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Physical and Mathematical Analysis of the Radial Water/ Oil Displacement in Porous Media and Determination of Hydrodynamic Parameters as well as Time of Exploitation of the Layer

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Abstract: *Displacement of oil with water is a process that is often encountered in the exploitation of oil fields. In these natural conditions, this process takes place in the layers that work in the water drive regime (with pressure), also it is created in the layers where water is injected to keep the energy of the layer constant, or to raise it. Therefore, the problem of displacing oil with water is of great practical importance. The main objective of this paper is to present a link between the physical context of the filtration of the fluid in the porous medium in the case of displacement of oil with water and its mathematical solution in the determination of the hydrodynamic parameters and in the determination of the time needed for the water-oil contact to move from its initial position to another position in the layer, as well as the time of full exploitation of the layer.*

Keywords: compressibility, radial flow, reservoir, pay zone, equation**,** pressure, time, exploitation.

INTRODUCTION

The very fine voids and channels which together make up the space where the liquid moves in the oil-bearing rock form a tangled network spread in a different way and with cross-sections that vary according to the geometric shape. Therefore, starting from the practical meaning of the word, by agreement, we will call the movement of fluids in porous media and fractures filtration [1]. One of the main disciplines in reservoir engineering is the flow of fluids in porous media, as well as the determination of some of the parameters such as the number of phases present, flow regimes, fluid compressibility and the geometry of flow systems plays a key role in determining of different hydrodynamic parameters [2,3]. In an underground oil and gas reservoir, the fluid flow that occurs in it is affected by its shape [3]. Despite the irregular shapes of reservoir contours, their rigorous mathematical description is possible using numerical simulators [4]. However, for convenience in the various calculations used during well testing, fluid flow geometry can be classified into three forms; radial flow, linear flow and spherical flow [5,6].

In this work, an oil bearing bed with a circular feeding contour (radial flow) is taken into consideration, in which we distinguish two areas, the oil-bearing area and the water-bearing area. The water-oil contact is also circular and concentric with the feeder contour. The layer is thin, with a thickness of h, and it is exploitated from an imperfect well in the center of the bed. The schematic presentation of the problem is given in figure 1 as following:

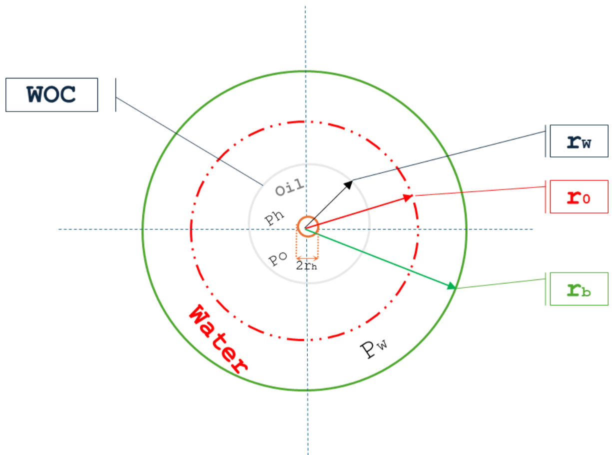

Figure 1*-Schematic representation of the Radial Water / Oil Displacement (Author).*

Consider Ø, k, and F, respectively: Porosity, permeability and cross-sectional area. In the initial conditions, the water-oil contact (separation boundary), which we considered circular and concentric with the feeding contour. In another moment, the contact distance is r_W. We denote by P_O and P_W the pressures at any point of the layer, respectively in the oil-saturated zone and in the water-saturated zone, as well as with μ_O and μ_W respectively oil viscosity and water viscosity.
Taking into account that the flow of the fluid is considered a radial flow and considering the fluid to be incompressible, we first convert the coordinates from cylindrical coordinates to Cartesian coordinates as following:

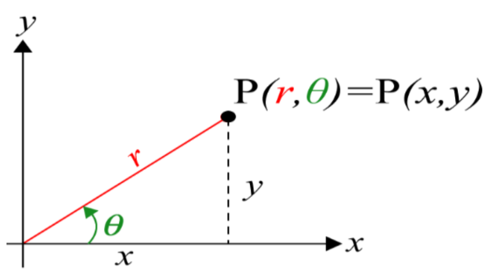

Figure 2*-Cylindrical Coordinate to Cartesian Coordinate [11].*

$\frac{\times}{r} = \cos\theta \rightarrow \times = r * \cos\theta$ and $\frac{\gamma}{r} = \sin\theta \rightarrow \times = r * \sin\theta$

$$\begin{cases} \times^2 = \mathrm{r}^2 * \cos^2\theta \\ \gamma^2 = \mathrm{r}^2 * \sin^2\theta \end{cases} \rightarrow \times^2 + \gamma^2 = r^2 * (cos^2\theta + sin^2\theta) \leftrightarrow \times^2 + \gamma^2 = r^2 \rightarrow r = (\times^2 + \gamma^2)^{\frac{1}{2}}$$

Based on the incompressible fluid equation and the coordinate transformation made above, taking into account the oil-saturated zone and the water-saturated zone, the solution of the basic filtration equation is achieved as shown in equation 3 as following:

$$\nabla^2 \mathrm{P} = 0 \quad (1)$$

$$\nabla^2 P_W = 0 \rightarrow \frac{\partial^2 P_W}{\partial \times^2} + \frac{\partial^2 P_W}{\partial \gamma^2} + \frac{\partial^2 P_W}{\partial z^2} = 0$$

$$\nabla^2 P_O = 0 \rightarrow \frac{\partial^2 P_O}{\partial \times^2} + \frac{\partial^2 P_O}{\partial \gamma^2} + \frac{\partial^2 P_O}{\partial z^2} = 0$$

$$\frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial}{\partial \times}\left(\frac{\partial P_W}{\partial \times}\right) \rightarrow \frac{\partial P_W}{\partial \times} = \frac{\partial P_W}{\partial r} * \frac{\partial r}{\partial \times}$$

$$\frac{\partial r}{\partial \times} = \frac{\partial}{\partial \times}(\times^2 + \gamma^2)^{\frac{1}{2}} \rightarrow \frac{\partial r}{\partial \times} = \frac{1}{2}(\times^2 + \gamma^2)^{\frac{1}{2}-1} * 2\times$$

$$\frac{\partial r}{\partial \times} = \frac{1}{\times}(\times^2 + \gamma^2)^{-\frac{1}{2}} = \frac{\times}{(\times^2 + \gamma^2)^{\frac{1}{2}}} = \frac{\times}{r}$$

$$\frac{\partial P_W}{\partial \times} = \frac{\partial P_W}{\partial r} * \frac{\partial r}{\partial \times} = \frac{\partial P_W}{\partial r} * \frac{\times}{r}$$

$$\frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial}{\partial \times}\left(\frac{\partial P_W}{\partial \times}\right) = \frac{\partial}{\partial \times}\left(\frac{\partial P_W}{\partial r} * \frac{\times}{r}\right) = \frac{\partial}{\partial \times}\left(\frac{\partial P_W}{\partial r}\right) * \frac{\times}{r} + \frac{\partial}{\partial \times}\left(\frac{\times}{r}\right) * \frac{\partial P_W}{\partial r}$$

$$\frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial}{\partial r}\left(\frac{\partial P_W}{\partial r}\right) * \frac{\partial r}{\partial \times} * \frac{\times}{r} + \left(\frac{r - \frac{\partial r}{\partial \times} * \times}{r^2}\right) * \frac{\partial P_W}{\partial r}$$

$$\frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\times}{r} * \frac{\times}{r} + \left(\frac{r - \frac{\times}{r} * \times}{r^2}\right) * \frac{\partial P_W}{\partial r} \leftrightarrow \frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\times^2}{r^2} + \left(\frac{r^2 - \times^2}{r^3}\right) * \frac{\partial P_W}{\partial r}$$

$$\frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\times^2}{r^2} + \left(\frac{\times^2 + \gamma^2 - \times^2}{r^3}\right) * \frac{\partial P_W}{\partial r} \leftrightarrow \frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\times^2}{r^2} + \frac{\gamma^2}{r^3} * \frac{\partial P_W}{\partial r}$$

$$\boldsymbol{\frac{\partial^2 P_W}{\partial \times^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\times^2}{r^2} + \frac{\gamma^2}{r^3} * \frac{\partial P_W}{\partial r} \text{ and } \frac{\partial^2 P_W}{\partial \gamma^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\gamma^2}{r^2} + \frac{\times^2}{r^3} * \frac{\partial P_W}{\partial r}} \quad (2)$$

$$\frac{\partial^2 P_W}{\partial \times^2} + \frac{\partial^2 P_W}{\partial \gamma^2} = \frac{\partial^2 P_W}{\partial r^2} * \frac{\times^2}{r^2} + \frac{\gamma^2}{r^3} * \frac{\partial P_W}{\partial r} + \frac{\partial^2 P_W}{\partial r^2} * \frac{\gamma^2}{r^2} + \frac{\times^2}{r^3} * \frac{\partial P_W}{\partial r}$$

$$\frac{\partial^2 P_W}{\partial \times^2} + \frac{\partial^2 P_W}{\partial \gamma^2} = \left(\frac{\times^2}{r^2} + \frac{\gamma^2}{r^2}\right) * \frac{\partial^2 P_W}{\partial r^2} + \left(\frac{\times^2}{r^3} + \frac{\gamma^2}{r^3}\right) * \frac{\partial P_W}{\partial r}$$

$$\frac{\partial^2 P_W}{\partial \times^2} + \frac{\partial^2 P_W}{\partial \gamma^2} = \left(\frac{r^2}{r^2}\right) * \frac{\partial^2 P_W}{\partial r^2} + \left(\frac{r^2}{r^3}\right) * \frac{\partial P_W}{\partial r} \leftrightarrow \frac{\partial^2 P_W}{\partial \times^2} + \frac{\partial^2 P_W}{\partial \gamma^2} = \frac{\partial^2 P_W}{\partial r^2} + \frac{1}{r}\frac{\partial P_W}{* \partial r}$$

$$\frac{\partial^2 P_W}{\partial r^2} + \frac{1}{r}\frac{\partial P_W}{\partial r} = 0 \leftrightarrow \frac{1}{r} * \frac{\partial}{\partial r}\left(r * \frac{\partial P_W}{\partial r}\right) = 0$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}(r) * \frac{\partial P_W}{\partial r} + r * \frac{\partial}{\partial r} * \left(\frac{\partial P_W}{\partial r}\right)\right] = 0 = \frac{\partial^2 P_W}{\partial r^2} + \frac{1}{r}\frac{\partial P_W}{* \partial r}$$

$$\boldsymbol{\frac{1}{r} * \frac{\partial}{\partial r}\left(r * \frac{\partial P_W}{\partial r}\right) = 0 \text{ and } \frac{1}{r} * \frac{\partial}{\partial r}\left(r * \frac{\partial P_O}{\partial r}\right) = 0} \quad (3)$$

For the case studied, equation 3 is the basic filtering equation. Based on this equation, taking into account the boundary conditions as well as the initial conditions set for both the oil-saturated zone and the water-saturated zone, it becomes possible to study and determine the parameters such as flow rate and the pressure distribution field in each zone, the displacement or change of the initial oil-water position, determination the velocity of filtration and the important parameter which is time, as in the case of determining the time required for the oil-water contact to move from its position. initial as well as the full time of use of the layer.

METHODOLOGY

Based on Eq. 3 above, setting the initial and boundary conditions, taking into account the effective permeability (phase), for water in the presence of residual oil, the effective permeability for oil in the presence of bound water, it is possible to determine the constants of found as follows a1, a2, b1 and b2, the replacement and combination of which with the other equations discussed above makes it possible to study and determine the hydrodynamic parameters. Following equation 3 and setting the initial and boundary conditions, the solution is given as following:

$$\boldsymbol{\frac{1}{r} * \frac{\partial}{\partial r}\left(r * \frac{\partial P_W}{\partial r}\right) = 0; \quad r_i \le r \le r_w}$$

$$\boldsymbol{\frac{1}{r} * \frac{\partial}{\partial r}\left(r * \frac{\partial P_O}{\partial r}\right) = 0; \quad r_h \le r \le r_W}$$

$$r * \frac{\partial P_W}{\partial r} = a_1 \text{ and } r * \frac{\partial P_O}{\partial r} = a_2$$

$$r * \frac{\partial P_W}{\partial r} = a_1 \to \partial P_W = a_1 * \frac{\partial r}{r} \to P_W = a_1 * \ln r + b_1 \quad (4)$$

$$r * \frac{\partial P_O}{\partial r} = a_2 \to \partial P_O = a_2 * \frac{\partial r}{r} \to P_O = a_2 * \ln r + b_2 \quad (5)$$

$$r = r_W \text{ and } P_W = P_O$$

$$a_1 * \ln r_W + b_1 = a_2 * ln\, r_W + b_2$$

$$r = r_W \text{ and } v_W = v_O$$

$$v_W = \frac{k^*}{\mu_W} * \frac{\partial P_W}{\partial r} \text{ and } v_O = \frac{k^{**}}{\mu_O} * \frac{\partial P_O}{\partial r}$$

$$\frac{\partial P_W}{\partial r} = \frac{a_1}{r}; \ \frac{\partial P_O}{\partial r} = \frac{a_2}{r}$$

$$\frac{k^*}{\mu_W} * \frac{a_1}{r} = \frac{k^{**}}{\mu_O} * \frac{a_2}{r} \rightarrow a_2 = \left(\frac{k^*}{\mu_W} * \frac{\mu_O}{k^{**}}\right) * a_1 \leftrightarrow a_2 = \varphi * a_1$$

$$\mathrm{r} = r_b \rightarrow P_b = P_W \rightarrow P_b = a_1 * \ln r_b + b_1$$

$$\mathrm{r} = r_h \rightarrow P_O = P_h \rightarrow P_O = a_2 * \ln r_h + b_2$$

$$b_2 = P_h - a_2 * ln\, r_h$$

$$b_1 = P_b - a_1 * ln\, r_b$$

$$b_2 = P_h - \varphi * a_1 * ln\, r_h$$

$$b_1 = P_b - a_1 * ln\, r_b$$

$$a_1 * ln\, r_W + P_b - a_1 * ln\, r_b = \varphi * a_1 * ln\, r_W + P_h - \varphi * a_1 * ln\, r_h$$

$$P_b - P_h = \varphi * a_1 * ln\, r_W - \varphi * a_1 * ln\, r_h + a_1 * ln\, r_b - a_1 * ln\, r_W$$

$$P_b - P_h = \varphi * a_1 * \left(\ln\frac{r_W}{r_h}\right) + a_1 * \left(ln\frac{r_b}{r_W}\right) = a_1 * \left(ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}\right)$$

$$\boldsymbol{a_1} = \frac{\boldsymbol{P_b - P_h}}{\boldsymbol{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}}} \quad (6)$$

$$\boldsymbol{a_2} = \boldsymbol{\varphi} * \frac{\boldsymbol{P_b - P_h}}{\boldsymbol{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}}} \quad (7)$$

$$\boldsymbol{b_1 = P_b} - \frac{\boldsymbol{P_b - P_h}}{\boldsymbol{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}}} * \boldsymbol{ln\, r_b} \quad (8)$$

$$\boldsymbol{b_2 = P_h - \varphi} * \frac{\boldsymbol{P_b - P_h}}{\boldsymbol{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}}} * \boldsymbol{ln\, r_h} \quad (9)$$

RESULTS AND DISCUSSION

All the above equations, the treatment in both physical and mathematical contexts, both for the water-saturated zone and for the oil-saturated zone, considering the radial flow in the case of incompressible fluid, the study of the geometry of fluid motion by transforming the coordinates from cylindrical coordinates to Cartesian coordinates, replacing the basic equations mainly of pressure distribution in the case of incompressible fluids and radial flow, defining the four constants a_1 , a_2 , b_1 and b_2, all these parameters of mentioned above are the main key that help to solve this study regarding the displacement of oil from water in porous environments [7,8].
Replacing the values of the constants a_1 , a_2 , b_1 and b_2, in equations 4 and 5 will give expression as following:

$$P_W = P_b - \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * ln\, r_b + \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * ln\, r$$

$$P_W = P_b - \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * (ln\, r_b - \ln r)$$

$$\boldsymbol{P_W = P_b} - \frac{\boldsymbol{P_b - P_h}}{\boldsymbol{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}}} * \boldsymbol{ln\frac{r_b}{r}} \rightarrow \textbf{\textit{In the water bearing zone}} \quad (10)$$

$$P_O = P_h - \frac{\varphi * (P_b - P_h)}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * ln\, r_h + \frac{\varphi * (P_b - P_h)}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \ln r$$

$$P_O = P_h + \frac{\varphi * (P_b - P_h)}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * (ln\, r - ln\, r_h)$$

$$\boldsymbol{P_O = P_h + \frac{\varphi*(P_b - P_h)}{ln\frac{r_b}{r_W}+\varphi*ln\frac{r_W}{r_h}} * \ln\frac{r}{r_h} \rightarrow In\ the\ oil\ bearing\ zone} \qquad (11)$$

From Eq. (10) and (11), it appears that the magnitude of the pressure at any point of the layer depends not only on the coordinates of this point, but also on the position of the Water oil-contact. Changing pressure gradients:

$$\frac{\partial P_W}{\partial r} = \frac{\partial}{\partial r}\left(P_b - \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * ln\frac{r_b}{r}\right) = -\frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \left(-\frac{1}{r}\right)$$

$$\frac{\partial P_W}{\partial r} = \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \left(\frac{1}{r}\right)$$

$$\frac{\partial P_O}{\partial r} = \frac{\partial}{\partial r}\left(P_h + \frac{\varphi * (P_b - P_h)}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * ln\frac{r}{r_h}\right) = \frac{\varphi * (P_b - P_h)}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \frac{1}{r}$$

$$\frac{\partial P_O}{\partial r} = \frac{\varphi * (P_b - P_h)}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \frac{1}{r}$$

$$\frac{\partial P_W}{\partial r} \Big/ \frac{\partial P_O}{\partial r} = \frac{P_b - P_h}{\left(ln\frac{r_b}{r_W}+\varphi*ln\frac{r_W}{r_h}\right)*r} * \frac{\left(ln\frac{r_b}{r_W}+\varphi*ln\frac{r_W}{r_h}\right)*r}{\varphi*(P_b - P_h)} = \frac{1}{\varphi} \qquad (12)$$

The mathematical break is on point $\frac{1}{\varphi}$.

$$v_W = \frac{k^*}{\mu_W} * \frac{\partial P_W}{\partial r}$$

$$v_W = \frac{k^*}{\mu_W} * \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \left(\frac{1}{r}\right)$$

$$v_O = \frac{k^{**}}{\mu_O} * \frac{\partial P_O}{\partial r}$$

$$v_O = \frac{k^{**}}{\mu_O} * \frac{\varphi*(P_b - P_h)}{ln\frac{r_b}{r_W}+\varphi*ln\frac{r_W}{r_h}} * \frac{1}{r} \qquad (13)$$

The flow rate for well will be:

$$Q = v * F$$

$$Q = 2\pi * r * h * \frac{k^* * (P_b - P_h)}{\mu_W * \left(ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}\right)} * \frac{1}{r}$$

$$\boldsymbol{Q} = \frac{2\pi * k^* * h * (P_b - P_h)}{\mu_w * \left(ln\frac{r_b}{r_w} + \varphi * ln\frac{r_w}{r_h}\right)} \qquad (14)$$

From the analysis of the filtering parameters expressed in the equations above, it appears that for φ =1, the radial displacement process is an undecided process. The radius r_W decreases with time. The piezometric curve has a bend exactly for $r = r_W$. The bending rate is given by the ratio of the pressure gradients in both areas and is equal to φ. If we transform the flow rate formula into the form $\boldsymbol{Q} = \frac{2\pi * k^* * h * (P_b - P_h)}{\mu_w * ln\frac{r_b * r_w^{(\varphi-1)}}{r_h^{(\varphi)}}}$, it is clear that for $\varphi > 1$ with the reduction of the contact radius, the flow rate of the well increases. This phenomenon is also observed in the practice of exploitation underground oil and gas reservoirs [7]. From a physical point of view, this is related to the reduction of the overall hydraulic resistances in the formation, resulting from the advancement of the water-oil contact. To find the contact displacement law, the following formula applies:

$$v_W = \frac{k^*}{\mu_W} * \frac{P_b - P_h}{ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}} * \left(\frac{1}{r}\right) \qquad (15)$$

Based on the above formula, the displacement velocity of the contact is expressed by the differential equation:

$$\frac{dr_w}{dt} = \frac{v}{\emptyset * (1 - S_{wi} - S_{or})} = w \qquad (16)$$

$$\frac{dr_w}{dt} = \frac{1}{\emptyset * (1 - S_{wi} - S_{or})} * \frac{k^* * (P_b - P_h)}{\mu_W * \left(ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}\right) * r} \qquad (17)$$

By integrating the equation in the limits set as below and continuing the further solution, the time necessary for the displacement of the oil-water contact and the full time of exploitation of the layer is reched as following:

$t = 0 \rightarrow r_W = r_0$

$t = t \rightarrow r_W = r_W$

Based on Eq. (17) by making the appropriate combinations and substitutions with respect to dt and then continuing to integrate with the appropriate bounds determined, the following solution is obtained:

$$dt = \frac{\emptyset * (1 - S_{wi} - S_{or})}{k^* * (P_b - P_h)} * \left[\mu_W * \left(ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}\right)\right] * r * dr_W$$

$$\int_0^t dt = \frac{\emptyset * (1 - S_{wi} - S_{or})}{k^* * (P_b - P_h)} * \int_{r_0}^{r_W} \left[\mu_W * \left(ln\frac{r_b}{r_W} + \varphi * ln\frac{r_W}{r_h}\right)\right] * r * dr_W$$

$$t = \frac{\emptyset * (1 - S_{wi} - S_{or}) * \mu_W}{k^* * (P_b - P_h)} * \left[\int_{r_0}^{r_W} ln\frac{r_b}{r_W} * r_W * dr_W + \int_{r_0}^{r_W} \varphi * ln\frac{r_W}{r_h} * r_W * dr_W\right] \qquad (18)$$

$$\int_{r_0}^{r_W} ln\frac{r_b}{r_W} * r_W * dr_W = \left[ln\frac{r_b}{r_W} * \frac{{r_W}^2}{2}\right] \Big/_{r_0}^{r_W} - \int_{r_0}^{r_W} \frac{{r_W}^2}{2} * \left(-\frac{1}{r_W}\right) * dr_W$$

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$$\int_{r_0}^{r_W} ln\frac{r_b}{r_W} * r_W * dr_W = \left[ln\frac{r_b}{r_W} * \frac{{r_W}^2}{2}\right] \Big/_{r_0}^{r_W} + \left(\frac{{r_W}^2}{4}\right) \Big/_{r_0}^{r_W}$$

$$\int_{r_0}^{r_W} ln\frac{r_b}{r_W} * r_W * dr_W = \left[ln\frac{r_b}{r_W} * \frac{{r_W}^2}{2} - ln\frac{r_b}{r_0} * \frac{{r_0}^2}{2} + \frac{{r_W}^2 - {r_0}^2}{4}\right]$$

$$\int_{r_0}^{r_W} ln\frac{r_b}{r_W} * r_W * dr_W = \left[ln\frac{r_b}{r_W} * \frac{{r_W}^2}{2} - ln\frac{r_b}{r_0} * \frac{{r_0}^2}{2} + \frac{{r_W}^2 - {r_0}^2}{4}\right] = I_1 \quad (19)$$

$$\int_{r_0}^{r_W} \varphi * ln\frac{r_W}{r_h} * r_W * dr_W = \left[\varphi * ln\frac{r_W}{r_h} * \frac{{r_W}^2}{2}\right] \Big/_{r_0}^{r_W} - \int_{r_0}^{r_W} \varphi * \frac{{r_W}^2}{2} * \frac{1}{r_W} * dr_W$$

$$\int_{r_0}^{r_W} \varphi * ln\frac{r_W}{r_h} * r_W * dr_W = \varphi * \left[ln\frac{r_W}{r_h} * \frac{{r_W}^2}{2}\right] \Big/_{r_0}^{r_W} - \int_{r_0}^{r_W} \frac{{r_W}^2}{2} * dr_W$$

$$\int_{r_0}^{r_W} \varphi * ln\frac{r_W}{r_h} * r_W * dr_W = \varphi * \left[ln\frac{r_W}{r_h} * \frac{{r_W}^2}{2} - ln\frac{r_0}{r_h} * \frac{{r_h}^2}{2} - \frac{{r_W}^2 - {r_0}^2}{4}\right] = I_2 \quad (20)$$

Substitute I_1 and I_2 (equation 19 and 20) in the above equation 18 the expression as following is obtained:

$$t = \frac{\emptyset * (1 - S_{wc} - S_{or}) * \mu_W}{k^* * (P_b - P_h)} *$$
$$* \left[\int_{r_0}^{r_W} ln\frac{r_b}{r_W} * \frac{{r_W}^2}{2} - ln\frac{r_b}{r_0} * \frac{{r_0}^2}{2} + \frac{{r_W}^2 - {r_0}^2}{4} + \varphi * \left(ln\frac{r_W}{r_h} * \frac{{r_W}^2}{2} - ln\frac{r_0}{r_h} * \frac{{r_0}^2}{2}\right) - \frac{{r_W}^2 - {r_0}^2}{4} * \varphi\right]$$

$$t = \frac{\emptyset * (1 - S_{wi} - S_{or}) * \mu_W}{k^* * (P_b - P_h)} *$$
$$* \left[\frac{{r_W}^2}{2} * \left(ln\frac{r_b}{r_W} + \varphi * \ln\frac{r_W}{r_h}\right) - \frac{{r_0}^2}{2} * \left(\varphi * ln\frac{r_0}{r_h} + ln\frac{r_b}{r_0}\right) + \frac{{r_W}^2 - {r_0}^2}{4} * (1 - \varphi)\right] \quad (21)$$

Eq. (21) represents the time it takes for the contact to move from its initial position to another position in the layer. If in Eq. (21) the limit or limits for r_W=r_h are taken, then the time of full exploitation of the layer is:

$$t = \frac{\emptyset * (1 - S_{wi} - S_{or}) * \mu_W}{k^* * (P_b - P_h)} * \left[\frac{{r_h}^2}{2} * ln\frac{r_b}{r_h} - \frac{{r_0}^2}{2} * \left(\varphi * ln\frac{r_0}{r_h} + ln\frac{r_b}{r_0}\right) + \frac{{r_W}^2 - {r_0}^2}{4} * (1 - \varphi)\right]$$

$$t = \frac{\emptyset * (1 - S_{wi} - S_{or}) * \mu_W}{k^* * (P_b - P_h)} * \left[\frac{{r_h}^2}{2} * ln\frac{r_b}{r_h} - \frac{{r_0}^2}{2} * \left(\varphi * ln\frac{r_0}{r_h} + ln\frac{r_b}{r_0}\right)\right] \quad (22)$$

Equation 22 shows that the time it takes for the contour to move from one point to another is directly proportional to the porosity of the layer and inversely proportional to the coefficient of permeability and depression. This fact finds a simple physical interpretation. Since, with the increase in porosity, the amount of oil in the layer increases, then, to extract it under the same conditions, more time will be needed. The increase in the permeability of the layer and the depression affects the increase in the filtration velocity, i.e. in the reduction of the contact displacement time [8]. For φ =1, Eq. (22) is transformed into the well-known formula of the period of exploitation of a uniformly isotropic layer saturated with a homogeneous incompressible fluid.

$$T = \frac{\emptyset * \mu * ln\frac{r_b}{r_h}}{k^* * (P_b - P_h)} * ({r_h}^2 - {r_0}^2) = \frac{V}{Q} \quad (23)$$

From all the formulas calculated above in relation to the different hydrodynamic parameters, it is concluded that all the calculated parameters, the pressure of the layer at each of its points, the speed of filtration and the flow rate of the fluid are functions of time [9,10]. From a physical point of view, the legalities of the change in velocity of filtration and flow rate are simply explained. It is known that the movement of the fluid from the feeding contour towards the well is done under the action of the applied depression. Meanwhile, the size of the hydrodynamic resistances, which the lenfu encounters during the movement, depend on the size of the water-bearing area and the oil-bearing area.

CONCLUSION

From a hydrodynamic point of view, the problem of displacement of oil and gas with water is included in the general problem of the separation boundary of two immiscible liquids in porous media. and it is a problem that is often encountered in the practice of exploitation underground oil and gas reservoirs, as in the case of layers that work in the regime of water drive, as well as in the case when water is injected to keep the energy of the layer constant or to raise it that and this is the reason that the problem of pushing oil and gas with water has a great practical importance. The determination of the above hydrodynamic parameters not only express the physical concept of the movement and behavior of fluids in the layer under action and the existence of different conditions that develop in it, but also present the mathematical law, helping to approximate and model the best for efficient management to the reservoirs.

Future work

In the future, all these hydrodynamic parameters taken into account and determined analytically, as well as their relationship expressed on the basis of the physical concept of fluid mobility in the porous medium, make it possible not only to solve the displacement problem of oil with water in the case of underground oil and gas reservoirs, but also the variables involved in this detailed solution, which are expressed as functions of various variables, help to solve many problems encountered in the displacement of oil or in modeling defined during hydrodynamics analysis of the well.

Nomenclature

$\emptyset \rightarrow porosity$
$k \rightarrow permeability$
$F \rightarrow \text{cross} - \text{sectional area}$
$\mathrm{P_O} \rightarrow pressure\ in\ the\ oil\ saturated\ zone$
$\mathrm{P_W} \rightarrow pressure\ in\ the\ water\ saturated\ zone$
$P_b \rightarrow boundary\ pressure$

$P_h \rightarrow hole\ pressure$
$\mu_O \rightarrow oil\ viscosity$
$\mu_W \rightarrow water\ viscosity$
$r_b \rightarrow boundary\ radius$
$r_W \rightarrow current\ radius\ of\ water - oil\ contact$
$r_h \rightarrow hole\ radius$
$\varphi \rightarrow mobility\ coefficient$
$S_{wc} \rightarrow initial\ water\ saturation$
$S_{or} \rightarrow residual\ oil\ saturation$
$Q \rightarrow flow\ rate$
$v_W \rightarrow filtration\ velocity\ for\ the\ water\ phase$
$v_O \rightarrow filtration\ velocity\ for\ the\ oil\ \ phase$
$w \rightarrow displacement\ velocity\ of\ the\ contact$
$k^* \rightarrow effective\ permeability\ to\ water$
$k^{**} \rightarrow effective\ permeability\ to\ oil$
$r_0 \rightarrow the\ radius\ of\ the\ initial\ oil - water\ contact\ position$

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