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Modeling The Transmission Dynamics of Lassa Fever in a Heterogeneous Population Using a Deterministic Approach

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Abstract: *This study presents a deterministic mathematical model for the transmission dynamics of lassa fever in a heterogeneous population in Nigeria, incorporating both human and rodent compartments. The human population is divided into the susceptible, exposed, asymptomatic, infective and recovered classes. While, the rodent population consist of susceptible and infective classes. The model is analyses quantitatively to establish the positivity, boundedness and feasibility of solutions. The disease-free equilibrium (DFE) and endemic equilibrium (EE) are obtained and their stability is investigated using the Jacobian matrix and Routh-Hurwitz criteria. The basic reproduction number R_0 is derived using the next generational matrix method and is used as a threshold parameter to determine disease persistence. Sensitivity analysis is performed to identity key parameters influencing of the spread of the disease. Numerical simulations are carried out to illustrate the impact of these parameters to evaluate the effectiveness of control strategies such as hygiene, rodent control and treatment. The result shows that lassa fever can be controlled when $R_0 < 1$, and that a combination of intervention strategies significantly reduces disease transmission. This study provides useful insights for public health planning and policy formulation in controlling lassa fever outbreak in Nigeria.*

Keywords: population, sensitivity, Lassa fever, reproduction number, stability, endemic, equilibrium

INTRODUCTION

Lassa fever is an acute viral hemorrhagic fever first isolated in a town called Lassa in the Yedseram valley in Borno State of the Northern part of Nigeria in 1969 (Tara, 2004). Lassa fever is endemic in Nigeria, Liberia, Sierra Leone, Guinea and other west Africa countries, affecting about 2-3 Million persons with 5000-10,000 fatalities annually (McCornick et al, 1970). The lassa virus was isolated from a patients blood in 1970. Scientist classified Lassa as a member of the *Arenaviridae family* called the *Genus Arenavirus.* Its an animal-borne disease, which means it is transmitted from animals to humans, primary through contact with the urine of faeces of infected mastomys rat (Frame, 1970). The disease predominantly occurs during the dry season from December to April, when rodents are likely to move indoors in search of food and refuge according to (World Health Organization, 2004).

Promed (2006) reported outbreaks in some cities of West Africa countries of Sierra leone, Liberia, Guinea, in other countries like Togo, Ivory Coast, Benin and Ghana, they are no records. Though isolated cases has shown proofs of the disease circulation (Gunther, et al, 2001). Historically, two major endemic zones have been recognized which is the Mano River region which comprises of (Guinea, Sierra leone and Liberia) and Nigeria. person-to person transmission may occur primarily through in healthcare settings , where the virus may also be spread by contaminated medical equipments such as re-used needles and personal protective equipment (PPE) according to (World Health Organization, 2024).

Lassa fever is a zoonotic disease, meaning it can be transmitted from infected animals to humans. The disease is caused by single-stranded Ribonuclei Acid (RNA) virus, and its endemic in West Africa countries (McCornick, 1970). The disease is caused by lassa virus, an arenavirus whose natural reservoir is the multimammate rat species known as *Mastomys natalensis (*fisher-Hoch et al, 1995). Because certain varieties of *Mastomys* often live in human homes which makes the virus easily transmissible to humans. Transmission occurs through direct contact with rat urine, faeces and saliva also contact with excretion or secretion infected materials or by ingestion of excretion contaminated foods, household items and environment. Individuals can also be exposed by skin breaks, and by mucous membranes from aerosol transmission from dust-borne particles. In some areas where rodents are used like a food source such as bush meats, can provide additional exposure to the infected rat blood, as well as allowing ingestion of potentially contaminated meat (Eze et al, 2010).

Lassa virus can be transmited easily in humans, through human to human by aerosol transmission like Coughing and Sneezing or direct contact with human blood, human fluid secretion like semen, vaginal fluids and urine (Richmond, 2000). The incubation period of lassa fever ranges from 2-21 days, the onset of the disease is usually gradual, starting with fever, overall weakness, headache, malaise and hypertension. In some patient neurological problems may occur which includes hear loss which might be temporal or permanent (omalibu et al, 2005).

It can be difficult to clinically distinguish lassa fever from other infectious disease like malaria, typhoid fever, shigellosis, yellow fever and other hemorrhagic fevers, especially in the early stages of the disease. Confirmations that symptoms are caused by Lassa virus are made using the following diagnostic methods such as the reverse transcriptase (RT-PCR), enzyme-linked immunosorbent assay (ELISA) and antigen detection tests virus isolation by cell culture according to World Health Organization, (2024).

LITERATURE

Extensive theoretical research and mathematical modelling efforts have been undertaken to enhance the understanding of lassa virus. Nwasuka and Nwachukwu (2019) developed a non-linear deterministic model to investigate the transmission dynamics of lassa fever, focusing on separation of infected individuals and treatment as control measures. Their model assumes that humans acquire infection constant interaction with infected rodents and treatment is only administered to isolated cases. The authors derived the effective reproduction number (R_ef ıf), showing that when R_e ff < 1, the disease free equilibrium is locally and globally asymptomatically stable, implying the possibility of disease eradication under proper interventions.

In contrast, Bakare et al, (2019) Extended the modelling approach by incorporating seasonality into the dynamics of lassa fever transmission through a periodically forced non-autonomous systems of differential equation. The model captured fluctuations in mastomys rodent population, which serve as the main reservoir of infection. The analysis demonstrated that timely treatment with ribavirin is critical for reducing mortality, while non-pharmaceutical such as public health education, improved hygiene, isolation of cases and rodent control are equally essential for curbing morbidity and disease persistence. Olamuyiwa et al, (2020) investigated the tramsmision dynamics of lassa fever by developing a basic deterministic model, their analysis emphasize that human primarily contract the virus through exposure of food, household items contaminated with rodent excreta, the threshold parameters were derived to assess disease persistence, Using the next generation matrix.

Onuorah et al, (2016) developed a Lassa fever model using the sex structure approach. Their model represented the transmission dynamics of the Lassa fever disease using a set of ordinary differential equation. The total sum the population at a time is denoted by $N_h(t)$ was sub-divided into four (4) mutually exclusive sub-populations of susceptible males $S_1(t)$, infected males $I_1(t)$, Susceptible males $S_2(t)$ and Infected females $I_2(t)$ such that

$N_h(t) = S_1(t) + I_1(t) + S_2(t) + I_2(t)$. similarly the total number of natural reservoir/ host population at time t, denoted by $N_r(t)$ was sub-divided into dormant reservoir $R_1(t)$, active reservoir host $R_2(t)$, such that $N_r(t) = R_1(t) + R_2(t)$. Their model had the following assumptions. Susceptible individuals male or female can be infected by interaction with the active reservoir (Mastomys Natalensis) and their sexual interaction with opposite sex. Two major controls were considered, the use of condom to reduce contact by sexual interaction and the use of pesticides to kill the natural reservoir (mastomys natalensis). Finally, horizontal transmission for huma and vertical transmission for the reservoir.

Doohan et al, (2024) conducted a sytematic review and meta-analysis to synthesize knowledge on lassa outbreaks, epidemiological parameters and mathematical models which covers 193 publications which produced 440 parameter estimate. The authors extracted estimates related to seroprevelance, transmissibility, epidemiological delays, severity, risk factors and performed pooled analyses of case-fatality ratios (CFRs). Their findings show that although lassa fever remains endemic in West Africa, seroprevalence estimates are geographically inconsistent ranging from *2.6%* to *58*.2%, highlighting considerable spatial uncertainty in infection risk. Only four reproduction number R_o estimates were identified, with values between *1.13* and *1.40*, reflecting the scarcity of robust transmission studies. The meta-analysis estimated a pooled CFR of *35.5% (95% CI: 25.8-2.2)* and *an* average symptom on-set to hospital-admission delay of 8-19 days (*95% CI*:7.*31-9.06*), both suggesting substantial gaps in timely case detection and care.

In this paper, we extended an earlier work on Doohan et al, (2024). Specifically, we included a schematic diagram, sensitivity analysis, numerical computation of the basic reproductive number R_0 numerical simulation and extended the human population compartment which comprises of Susceptible (S_h), Exposed (E_h) Asymptomatic infective (A_h), Infective (I_h) and Recovered (R_h).

MATERIALS AND METHODS

Parameters of the Model

N_h Total population of human population
N_r Total population of rodent population
S_h Humans Susceptible to Lassa fever
S_r Rats Susceptible to Lassa fever
E_h Humans Exposed to Lassa fever
A_h Humans infected with Lassa fever and are Asymptomatic
I_h Infective humans that are Symptomatic with Lassa fever
I_r Infected rats with Lassa fever
β_h Human birth rate
μ_h Human natural death rate
ϕ_h Rate from recovered humans back to susceptible human population
w_h Death caused by the disease
ω_h Rate at which asymptomatic humans get recovered
$л_h$ Rate at which Susceptible humans becomes exposed
k_h Rate at which asymptomatic individuals becomes infectious
δ_h Rate at which infectious humans becomes recovered
α_h Rate at which exposed humans becomes asymptomatically infective
R_o Reproductive number
β_r Rat birth rate
μ_r Rat natural death rate
$л_r$ Rate of susceptible rat to infective rat
Θ_h Correlation from the infective rat to the susceptible human

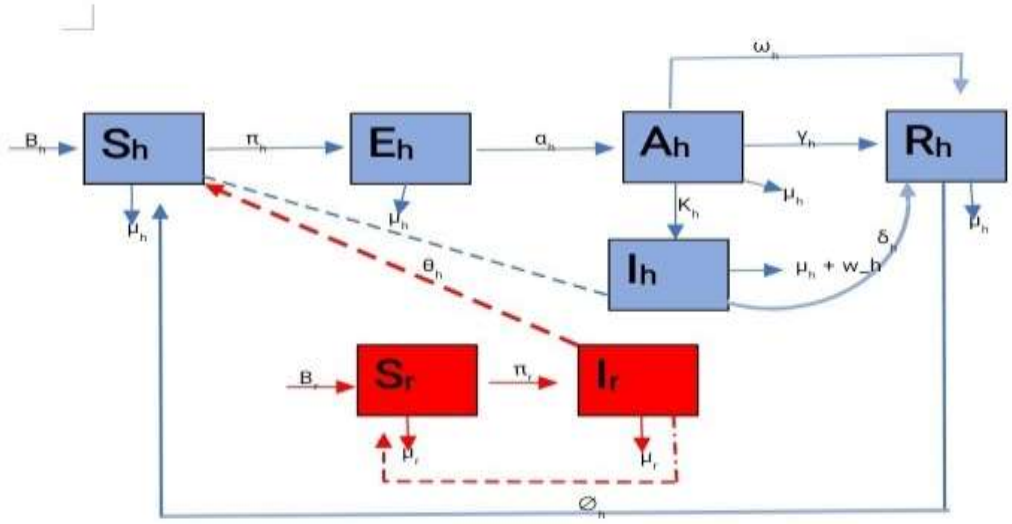

Figure 1. A schematic description of the model

The model

From the assumptions and the schematic diagram above we have the following equations for the population

$$\frac{dS_h}{dt} = B_h + \phi_h R_h - \Lambda_h S_h E_h - \Theta_h I_r - \mu_h S_h \quad (1)$$

$$\frac{dE}{dt} = \Lambda_h S_h E_h - \alpha_h E_h - \mu_h E_h \quad (2)$$

$$\frac{dA}{dt} = \alpha_h E_h - (\omega_h + \gamma_h + \mu_h + k_h) A_h \quad (3)$$

$$\frac{dI}{dt} = k_h A_h - (\mu_h + W_h + \delta_h) I_h \quad (4)$$

$$\frac{dR}{dt} = \gamma_h A_h + \delta_h I_h + A_h\omega_h + \varphi_h S_h - (\mu_h + \phi_h) R_h \quad (5)$$

$$\frac{dS_r}{dt} = B_r + \Lambda_r S_r I_r - \mu_r S_r \quad (6)$$

$$\frac{dI_r}{dt} = \Lambda S_r I_r + \Theta_r I_r - \mu_r I_r \quad (7)$$

Total human population is given as;

$N_h = S_h + E_h + A_h + I_h + R_h$

$$\frac{dN}{dt} = B_h - \omega_h I_h - \mu_h(S_h + E_h + A_h + I_h + R_h) \quad (8)$$

$$\frac{dN}{dt} = B_h - N\mu_h - \omega_h I_h \quad (9)$$

The total reservoir population is given as

$$N_r = B_r - \mu_r(S_r + I_r) \quad (10)$$

$\frac{dN_r}{dt} = B_r - N_r\mu_r$ (11)

Basic properties of the model

Theorem 1. *Presume that the equation is valid and every solution of the equation in the system of equation (9) with initial condition R^5_+ continues and remain in the compact set Ω as $t \to \infty$, the feasible solution, which is a positively invariant set of the model, is the given by*

$$\Omega = \{ (S_h, E_h, A_h, I_h, R_h) \in \mathbb{R}^5_+ : N(t) \leq \frac{B_h}{\mu_h} \} \quad (12)$$

Proof: From equation 9 we have

$$\frac{dN_h}{dt} = B_h - \mu_h N(t) - \omega_h I_h \quad (13)$$

Since $B_h > 0$, $\mu_h > 0$, $\omega_h > 0$, $I_h > 0$ and $N > 0$, we have

$$\frac{dN_h}{dt} = B_h - \mu_h N_h(t) - \omega_h I_h \leq B_h - \beta N(t) \quad (14)$$

$$\frac{dN_h}{dt} \leq B_h - \omega_h N(t) \quad (15)$$

$$\frac{dN_h}{dt} + \mu_h N_h(t) \leq B_h \quad (16)$$

Equation (16) is a linear first order differential inequality, it will be solved following the integration factor approach. It integration factor is

$I = e^{\int uh\, t\, dt} = e^{uht}$,

Multiplying through by the integrating factor $e^{uh\, t}$,

$$\frac{dN_h}{dt} . e^{uh\, t} + \mu_h N(t).e^{uht} \leq B_h.e^{uht} \quad (17)$$

$$\frac{d}{dt}[e^{uht} N(t)] \leq B_h e^{uht} \quad (18)$$

Integrating the inequality on both sides and applying the initial condition at t = 0, we obtain ,

$$\int \frac{d}{dt}[e^{uht} N(t)]\, dt \leq B_h \int e^{ut}\, dt \quad (19)$$

$$N_h(t) \leq N_h(0)e^{-uht} + \frac{B_h}{\mu}[1- e^{-ut}] \quad (20)$$

Hence, as $t \to \infty$, $N_h(0)e^{-uht} + \frac{B_h}{\mu_h}[1- e^{-uht}] \to \frac{B_h}{\mu_h}$ therefore,

$$N_h(t) \leq \frac{B_h}{\mu_h} \quad (21)$$

Theorem 2 . *Presume that the equation is valid and that every solution of the equation in the system of equations (11) with the initial condition in R^2_+ continues and remains in the compact set Ω as $t \to \infty$. Then the feasible solution which is a positive invariant set of the model is given by*

$$\Omega = \{(S_r, I_r) \in \mathbb{R}^2_+ : N(t) \leq \frac{B_r}{\mu_r} \} \quad (22)$$

Proof: from equation (11) , we have

$$\frac{dN_r}{dt} = B_r - \mu_r N(t) \quad (23)$$

Since $B_r > 0$, $\mu_r > 0$, $\omega_r > 0$ and $N > 0$, we have

$$\frac{dN_r}{dt} = B_r - \mu_r N_r(t) \leq B_r - \mu_r N_r(t) \quad (24)$$

$$\frac{dN_r}{dt} \leq B_r - \mu_r N(t) \quad (25)$$

$$\frac{dN_r}{dt} + \mu_r N_r(t) \leq B_r \quad (26)$$

Inequalities in equation (26) is a linear first order differential inequalities, it will be solved following the integration factor approach, it integration factor is

$I = e^{\int \mu_r t\, dt} = e^{\mu_r t}$,

Multiply through by the integrating factor $e^{\mu_r t}$

$\frac{dN_r}{dt} . e^{\mu_r t} + \mu_r N(t). e^{\mu_r t} \leq B_r . e^{\mu_r t}$ (27)

$\frac{d}{dt} [e^{\mu t} N_r(t)] \leq B_r e^{\mu_r t}$ (28)

Integrating the inequalities on both side and applying the initial condition at t = 0, we obtain

$\int \frac{d}{dt}[e^{\mu_r t} N(t)\, dt \leq B_r \int e^{\mu t}\, dt$ (29)

$N_r(t) \leq N_r(0)e^{-\mu t} + \frac{B_r}{\mu} [1- e^{-\mu t}]$ (30)

Hence, as $t \rightarrow \infty$, $N_r(0)e^{-\mu t} + \frac{B_r}{\mu_r}[1 - e^{-\mu t}] \rightarrow \frac{B_r}{\mu_r}$ therefore,

$N_r(t) \leq \frac{B_r}{\mu_r}$ (31)

Disease-Free Equilibrium (DFE) Of The Human Compartment

$B_h + \phi_h R_h^+ - \pi_h S_h^+ E^+ - \theta_h I_r^+ - \mu_h S_h^+ = 0$

$\pi_h S_h^+ E_h - \alpha E_h^+ - \mu_h E_h^+ - \mu_h E_h^+ = 0$

$\alpha E_h^+ - (\omega_h - \gamma_h - \mu_h + k_h) A_h^+ = 0$

$k_h A_h^+ - (\mu_h + w_h + \delta_h) I_h^+ = 0$

$\gamma_h A_h^+ + \delta_h I_h + \omega_h A_h^+ + \varphi_h S_h^+ - (\mu_h + \phi_h) R_h^+ = 0$

$\frac{dS_h}{dt} = \frac{dE_h}{dt} = \frac{dA_h}{dt} = \frac{dI_h}{dt} = \frac{dR_h}{dt} = 0$

Local Stability Analysis Of The Disease-Free Equilibrium (DFE)

For us to analyze our disease free equilibrium for stability we first obtain the Jacobian matrix at disease free equilibrium and basic reproduction number of our model. $^+$

The solution of $|J - \lambda I|$ gives the Eigenvalues as

$\lambda_1 = -(\mu_h + \pi_h)$,

$\lambda_2 = \pi_h \left(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h+\pi} \right) - (\alpha_h + \mu_h)$,

$\lambda_3 = -(\omega_h + \gamma_h + \mu_h + k_h)$,

$\lambda_4 = -(\mu_h + w_h + \mu_h + \delta_h)$

$\lambda_5 = -(\mu_h + \phi_h)$

Then the (DFE) P^+ is asymptomatically stable if $\lambda i < 0$, $i = 1,2,3,4,5$ are unstable if at least one of $\lambda i > 0$ for all ϕ_h, ω_h, B_h, μ_h, π_h, δ_h.

Basic Reproductive Number (R_o)

One of the most important concerns about any infectious disease is its ability to invade a population. Many epidemiological models have a disease free equilibrium (DFE) at which the population remains in the absence of the disease. These models usually have a threshold parameter, known as the basic reproductive number R_o such that when $R_o < 1$, then the DFE is locally asymptomatically stable and the disease cannot invade the population, but if $R_o < 1$, then the DFE is unstable and invasion is always possible see (Hethcote, 1978).

The basic reproductive number R_o, is defined as the average number of secondary infection produced by a single infected individual during their infectious in a completely susceptible population. We use the next generation matrix approach as described by (Driessche and Wathmough, 2005) to derive our basic reproductive disease.
Here, the basic reproduction number R_o is given by;
$R_o = \rho(FV^{-1})$, since the system is block diagnonal, we have:

$$V = \begin{pmatrix} q_1 & 0 & 0 \\ -\alpha_h & q_2 & 0 \\ 0 & -K_h & q_3 \end{pmatrix},$$

$$V^{-1} = \begin{pmatrix} \frac{1}{q_1} & 0 & 0 \\ \frac{\alpha_h}{q_1 q_2} & \frac{1}{q_2} & 0 \\ \frac{\alpha_h K_h}{q_1 q_2 q_3} & \frac{K_h}{q_2 q_3} & \frac{1}{q_3} \end{pmatrix}$$

$R_o = \max(R_h, R_r)$
Human Reproduction Number
$R_h = \frac{\pi_h S_h \alpha_h K_h}{(\alpha_h + \mu_h)(\omega_h + \gamma_h + \mu_h + k_h)(\mu_h + W_h + \delta_h)}$, substituting $S_h = \frac{B_h}{\mu_r}$;
$R_h = \frac{\pi_h B_h \alpha_h K_h}{\mu_h(\alpha_h + \mu_h)(\omega_h + \gamma_h + \mu_h + K_h)(\mu_h + W_h + \delta_h)}$

Rodent Reproduction number
$R_r = \frac{\pi_r S_r}{\mu_r + \theta_r}$, substituting $S_r = \frac{B_r}{\mu_h}$;
$R_r = \frac{\pi_r B_r}{\mu_r(\mu_r + \theta_r)}$
Substituting R_h and R_r into $R_o = \max(R_h, R_r)$, we obtain
$R_o = \max\left(\frac{\pi_h B_h \alpha_h k_h}{\mu_h(\alpha_h + \mu_h)(\omega_h + \gamma_h + \mu_h + K_h)(\mu_h + W_h + \delta_h)}, \frac{\pi_r B_r}{\mu_r(\mu_r + \theta_r)}\right)$

Global Stability of DFE of human compartment
In this section, we prove the global stability of DFE E^+ when the basic reproductive number is less than or equal to unity.
Theorem 4.13.1. *The DFE E^+ of the equilibrium (4.11) is globally asymptotically stable in* Ω *if* $R_0 \leq 1$
Proof: Since $R_0 \leq 1$, it follows that there exists $\epsilon_0 > 0$ such that

$$B_h + \emptyset h R_h - \pi_h S_h E_h - \theta_h I_h - \mu_h S_h = 0 \qquad (32)$$

$$B_h + \varnothing_h \left(\frac{\varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} + \epsilon_0 \right) - \pi_h S_h(0) - \theta_h(0) - \mu_h S_h < 0 \tag{33}$$

in view of the first equation in the system of equation

$$\frac{ds(t)}{dt} \leq B_h + \varnothing_h \left(\frac{\varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) - \mu_h S_h \tag{34}$$

$$\frac{ds(t)}{dt} \leq B_h + \varnothing_h \left(\frac{\varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) - \mu_h S_h < 0$$

$$\frac{ds(t)}{dt} \leq B_h + \varnothing_h \left(\frac{\varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) - \mu_h S_h$$

$$\frac{ds(t)}{dt} + \mu_h S_h \leq B_h + \varnothing_h \left(\frac{\varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) \tag{35}$$

Integrating Factor

$e\mathrm{R}(\mu)dt \leq e(\mu)t,$

multiply both equation by the integrating factor

$$e^{(\mu)t}.\frac{ds(t)}{dt} + \mu_h S_h.e^{(\mu)t} \leq e^{(\mu)t}.B_h + \varnothing_h \left(\frac{\varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right).e^{(\mu)t}$$

$$\int \frac{ds(t)}{dt} S(t) e^{(\mu)t} \leq \left(B_h + \frac{\varnothing_h \varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) \int e^{(\mu)dt}$$

$$S(t)e^{(\mu)t} \leq \left(B_h + \frac{\varnothing_h \varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) \frac{e^{(\mu)t}}{(\mu)t} + c$$

divide through by the coefficient of S(t)

$$S(t) \leq \left(B_h + \frac{\varnothing_h \varphi_h B_h}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right) \frac{e^{(\mu)t}}{(e^{\mu})} + ce^{\mu}$$

Hence, as $t \to \infty$.

$$S(t) \leq \frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \tag{36}$$

consider the first equation

$$\frac{dE}{dt} = \pi_h S_h^+ E_h^+ - \alpha_h E_h^+ - \mu_h E^+ \tag{37}$$

$$\frac{dE}{dt} \leq \pi_h S_h^+ E_h^+ - (\alpha_h E_h^+ + \mu_h) E_h^+,$$

$$\frac{dE}{dt} \leq -(\alpha_h E_h^+ + \mu_h) E_h^+ + \pi_h S_h^+ E_h^+$$

$$\frac{dE}{dt} \leq -(\alpha_h + \mu_h + \pi_h \left(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right)) E_h^+$$

$$\frac{dE^+(t)}{dt} + (\alpha_h + \mu_h + \pi_h \left(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h} \right)) E_h^+ \leq 0$$

by using the IF, we obtained

$$e^{\int}(\alpha_h+\mu_h+\pi_h(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varnothing_h\varphi_h}))dt = e^{(\alpha_h+\mu_h+\pi_h(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varnothing_h\varphi_h}))t}$$

multiply by the integrating factor

$$\int[\frac{d}{dt}e^{(\alpha_h+\mu_h+\pi_h(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varnothing_h\varphi_h}))}E^+(t)] = 0$$

$$e^{\left(\alpha_h+\mu_h+\pi_h\left(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varphi_h\phi_h}\right)\right)t}\frac{dE_h^+(t)}{dt}+e^{\left(\alpha_h+\mu_h+\pi_h\left(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varphi_h\phi_h}\right)\right)t}\left(\alpha_h+\mu_h+\pi_h\left(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varphi_h\phi_h}\right)\right)$$
$$e^{(\alpha_h+\mu_h+\pi_h(\frac{B_h(\mu_h+\phi_h)}{\mu_h(\mu_h+\phi_h)-\varnothing_h\varphi_h}))}E(t) = 0$$

Dividing both side by the integrating factor and taking limit as t → ∞.

$$\lim_{x\to 0} E^+(t) = 0 \qquad (37)$$

The Third equation in the system

$$\alpha_h E_h^+ - (\omega_h+\gamma_h+\mu_h+K_h)A^+_h = 0 \qquad (38)$$

$$\frac{dA_h^+(t)}{dt} \le \alpha_h(0) - (\omega_h+\gamma_h+\mu_h+K_h)A_h^+$$

$$\frac{dA_h^+(t)}{dt} + (\omega_h+\gamma_h+\mu_h+K_h)A_h^+ = 0$$

By using the IF, we obtained

$$e\mathrm{R}(\omega h+\gamma h+\mu h+Kh)dt = e(\omega h+\gamma h+\mu h+Kh)t$$

multiply by the integrating factor

$$e^{(\omega_h+\gamma_h+\mu_h+K_h)}t.\frac{dA_h^+(t)}{dt} + e^{(\omega_h+\gamma_h+\mu_h+K_h)}t.(\omega_h+\gamma_h+\mu_h+K_h)A_h^+ = 0$$

$$\int[\frac{d}{dt}e^{(\omega_h+\gamma_h+\mu_h+K_h)}E_h^+(t)] = 0$$

$$e^{(\omega_h+\gamma_h+\mu_h+K_h)}A_h^+(t) = 0$$

Dividing both side by the integrating factor and taking limit as t → ∞.

$$\lim_{x\to 0} A_h^+(t) = 0 \qquad (39)$$

Consider the fourth

$$\kappa_h A_h^+ - (\mu_h+W_h+\delta_h)I_h^+ = 0 \qquad (40)$$

$$\frac{dI_h^+(t)}{dt} + (\mu_h+W_h+\delta_h)I_h^+ = 0$$

by using the IF, we obtained

$$e\mathrm{R}(\mu_h+W_h+\delta_h)I_h+dt = e(\mu h+Wh+\delta h)Ih+t$$

multiply by the integrating factor

$$e^{(\mu_h+W_h+\delta_h)I_h^+}t.\frac{dI(t)}{dt} + e^{(\mu_h+W_h+\delta_h)I_h^+}t.(\mu_h+W_h+\delta_h)I^+(t) = 0$$

$$\int[\frac{d}{dt}e^{(\mu_h+W_h+\delta_h)}I(t)] = 0$$

$$e^{(\beta+\alpha+\theta)}I(t) = 0$$

Dividing both side by the integrating factor and taking limit as t → ∞.

$$\lim I^{+}(t) = 0 \; x \to 0 \tag{41}$$

consider the first equation

$$\frac{dE}{dt} = \pi_h S_h^+ E_h^+ - \alpha_h E_h^+ - \mu_h E^+ \tag{42}$$

$$\frac{dE}{dt} \leq \pi_h S_h^+ E_h^+ - (\alpha_h E_h^+ + \mu_h) E_h^+,$$

$$\frac{dE}{dt} \leq -(\alpha_h E_h^+ + \mu_h) E_h^+ + \pi_h S_h^+ E_h^+$$

$$\frac{dE}{dt} \leq -(\alpha_h + \mu_h + \pi_h(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h}))E_h^+$$

$$\frac{dE^+(t)}{dt} + (\alpha_h + \mu_h + \pi_h(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h}))E_h^+ \leq 0$$

by using the IF, we obtained

$$e^{\int}(\alpha_h + \mu_h + \pi_h(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h}))dt = e^{(\alpha_h + \mu_h + \pi_h(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h}))t}$$

multiply by the integrating factor

$$e^{\left(\alpha_h + \mu_h + \pi_h\left(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varphi_h \phi_h}\right)\right)t} \frac{dE_h^+(t)}{dt} + e^{\left(\alpha_h + \mu_h + \pi_h\left(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varphi_h \phi_h}\right)\right)t} \left(\alpha_h + \mu_h + \pi_h \left(\frac{B_h(\mu_h -}{\mu_h(\mu_h + \phi_h}\right.\right.$$

$$\int [\frac{d}{dt} e^{(\alpha_h + \mu_h + \pi_h(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h}))} E^+(t)] = 0$$

$$e^{(\alpha_h + \mu_h + \pi_h(\frac{B_h(\mu_h + \phi_h)}{\mu_h(\mu_h + \phi_h) - \varnothing_h \varphi_h}))} E(t) = 0$$

Dividing both side by the integrating factor and taking limit as t → ∞.

$$\lim E^{+}(t) = 0 \tag{42}$$

$x \to 0$

The Third equation in the system

$$\alpha_h {E_h}^+ - (\omega_h + \gamma_h + \mu_h + K_h) {A^+}_h = 0 \tag{43}$$

$$\frac{dA_h^+(t)}{dt} \leq \alpha_h(0) - (\omega_h + \gamma_h + \mu_h + K_h) A_h^+$$

$$\frac{dA_h^+(t)}{dt} + (\omega_h + \gamma_h + \mu_h + K_h) A_h^+ = 0$$

by using the IF, we obtained

$e\mathrm{R}(\omega h + \gamma h + \mu h + Kh)dt = e(\omega h + \gamma h + \mu h + Kh)t$

multiply by the integrating factor

$$e^{(\omega_h + \gamma_h + \mu_h + K_h)t} . \frac{dA_h^+(t)}{dt} + e^{(\omega_h + \gamma_h + \mu_h + K_h)t} . (\omega_h + \gamma_h + \mu_h + K_h) A_h^+ = 0$$

$$\int [\frac{d}{dt} e^{(\omega_h + \gamma_h + \mu_h + K_h)} E_h^+(t)] = 0$$

$$e^{(\omega_h + \gamma_h + \mu_h + K_h)} A_h^+(t) = 0$$

Dividing both side by the integrating factor and taking limit as t → ∞.

$$\lim_{x \to 0} A_h^+(t) = 0 \tag{44}$$

Consider the fourth

$$\kappa_h A_h^+ - (\mu_h + W_h + \delta_h) I_h^+ = 0 \qquad (45)$$
$$\frac{dI_h^+(t)}{dt} + (\mu_h + W_h + \delta_h) I_h^+ = 0$$

by using the IF, we obtained

$eR(\mu_h + W_h + \delta_h)I_h + dt = e(\mu h + Wh + \delta h)Ih + t$

multiply by the integrating factor

$$e^{(\mu_h+W_h+\delta_h)I_h^+} t.\frac{dI(t)}{dt} + e^{(\mu_h+W_h+\delta_h)I_h^+} t.(\mu_h + W_h + \delta_h) I^+(t) = 0$$
$$\int [\frac{d}{dt} e^{(\mu_h+W_h+\delta_h)} I(t)] = 0$$

$$e^{(\beta+\alpha+\theta)} I(t) = 0$$

Dividing both side by the integrating factor and taking limit as t $\rightarrow \infty$.

$$\lim I^+(t) = 0 \; x \rightarrow 0 \qquad (46)$$

Considering the fifty equation

$$\gamma hA + h + \delta hIh + + \omega hA + h + \varphi hSh + - (\mu h + \emptyset h)Rh + = 0 \qquad (47)$$
$$\frac{dR_h^+(t)}{dt} \leq \gamma_h(0) + \delta_h(0) + \omega_h(0) + \varphi_h S_h^+ - (\mu_h + \varnothing_h) R_h^+ = 0$$
$$\frac{dR_h^+(t)}{dt} \leq -(\mu_h + \varnothing_h) R_h^+ + \varphi_h S_h^+$$
$$\frac{dR_h^+(t)}{dt} + (\mu_h + \varnothing_h) R \leq \varphi_h (\frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \varnothing_h) - \varnothing_h \varphi_h})$$

Integrating Factor

$eR(\mu h + \emptyset h)dt \leq e(\mu h + \emptyset h)t$

,

multiply both equation by the integrating factor

$$e^{(\mu_h+\varnothing_h)t}.\frac{dR_h^+(t)}{dt} + e^{(\mu_h+\varnothing_h)t}.(\mu_h + \varnothing_h) R \leq e^{(\mu_h+\varnothing_h)t}.\varphi_h (\frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \varnothing_h) - \varnothing_h \varphi_h})$$
$$\int [\frac{dR_h^+(t)}{dt} R_h^+(t) e^{(\mu_h+\varnothing_h)t}] \leq \varphi_h (\frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \varnothing_h) - \varnothing_h \varphi_h})$$
$$e^{(\mu_h+\varnothing_h)t} R(t) \leq \varphi_h (\frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \varnothing_h) - \varnothing_h \varphi_h}).e^{(\mu_h+\varnothing_h)t}) + C$$

divide through by the coefficient of R(t)

$$R_h^+(t) \leq \varphi_h (\frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \varnothing_h) - \varnothing_h \varphi_h}) + Ce^{-(\mu_h+\varnothing_h)t}$$

Hence, as t $\rightarrow \infty$.

$$R_h^+(t) \leq \varphi_h \left(\frac{B_h(\mu_h + \varnothing_h)}{\mu_h(\mu_h + \varnothing_h) - \varnothing_h \varphi_h} \right) \tag{48}$$

the above equation, signifies a complete proof. Therefore, the DFE is globally stable

Global Stability of DFE of Rat Compartment

In this section, we prove the global stability of DFE E^+ when the basic reproductive number of the rat compartment is less than or equal to unity.

Theorem 4.14.1. *The DFE E^+ of the equilibrium is globally asymptotically stable in* Ω *if* $R_0 \leq 1$

Proof: Since $R_0 \leq 1$, it follows that there exists $\epsilon_0 > 0$ such that

$$\begin{aligned} B_r^+ + \pi_r S_r^+ I_r^+ - \mu_r^+ S_r^+ &= 0 \\ B_r^+ + \pi_r S_r^+(0) - \mu_r^+ S_r^+ &< 0 \end{aligned} \tag{49}$$

in view of the first equation in the system of equation

$$\frac{ds(t)}{dt} B_r - \mu_r S_r$$

$$\frac{ds(t)}{dt} \leq B_r - \mu_r S_r < 0$$

$$\frac{ds(t)}{dt} \leq B_r - \mu_r S_r)$$

$$\frac{ds(t)}{dt} + \mu_r S_r \leq B_r$$

Integrating Factor

$e^{\mathrm{R}}\mu_r dt \leq e^{\mu r} t,$

multiply both equation by the integrating factor

$$e^{\mu_r} . \frac{ds(t)}{dt} + e^{\mu_r} . \mu_r S_r \leq B_r . e^{\mu_r}$$

$$\int \frac{ds(t)}{dt} S(t) e^{\mu_r t} \leq B_r \int e^{(\mu_r t)} dt$$

$$\int \frac{ds(t)}{dt} S(t) e^{\mu_r t} \leq B_r \int e^{(\mu_r t)} dt + c$$

divide through by the coefficient of S(t)

$$S(t) e^{(\mu_r t)} \leq B_r \frac{1}{(\mu_r)} e^{\mu_r} + c e^{-\mu_r}$$

Hence, as t $\rightarrow \infty$.

$$S(t) \leq \frac{B_r}{\mu_r} \tag{50}$$

Consider the first equation:

$$\begin{aligned} \frac{dI_r^+}{dt} &= \pi_r S_r^+ I_r^+ + \theta_r I_r^+ - \mu_r I_r^+ \\ \frac{dI_r^+}{dt} &= \pi_r S_r^+(0) + (\theta_r - \mu_r)(0) \end{aligned} \tag{51}$$

Taking the limit as $t \rightarrow \infty$, we obtain

$$\lim_{t\to\infty} I_h^{+}(t) = 0. \tag{52}$$

solution $E^* = (S^*,E^*,L^*,A^*,R^*) \neq (0,0,0,0,0)$ of the above equation is given by

$$S_h^* = \frac{\alpha_h + \mu_h}{\pi_h} \tag{5}$$

$$A_h^* = \frac{\alpha_h}{\omega_h + \gamma_h + \mu_h + K_h} \cdot \frac{\theta_h I_r^+ + \mu_h \frac{\alpha_h + \mu_h}{\pi_h} - B_h - \frac{\varnothing_h \varphi_h (\alpha_h + \mu_h)}{\pi_h (\mu_h + \varnothing_h)}}{\frac{\varnothing_h}{\mu_h + \varnothing_h}\left[\frac{(\gamma_h + \omega_h)\alpha_h}{\omega_h + \gamma_h + \mu_h + K_h} + \frac{\delta_h \kappa_h \alpha_h}{(\mu_h + W_h + \delta_h)(\omega_h + \gamma_h + \mu_h + K_h)}\right] - (\alpha_h + \mu_h)} \tag{4.120}$$

$$I_h^* = \frac{\kappa_h \alpha_h}{(\mu_h + W_h + \delta_h)(\omega_h + \gamma_h + \mu_h + K_h)} \cdot \frac{\theta_h I_r^+ + \mu_h \frac{\alpha_h + \mu_h}{\pi_h} - B_h - \frac{\varnothing_h \varphi_h (\alpha_h + \mu_h)}{\pi_h (\mu_h + \varnothing_h)}}{\frac{\varnothing_h}{\mu_h + \varnothing_h}\left[\frac{(\gamma_h + \omega_h)\alpha_h}{\omega_h + \gamma_h + \mu_h + K_h} + \frac{\delta_h \kappa_h \alpha_h}{(\mu_h + W_h + \delta_h)(\omega_h + \gamma_h + \mu_h + K_h)}\right] - (\alpha_h + \mu_h)} \tag{4.121}$$

$$R_h^* = \frac{1}{\mu_h + \varnothing_h}\left\{\alpha_h \left[\frac{\gamma_h + \omega_h}{\omega_h + \gamma_h + \mu_h + K_h} + \frac{\delta_h \kappa_h}{(\mu_h + W_h + \delta_h)(\omega_h + \gamma_h + \mu_h + K_h)}\right] \times \frac{B_h - \mu_h \frac{\alpha_h + \mu_h}{\pi_h} + \frac{\varphi_h \varnothing_h}{\mu_h + \varnothing_h} \cdot \frac{\alpha_h + \mu_h}{\pi_h}}{(\alpha_h + \mu_h) + \theta_h \frac{\kappa_h \alpha_h}{(\mu_h + W_h + \delta_h)(\omega_h + \gamma_h + \mu_h + K_h)} - \frac{\alpha_h \varnothing_h}{\mu_h + \varnothing_h}\left[\frac{\gamma_h + \omega_h}{\omega_h + \gamma_h + \mu_h + K_h} + \frac{\delta_h \kappa_h}{(\mu_h + W_h + \delta_h)(\omega_h + \gamma_h + \mu_h + K_h)}\right]} + \varphi_h \frac{\alpha_h + \mu_h}{\pi_h}\right\}.$$

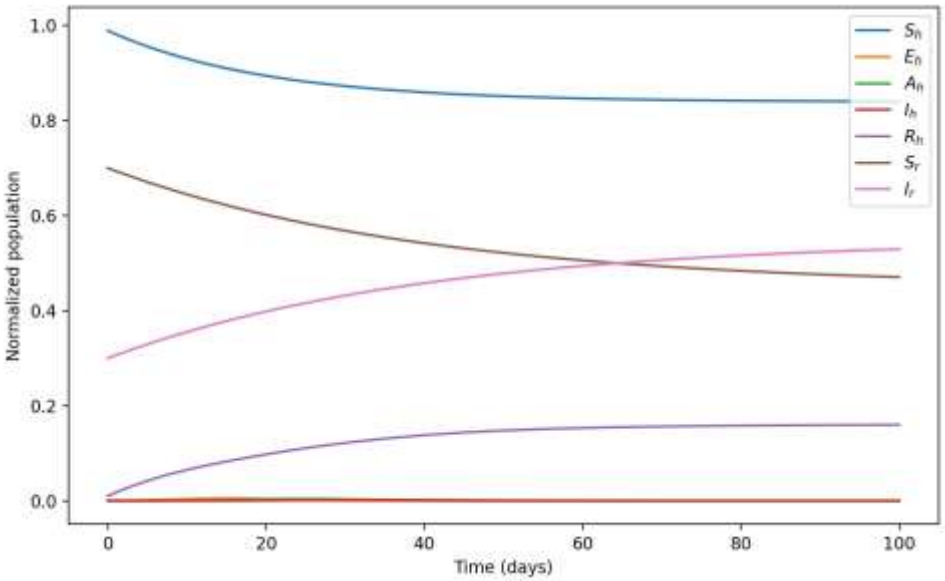

Figure 4.1: graphical representation of numerical stimulation for both human and rodent compartment.

Summary

This thesis analyzed the transmission dynamics of Lassa fever in Nigeria using a deterministic compartmental model that captures interactions between human and rodent populations. The human population is divided into susceptible, exposed, asymptomatic, symptomatic, and recovered classes, while the rodent population consists of susceptible and infected groups. The model incorporates human-to-human and rodent-to-human transmission, as well as asymptomatic spread and population heterogeneity. Analytical results established the positivity and boundedness of solutions, existence of a disease-free equilibrium, and the basic reproduction number R_0, which determines disease persistence. The disease-free equilibrium is stable when $R_0 < 1$ and unstable otherwise. Findings reveal that Lassa fever persists due to infected rodents, favorable environmental conditions, and delayed detection. The inclusion of asymptomatic individuals highlights their significant role in silent transmission. Sensitivity analysis shows that rodent-to-human transmission and rodent population dynamics are key drivers of the disease. Among control strategies, rodent control is most effective individually, while combined interventions yield the greatest reduction and potential elimination of the disease.

Contribution to Knowledge

This study contributes to the existing body of knowledge on the mathematical modelling of infectious diseases by developing a deterministic compartmental model for the transmission dynamics of Lassa fever in a heterogeneous population. The model incorporates both human and rodent populations and introduces a realistic transmission structure by accounting for different epidemiological classes within the human population, including susceptible, exposed, infectious, and recovered individuals. Unlike several existing models that assume population homogeneity, this study incorporates heterogeneity in transmission dynamics, thereby providing a more accurate representation of Lassa fever spread in endemic regions such as Nigeria. The qualitative analysis of the model establishes the positivity and boundedness of solutions, ensuring that the model is biologically meaningful. The disease-free

equilibrium is determined and its stability is investigated using standard mathematical techniques. Furthermore, a sensitivity analysis is performed on the model parameters in order to identify the key drivers of disease transmission. The results reveal the parameters that have the most significant impact on the basic reproduction number and overall disease dynamics, thereby highlighting the most influential factors in the spread and control of Lassa fever. The findings of this study provide theoretical insight into the conditions under which Lassa fever can either die out or persist in the population, and contribute to a deeper understanding of its transmission dynamics WHO (2024).

REFERENCES

Andersen, K. G., Shapiro, B. J., Matranga, C. B., Sealfon, R., Lin, A. E., Moses, L. M., Folarin, O. A., Goba, A., Odia, I., Ehiane, P. E., and Consortium, V. H. F. (2015). Clinical Sequencing Uncovers Origins and Evolution of Lassa Virus. *Cell*, 162(4):738–750.

Anderson, R. M. and May, R. M. (1991). *Infectious Diseases of Humans: Dynamics and Control*. Oxford University Press.

Asogun, D., Arogundade, B., Unuabonah, F., Olugbenro, O., Asogun, J., Aluede, F., and Ehichioya, D. (2025). A review of the epidemiology of Lassa fever in Nigeria: Trends, transmission patterns, and public health implications. *Microorganisms*, 13(6):1419.

Asogun, D. A., Gunther, S., Akpede, G. O., Ihekweazu, C., and Zumla, A. (2019). Lassa Fever: Epidemiology, Clinical Features, Diagnosis, Management and Prevention. *Infectious Disease Clinics of North America*, 33(4):933–951.

Auwal, A. M. and Usaini, S. (2025). Mathematical Analysis of Lassa Fever Epidemic Model Utilizing Nigeria Demographic Data. *Franklin Open*, 11:100276.

Bakare, E. A., Are, E. B., Abolarin, O. E., Osanyinlusi, S. A., Ngwu, B., and Ubaka, O. N. (2020). Mathematical modelling and analysis of transmission dynamics of Lassa fever: A seasonally forced model for Lassa fever transmission dynamics and control strategies. *Journal of Applied Mathematics*, 2020:6131708.

Begon, M., Townsend, C. R., and Harper, J. L. (2006). *Ecology: From Individuals to Ecosystems*. Blackwell Publishing, 4 edition.

Bonwitt, J., Mari Sae´z, A., Lamin, J., Ansumana, R., Dawson, M., Fichet-Calvet, E., and Brown, H. (2017). At Home with Mastomys and Rattus: Human-Rodent Interactions and Potential for Primary Transmission of Lassa Virus in Domestic Spaces. *The American Journal of Tropical Medicine and Hygiene*, 96(4):935–943.

Bowen, M. D., Rollin, P. E., Ksiazek, T. G., Hustad, H. L., Bausch, D. G., Demby, A. H., Bajani, M. D., Peters, C. J., and Nichol, S. T. (2000). Genetic Diversity among Lassa Virus Strains. *Journal of Virology*, 74(15):6992–7004.

Cummins, D., McCormick, J. B., Bennett, D., Samba, J. A., Farrar, B., Machin, S. J., FisherHoch, S. P., and Hallam, N. F. (1990). Acute Sensorineural Deafness in Lassa Fever. *JAMA*, 264(16):2093–2096.

Doohan, P., Jorgensen, D., Naidoo, T. M., McCain, K., Hicks, J. T., McCabe, R., Bhatia, S., Charniga, K., Cuomo-Dannenburg, G., Hamlet, A., Nash, R. K., Nikitin, D., Rawson, T., Sheppard, R. J., Unwin, H. J. T., van Elsland, S., Cori, A., Morgenstern, C., and Imai-Eaton, N. (2024). Lassa fever outbreaks, mathematical models, and disease parameters: A systematic review and meta-analysis. *The Lancet Global Health*, 12(12):e1962–e1972.

Fichet-Calvet, E. and Rogers, D. J. (2009). Risk Maps of Lassa Fever in West Africa. *PLoS Neglected Tropical Diseases*, 3(3):e388.

Fisher-Hoch, S. P., Tomori, O., Fakile, Y., and McCormick, J. B. (1995). Review of Cases of Nosocomial Lassa Fever in Nigeria. *The Lancet*, 345:1271–1274.

Frame, J. D. (1970). Lassa fever virus isolation and rodent transmission studies. *Transactions of the Royal Society of Tropical Medicine and Hygiene*, 64(6):1–8.

Garry, R. F. (2023). Lassa Fever: The Road Ahead. *Nature Reviews Microbiology*, 21:87–96.

Happi, A. N., Happi, C. T., and Schoepp, R. J. (2019). Lassa Fever Diagnostics: Past, Present and Future. *Current Opinion in Virology*, 37:132–138.

Hethcote, H. W. (2000). The Mathematics of Infectious Diseases. *SIAM Review*, 42(4):599– 653.

Ibrahim, M. A. and De´nes, A. (2021). A mathematical model for Lassa fever transmission dynamics in a seasonal environment with a view to the 2017-20 epidemic in Nigeria. *Nonlinear Analysis: Real World Applications*, 60:103310

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