International Journal of Mathematics and Statistics Studies, 13 (3), 42-59, 2025

Print ISSN: 2053-2229 (Print),

Online ISSN: 2053-2210 (Online)

Website: https://www.eajournals.org/

Publication of the European Centre for Research Training and Development -UK

Mathematics Concepts and Their Interactions

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doi: https://doi.org/10.37745/ijmss.13/vol13n34259 Published November 11, 2025

Citation: Kong L.S. (2025) Mathematics Concepts and Their Interactions, *International Journal of Mathematics and Statistics Studies,* 13 (3), 42-59

Abstract: *The aim of the paper is to define mathematics concepts and the interaction between them based on the unified framework of mathematics by the philosophy method proposed in "Map of Mathematics under Yin-Yang Theory". Firstly, develop & prove addition, subtraction, multiplication & division with the help of "form" & 'equation'; then define negative number with the help of the concept of opposite unit interval (-). After these efforts, the activity of proving commutative property of addition & multiplication is conducted. Next goals are to define continues number with the help of arrays of assembly of unit intervals and refine the definition of 'epsilon-delta' limit with the help of the newly defined continuity concept. Then refine definition of 1D, 2D & 3D space; from the introduced space, refine the definition of vector & combination of vectors. Lastly. explore the interaction between concepts of variable, limit, space & origin by the concept of graph and derivative.*

Keywords: mathematics framework, arithmetic concepts, limit, space, negative number, commutative properties of addition & multiplication

INTRODUCTION

This paper aims to discover mathematics concepts and the interactions between them based on a framework developed by Sau Kong from Yin-Yang philosophy perspective. [1]. Concepts of addition, subtraction, multiplication & division, defined based on the perspectives that arose from the mentioned framework, shall be discussed, followed by proof whenever necessary. Then the paper moves further by defining four (4) main concepts: variable, limit, space, and origin of said framework. Each of these concepts contributes to basic branches of mathematics, and the interactions between them further develop the subject of mathematics.

LITERATURE REVIEW

Figure 1 from the article "Map of Mathematics under Yin Yang Theory" [1] disclosed the concept of a unit interval "—"and its assembly shall associate or correspond to a number. The assembly of the unit intervals implicitly proved that 1+1=2

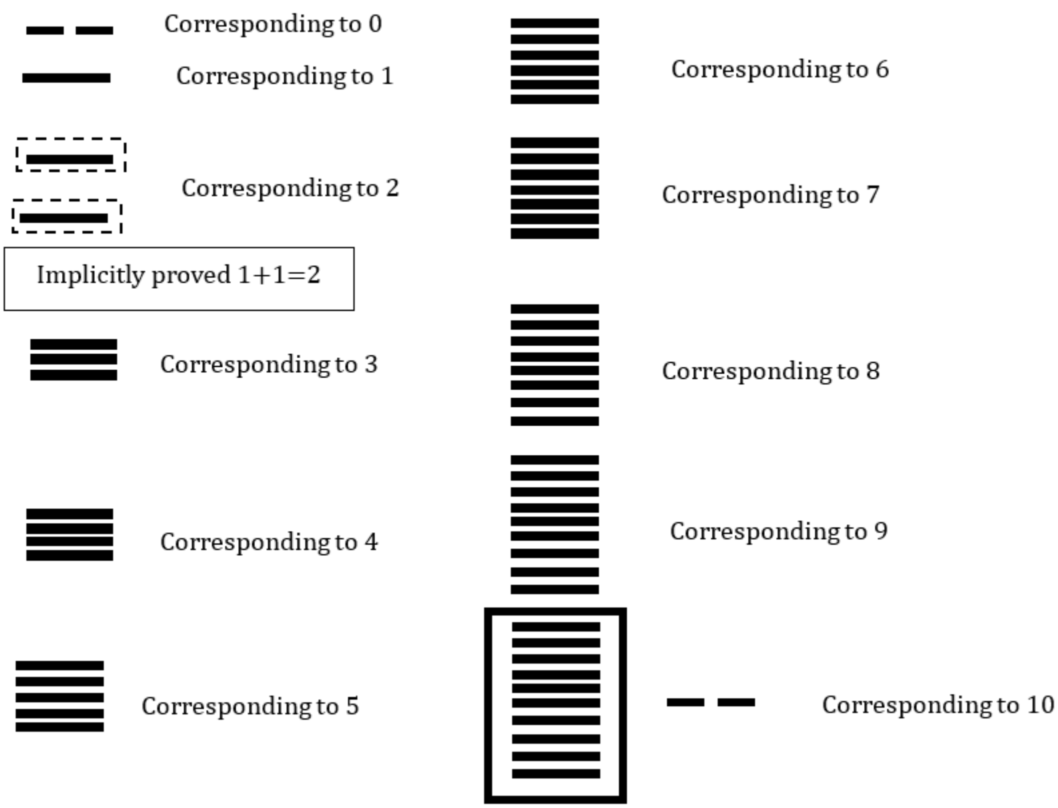


Figure 1: Arrays of assembly of unit intervals and their corresponding numbers implicitly proved 1+1 =2

METHODOLOGY

Define what is 'form" & "equation" in the context of Yin Yang Theory, hence, concepts of addition, subtraction, multiplication & division can then be defined based on such concepts & perspectives. By utilizing the Yin Yang multilevel reasoning property to further extend those defined concepts & perspectives in developing new concepts: variable, limit, space, and origin, and interaction between them.

Define Concepts of Addition, Subtraction, Multiplication & Division

The relationship between unit interval(s) gives rise to the concept of 'form', while the relationship between numbers gives rise to the concept of 'equation'. Form & equation are the two sides of the same coin.

Addition

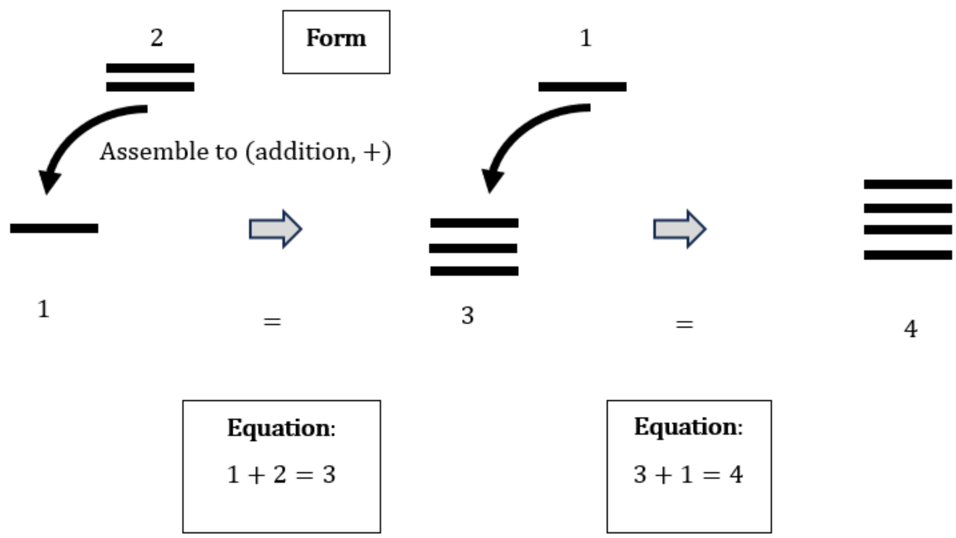


Figure 2: Concept of addition by utilizing 'form' & 'equation'

From figure 2, the concept of addition is defined as assembly process of unit interval/assembly of unit intervals. It can be represented in 'form' & 'equation'.

Subtraction

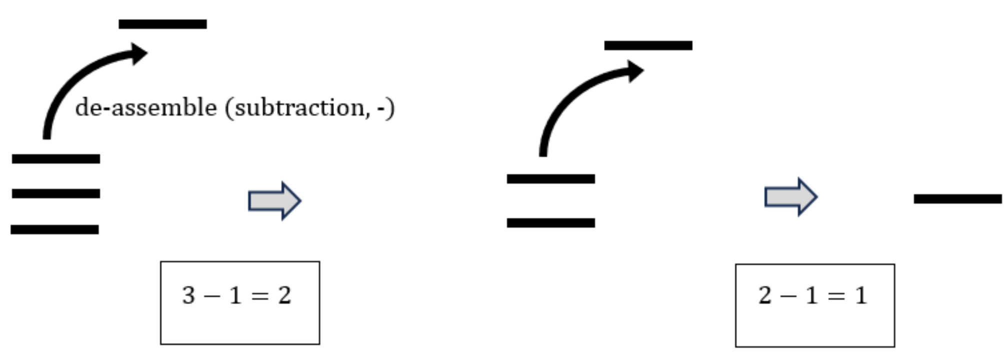


Figure 3: Concept of subtraction by utilizing 'form' & 'equation'

From figure 3, the concept of subtraction is defined as de-assembly process of unit interval/assembly of unit intervals. It can be represented in 'form' & 'equation'.

Multiplication

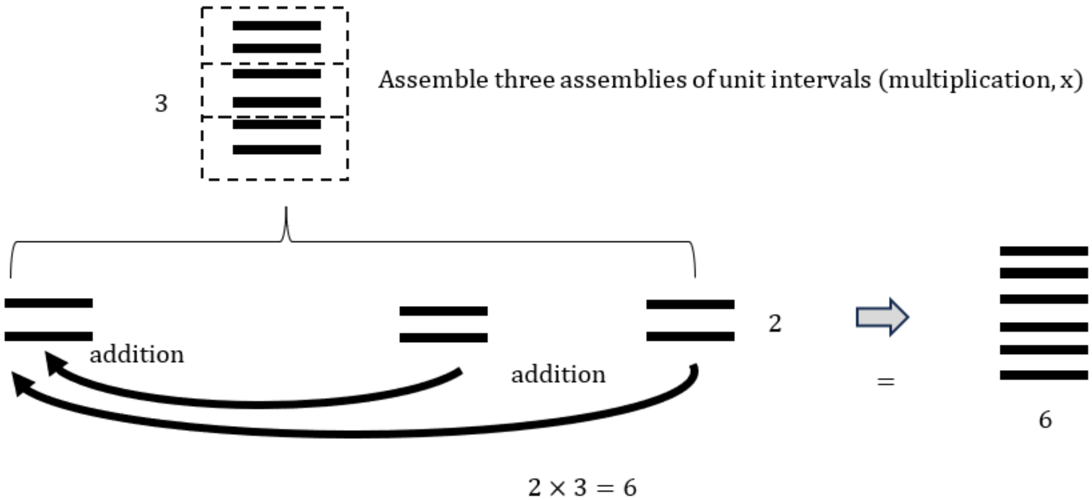


Figure 4: Concept of multiplication by utilizing ‘form’ & ‘equation’, example I

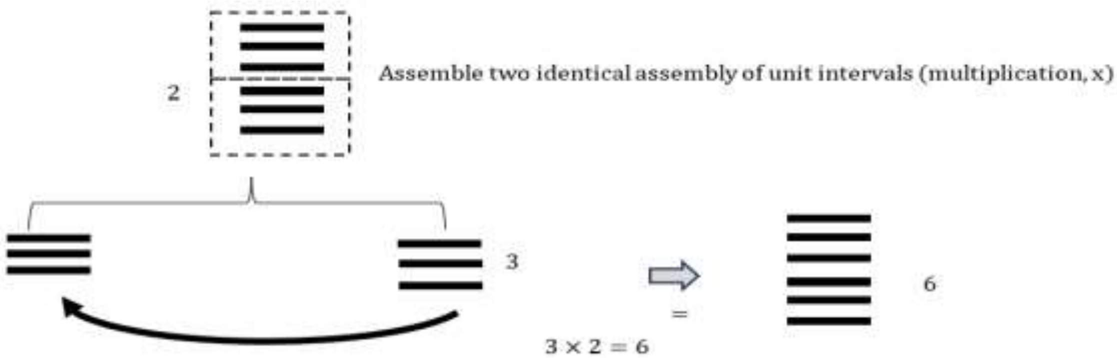


Figure 5: Concept of multiplication by utilizing ‘form’ & ‘equation’, example II

From figure 4 & 5, the concept of multiplication is defined as an assembly process of identical assemblies of unit intervals.

Division

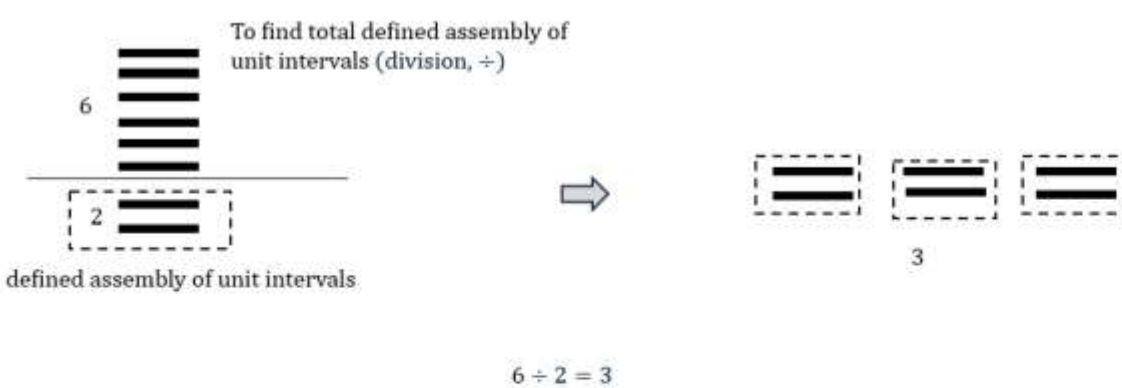
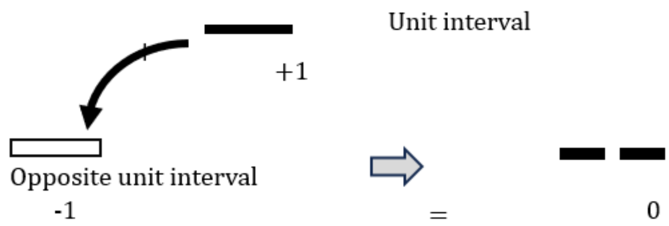


Figure 6: Concept of division by utilizing ‘form’ & ‘equation’

From figure 6, the concept of division is defined as finding the total defined assembly of unit intervals.

Negative Number

Negative number, in simple terms is the existence of opposite unit interval(s) in which, by adding with equal unit interval(s) shall give result of zero. Based on this new concept, the unit interval could be labeled as positive number. In equation, the symbol used to represent opposite unit interval(s) is "—", which is the same as the symbol of subtraction. In the same way, the symbol used to represent unit interval(s) is "+", which is the same as symbol of addition.

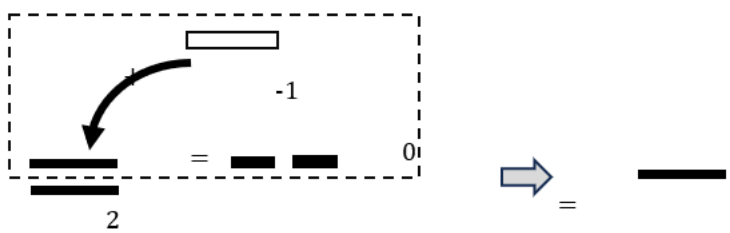


$$-1 + (+1) = -1 + 1 = 0$$

Figure 7: Concept of negative number by utilizing 'form' & 'equation'

5.1 Addition, & Subtraction by Negative Number

5.1.1 Addition by negative number



$$2 + (-1) = 1 + (1 + (-1)) = 1$$

Figure 8: Concept of addition by negative number by utilizing 'form' & 'equation', example I

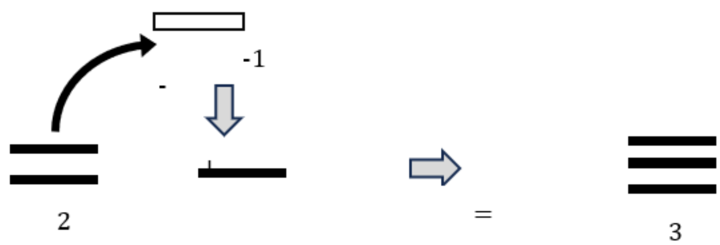
$$-2 + (-1) = -2 - 1 = -3$$

Figure 9: Concept of addition by negative number by utilizing 'form' & 'equation', example II

The addition of two negative numbers is building up the total, and the addition of equal negative number & positive number gives a result of zero.

Commutative property of addition
From figures 2,7,8&9, as addition of unit interval(s) itself or addition of opposite unit interval(s) itself is building up the total of the same kind; and the addition between equal total of unit interval(s) & opposite unit interval(s) gives result of zero; the addition doesn't depend on the orders of the addends. Hence, the commutative property of addition: $a + b = b + a$ had been proved.

Subtraction by negative number
Subtraction by negative number is defined as de-assembly process of opposite unit interval(s), which means to flip the opposite unit interval(s) to become unit interval(s) (+) for assemble to the minuend.



$2 - (-1) = 2 + 1 = 3$

Figure 10: Concept of subtraction by negative number by utilizing 'form' & 'equation', example I

$-2 - (-1) = -1 - 1 + 1 = -1$

Figure 11: Concept of subtraction by negative number by utilizing 'form' & 'equation', example II

Multiplication and Division by Negative Number

Multiplication by negative number
Multiplication by negative number as multiplicand is defined in the same way as multiplication by positive number specified in section 2.3: assembly process of identical assembly of opposite unit intervals, in which the multiplier must be positive.

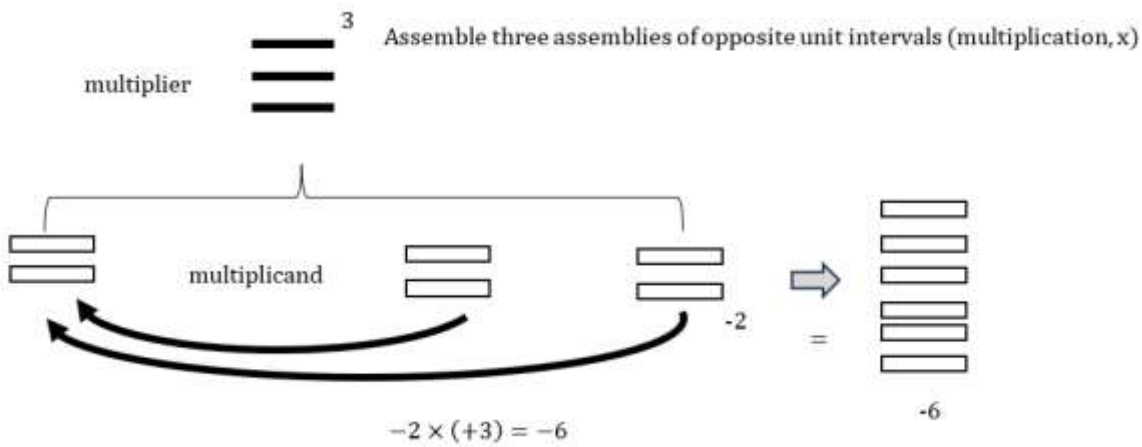


Figure 12: Concept of multiplication by negative number as multiplicand by utilizing ‘form’ & ‘equation’

In contrast, multiplication by negative number as multiplier is not valid as multiplier must be a positive number. Thus, multiplication by negative number as multiplier is defined as: assemble identical assemblies of unit intervals (+) followed by flipping to assembly of opposite unit intervals (-) or conversely, assemble identical assemblies of opposite unit intervals (-) followed by flipping to assembly of unit intervals (+).

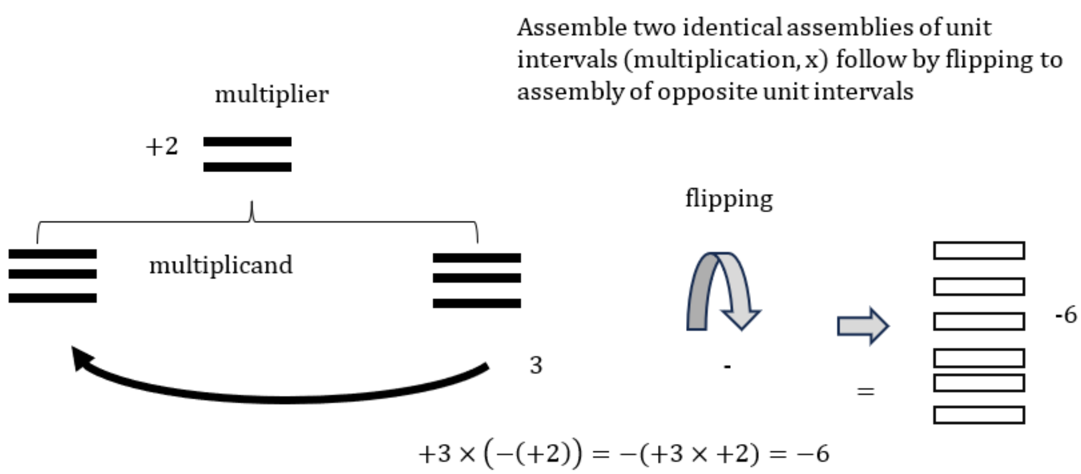


Figure 13: Concept of multiplication by negative number as multiplier by utilizing ‘form’ & ‘equation’, example I

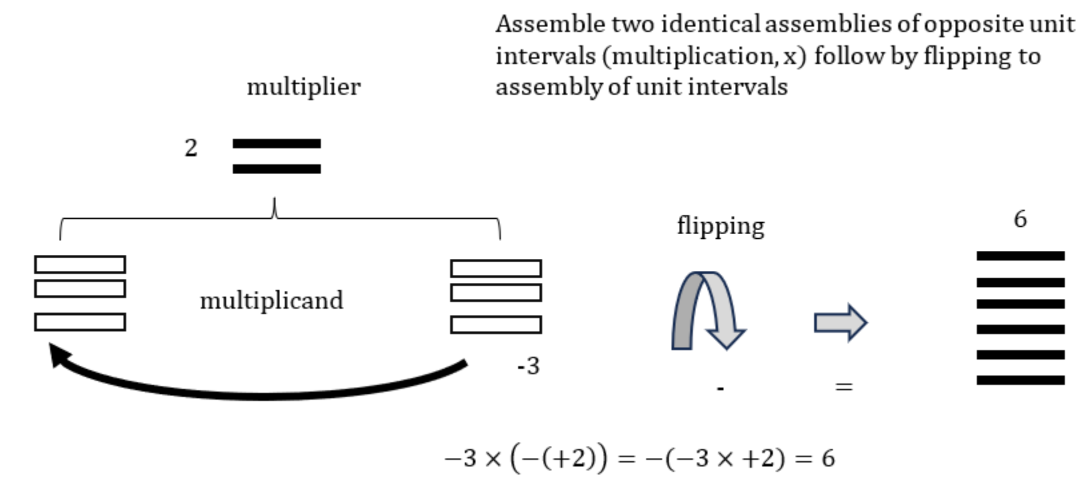


Figure 14: Concept of multiplication by negative number as multiplier by utilizing 'form' & 'equation', example II

Division by negative number

Division by numerator & denominator are not in same sign is not valid as quotient must be in positive number. Thus, division by numerator & denominator are not in same sign is defined as: to find the total defined assembly/the quotient followed by flipping the quotient to negative sign (-).

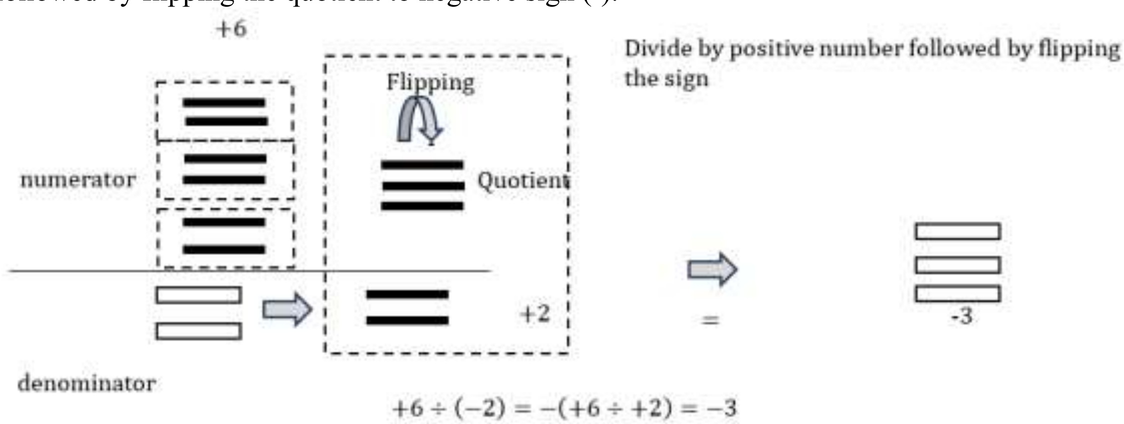


Figure 15: Concept of division with numerator & denominator are not in same sign by utilizing 'form' & 'equation', example I

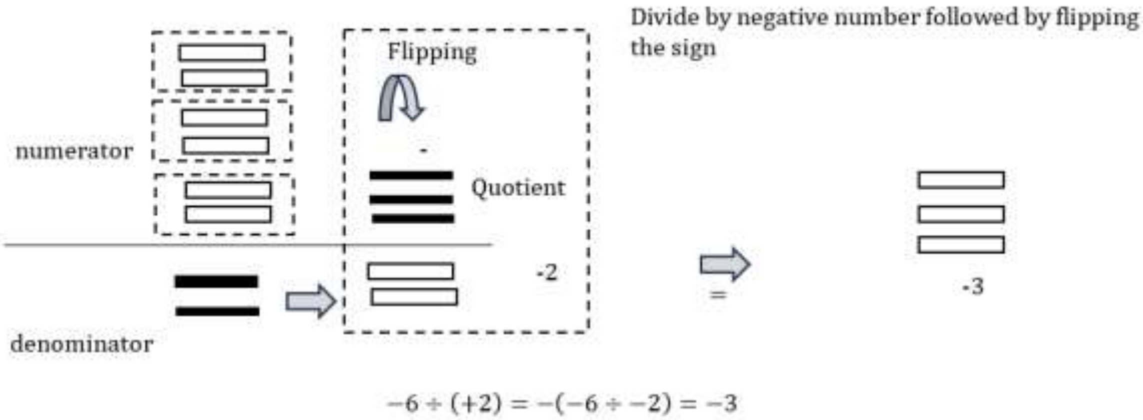
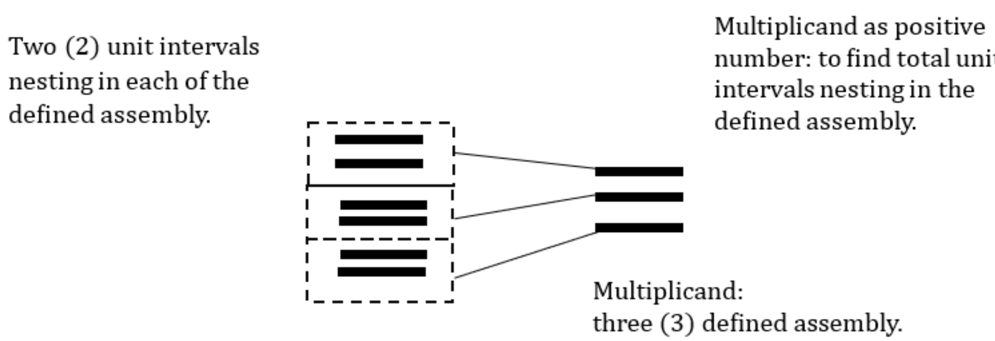


Figure 16: Concept of division with numerator & denominator are not in same sign by utilizing 'form' & 'equation', example II

Commutative property of multiplication

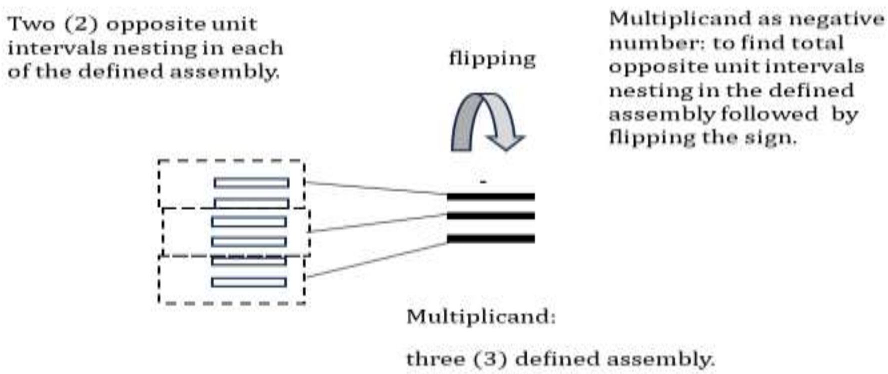
Commutative of multiplication $a \times b = b \times a$ simply is the consequence of defined concept of multiplication in another way, like the two sides of the same coin.

By using the 'nesting' concept, multiplicand must be a positive number; thus, multiplication by multiplicand as positive number is defined as to find total unit intervals (+) from known total defined assembly of unit intervals or to find total opposite unit intervals (-) from known total defined assembly of opposite unit intervals. Multiplication by multiplicand as negative number is defined as: to find total unit intervals (+) from known total defined assembly of unit intervals followed by flipping the sign to negative(-) or to find total opposite unit intervals (-) from known total defined assembly of opposite unit intervals followed by flipping the sign to positive(+).



$$3 \times (+2) = 6$$

Figure 17: Define multiplication by concept of "nesting',example I

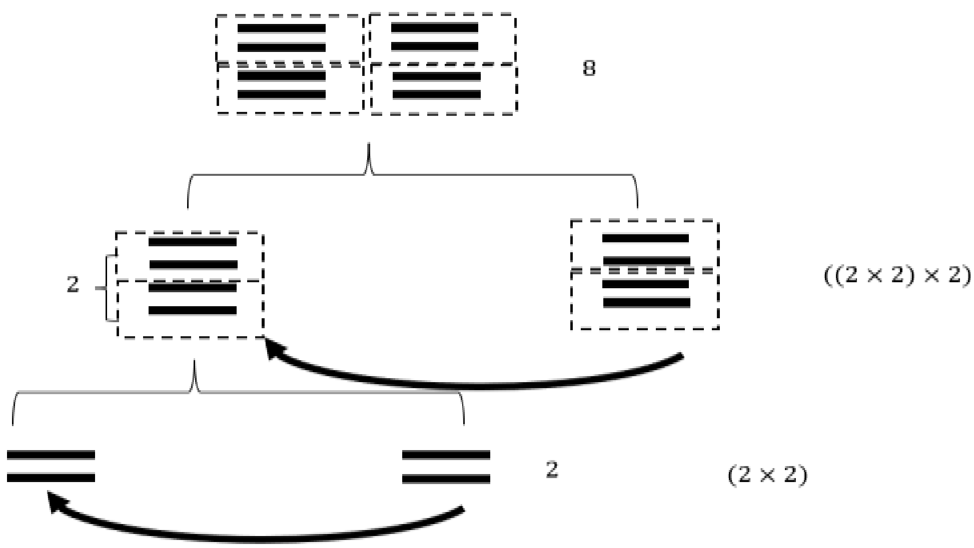


$$-3 \times (-2) = -(3 \times (-2)) = 6$$

Figure 18: Define multiplication by concept of "nesting' , example II

Power Numbers

By refer to figure 1, a new rule was discovered for the relationship between unit interval(s) or numbers. The multilevel multiplication of a number by itself as shown in figure 19 is called power number symbolized as $base^{exponent}$.



$$((2 \times 2) \times 2) = 8$$

$$2^2 \times 2 = 8$$

In which $base^{exponent}$ symbolize multiplying 2(base) by 2 times(exponent)

$$2^2 \times 2^1 = 8 \qquad (1)$$

$$2^3 = 8$$

Figure 19: Concept of power number by utilizing form' & 'equation'

Hence, from (1), $2^1 = 2$ had been proved.
How to prove $2^0 = 1$? The definition of 2^0 is commonly known as the result of extend the concept of division as shown below:

$$\frac{2^2}{2^1} = \frac{2 \times 2}{2} = 2^1 = 2^{2-1} \qquad (2)$$

$$\frac{2^2}{2^2} = 1 = 2^{2-2} \quad \text{Based on reasoning of (2)} \quad = 2^0 = 1$$

Concepts of Variable, Limit, Space & Origin

Concepts of variable, limit, space & origin shall be defined by extending the perspectives and concepts developed till now under the said framework summarized in figure 20 shown below [1].

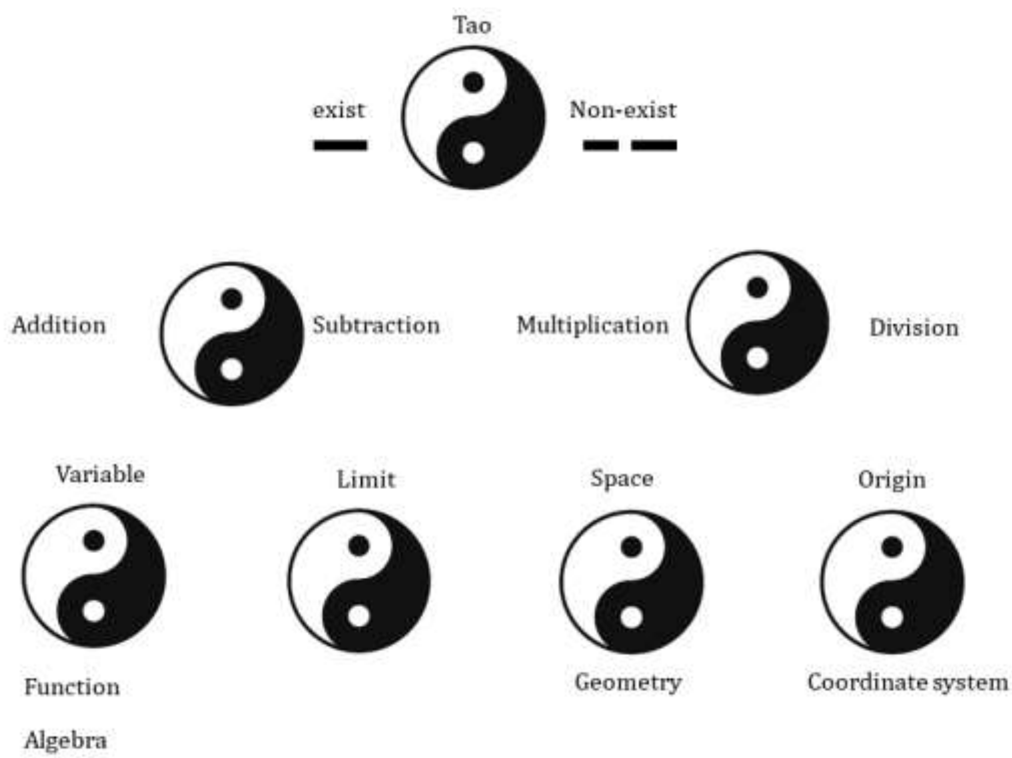


Figure 20: The perspectives of unit interval, assembly of unit intervals, addition, subtraction, multiplication, and division are extended to have perspectives of variable, limit, space and origin.[1]

Variable

This section shall focus on expression of concepts in "equation" rather than "form" as "equation" is more easily be comprehended and represented.

Variable is defined as the number shown in figure 1, can be represented by a symbol (typically a letter); where the value can be changed within a defined interval of numbers.

By introducing the concept of variable, an equation can be evolved as below:

$$2 + 3 = 6 \quad (1)$$

$$X+3=6 \quad (2)$$

X in equation (2) is a type of variable called unknown, in which the equation has specific value(s) to solve as solution. The type of equation (2) is classified as algebraic equation; the related operations is arithmetic (addition, subtraction, multiplication, division, root & exponent)

Continuity of numbers

Peano Axioms on natural numbers and continuity of numbers are defined and proved by the arrays of assembly of unit intervals in figure 1: the "form" of the arrays is not change, only the "length" of unit interval changes when the corresponding number changes:

(i) number begins as 1, followed by 2,3,4…….

OR

(ii) number begins as 0.01, followed by 0.02, 0.03,0.04…

OR

(iii) any other infinitesimally small number ones defined.

The choice is solely arbitrary. In all three cases, the following number is the immediate next value of the beginning number by referring to the "form" of the said arrays.

Function

The function described how a variable y changes by varying the other variable x by expressing the relationship between them. Leibniz formalized the concept of function by the definition: if a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, we say that y is a function of x [2]. The functions may, but not all expressed by 'equation'. Later, Leonhard Euler conceived the idea of denoting a function by letter [2] :

f is a rule

$f(x)$ is the output

x is the input

Other terms:

Domain: set of all possible value, x

Range: set of all possible output, $f(x)$

<u>Example of a function</u>

$$f(x) = 2x + 2 \quad [-\infty, \infty]$$

Arrays shown in figure 1 could be represented in function

$$f(x) = x \ \ \{1,2,3,4 \ldots.\}$$

OR

Represented in another type of function called sequence,

$$f(n) = 1 + (n - 1)1 \ \{1,2,3,4 \ldots.\}$$

Continuity of function

Prerequisite of the continuity of function, $f(x)$ is the continuity of the domain. The continuity of function must satisfy the definition of continuity of numbers in section 4.1.1.

Addition, subtraction, multiplication & division of function

Figure 20 shows the proposed framework of mathematics is represented in levels. Concepts of addition, subtraction, multiplication & division defined in level two of figure 20 may be applicable to new items, such as those defined in level three, but subject to the proofing process. The framework provides a method for extending the already defined concepts.

Limit

Like the concept of subtraction is related to the definition of addition, the concept of limit is lay on the concept of function.
The precise definition of two-sided limit by Karl Weierstrass is commonly called "epsilon-delta" definition.

"Let $f(x)$ be defined for all x in some open interval contains the number a,with the possible exception that $f(x)$ need not be defined at a. We will write

$$\lim_{x \to a} f(x) = L$$

if given any number $\epsilon > 0$ we can find a number $\delta > 0$ such that

$$|f(x) - L| < \epsilon \;\; \text{if} \;\; 0 < |x - a| < \delta\text{"} \quad [3]$$

Author aims to explain "epsilon-delta" by incorporating the continuity concept described in section 6.1.1.
For limit to exist:
(i)The delta, δ in input domain is equal to $|x - a|$. It must meet continuity.
(ii)The epsilon,ϵ in output range is equal to $|f(x) - L|$. It must meet continuity, too.
(iii) Relation of epsilon,ϵ and delta, δ is a ratio: $\frac{\varepsilon}{\delta}$
As delta, δ approach infinitesimally small, epsilon,ε must be infinitesimally small too.
The ratio mentioned in (iii) is the slope of tangent line at a if $f(x)$ defined at a

Space

Concept of space is defined as the ability to locate unit interval(s). One-dimensional (1D) Euclid space is defined as an imaginary line with infinite length. All unit interval(s) can only be located on this imaginary line; length is defined as finite envelope of 1D space.

Figure 21: 1D space

Two-dimensional (2D) Euclidean space is defined by three (3) equal-length lines joined at three points forming a geometry object called an equilateral triangle; the lines of the triangle extend the length infinitely to form an imaginary envelope of space called plane. It could locate point, line, triangle, and all shapes that possess the property of area; area is defined as a finite envelope of 2D space.

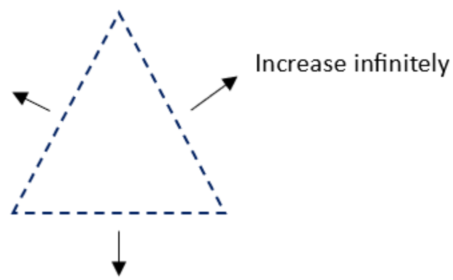


Figure 22: 2D space

Three-dimensional (3D) Euclidean space is defined by each point of an equilateral triangle joined to a common fourth point by using lines to form a regular tetrahedron; all six(6) lines extend the length infinitely to form an imaginary envelope of space. it could locate point, line, triangle, cube, and all shapes possess the property of volume; volume is defined as a finite envelope of 3D space.

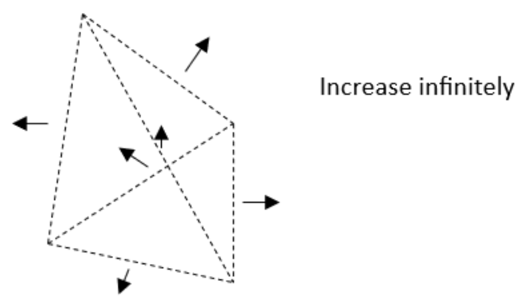


Figure 23: 3D space

6.3.1 Finite space envelope interacts with algebra

Concepts of length, area, and volume could be represented in algebraic function.

$$length, l = l$$
$$area, A = l \times l$$
$$volume, V = l \times l \times l$$

Other geometry concepts

Angle and arc are two unique concepts resulting from a specific locating pattern of unit interval(s) in space.
Angle, θ is defined in Euclidean geometry as the result of the interaction of two (2) lines emanating from the same point. It cannot be written as function of length.

Figure 24: Concept of angle

Arc is a section of a circle defined by radius & angle. Arc is represented in algebraic function by arc length. Arc length is a concept of approximation; it uses unit interval(s) to imitate the profile of arc.

$$arc\ length\ of\ a\ full\ circle, = 2\pi r$$

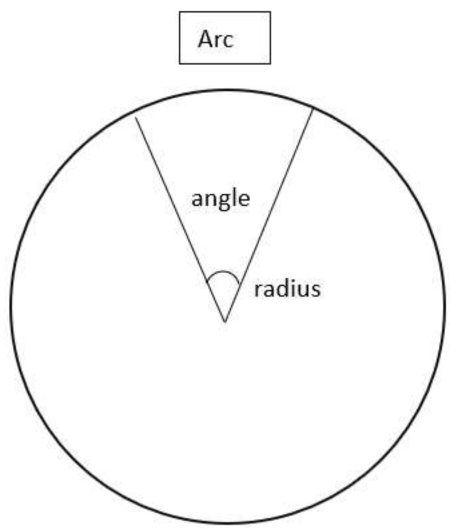


Figure 25: Concept of arc

The constant π till today is recognized as an irrational number, by logical interpretation, it is because the arc length is an approximation function.

Trigonometry

Three line segments forming a right angle-triangle, the relationship between the sides length & angle are functions, can be represented by algebraic equation.

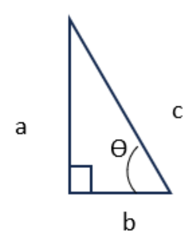


Figure 26: Right angle-triangle

$$\sin\theta = \frac{a}{c}$$
$$\cos\theta = \frac{b}{c}$$

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$$\tan\theta = \frac{a}{b}$$

Vector

Unit interval in Euclidean space has the property of connecting two selected points in space, named as a vector. By defining the starting point & the end point, besides having the property of length, such unit interval is claimed to have a directional property. The notation of the vector connecting point A to endpoint B is $\underset{AB}{\rightarrow}$

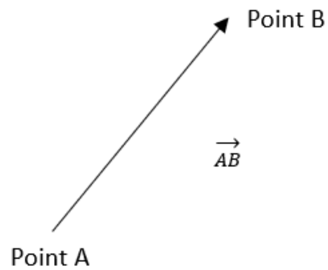

Figure 27: Concept of vector

Property of vector

$\underset{AB}{\rightarrow}$ is equal to $\underset{DE}{\rightarrow}$ if they are equal in length and meet the Euclid parallel postulate.

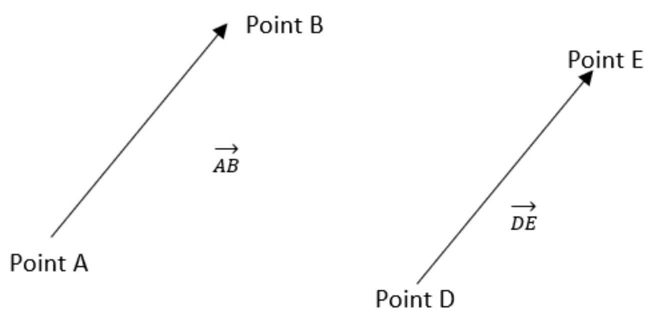

Figure 28: Equal vectors

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Combination of vectors

The endpoint of $\underset{AB}{\rightarrow}$ connects to the starting point of $\underset{BC}{\rightarrow}$ shall produce a new vector $\underset{AC}{\rightarrow}$ with a starting point at A and the endpoint at C; such reasoning is valid because the combination of $\underset{AB}{\rightarrow}$ & $\underset{BC}{\rightarrow}$ is connecting the same two points of the vector $\underset{AC}{\rightarrow}$.

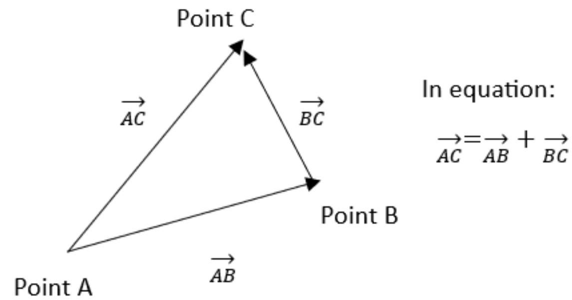

Figure 29: Combination of vectors

Origin

Like concept of limit lies on the concept of function, concept of origin lies on the concept of space. Descartes introduced the concept of origin when he developed the Cartesian coordinate system. Cartesian coordinate system in 2D space utilizes two axis lines intersecting at 90 degrees; the order pairs numbers from the two axis lines serve the purpose to distinguish points at 2D space. The origin is the intercepting point of the two axis lines.

More Interaction between concepts of variable, limit, space, origin

Graph

The algebra function equation introduced in section 6.1.2 may be represented in graph. The graph is a unique form of coordinate system, in which the X-axis corresponds to the input domain, and Y-axis corresponds to the output range. A continuous domain may result continuous function, $f(x)$, those output range marks as points, then join to form a chain of unit intervals called a curve. Thus, curve in space as geometric object can be represented as function equation.

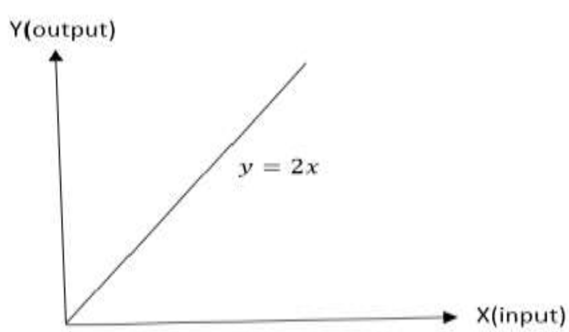

Figure 30: Curve represented as an algebraic equation

Calculus

Calculus is the branch of mathematics that consists of differential calculus & integral calculus. Modern calculus was discovered by Newton & Leibniz, but today calculus uses Leibniz's notation.

Differential Calculus

The concept of differential calculus is the further extension of the concept of limit, defining the instantaneous rate of change or derivative of a function. In short, derivative is to find the limit of the ratio 'epsilon-delta',$\frac{\varepsilon}{\delta}$ described in section 6.2 with below additional conditions:

(i) the function is defined at the point $x = a$

(ii) two-sided limits exist and are equal, forming a single tangent line at the point $x = a$.

$$\text{Derivative of function } f \text{ at point } a,\ f'(a) = \lim_{\delta \to 0} \frac{f(a+\delta) - f(a)}{\delta}$$

CONCLUSION

This paper has been developed and defined the concepts of mathematics based on perspectives found in the framework proposed by Sau Kong.[1]. The areas or concepts that been developed or been prove are : concepts of addition, subtraction, multiplication & division with the help of "form" & 'equation'; what is negative number with the help of concept of opposite unit interval; what is multiplication & division by negative number; commutative properties of addition & multiplication; what is continues number with the help of arrays of assembly of unit intervals; refine definition of 'epsilon-delta' limit with the help of newly defined continuity concept; refine definition of 1D, 2D & 3D Euclid space; refined definition of vector & combination of vectors; explored interaction between concepts of variable, limit, space & origin by the concepts of graph and derivative.

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