline $y = 0.5$, with zero velocity at the walls. This behavior is characteristic of pressure-driven laminar flow in a channel, governed by no-slip boundary conditions.

Each curve corresponds to a different value of the parameter δ, and a clear trend is observed: as δ increases from 2 to 10, the peak velocity at the center of the channel increases accordingly. This indicates that has a significant influence on the flow intensity.

The increasing velocity with higher δ suggests that δ governs a mechanism that reduces flow resistance or enhances momentum transport. Depending on the physical context, δ may represent permeability in porous media, a slip parameter in boundary-driven flows, or a term inversely related to magnetic field strength in magnetohydrodynamic systems.

Thus, the results demonstrate that increasing δ leads to a reduction in overall flow resistance, thereby allowing the fluid to achieve higher velocities under the same driving conditions. The system transitions from a more restricted flow regime (lower δ to a more enhanced or facilitated one (higher δ).

Figure 5 ; The oscillatory behavior induced by acoustic waves is clearly illustrated in the velocity profile shown in figure 5. Here, the velocity u(y) is plotted across the spatial domain at various time instances t=2,4,6,8,10, under fixed flow parameters. The sinusoidal nature of the velocity field is evident from the way the profile shifts over time. Initially, at t =2, the fluid moves in a negative direction with moderate amplitude. As time progresses, the velocity magnitude increases, reaching a peak negative value around t=10. This evolution reflects the periodic reversal and amplification of flow direction governed by the sine term sin function in the governing equations.

CONCLUSION

The figure 5 demonstrates how the phase angle ϕ and the angular frequency ω modulate the flow oscillations over time. Each curve corresponds to a distinct phase in the oscillatory cycle, illustrating how temporal control over the wave can lead to significant changes in the flow field. This supports the broader idea that by tuning wave parameters especially phase and timing engineers can influence flow behavior in porous media, a concept fundamental to applications like enhanced oil recovery and microscale mixing.

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