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Off Grid Initial Value Solver for First Order Ordinary Differential Equations in Block Form Using Chebyshev Polynomial as Basis Function

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ABSTRACT: *The importance of numerical solution to differential equations cannot be overemphasized. It has been observed that the analytic method of solution to some differential equation often became laborious and practicably impossible. In order to circumvent this problem, then the introduction of an approximate solution became inevitable. This paper focuses on the derivation and application of an appropriate continuous linear multistep method in a block form in solving first order ordinary differential equations by collocating at some selected off grid points and interpolating at only one grid point. To achieve this, Chebyshev polynomial is used as basis function. Some basic properties of Multistep methods were critically examined such as order, consistency, zero stability and region of absolute stability and the level of accuracy of the method was equally compared with an existing method and was found out to performs better that the method compared with.*

Keywords: Collocation, Interpolation, Chebyshev polynomial, Consistency, Multistep.

INTRODUCTION

This paper focuses on the derivation and application of Linear Multistep Method in Block form in solving the differential equations of the form

$$y'(x) = f(x, y); y(x_0) = y_0 \quad (1)$$

where f is continuous and satisfies Lipchitz's condition that guarantees the uniqueness and existence of solution to differential equation [11]. Many authors have worked extensively on formulation an appropriate Multistep method in solving equation (1) using different approaches and method which include Power Series Method. Some of these authors include [2,7,8,12, 13, 14]. However, some used Chebyshev polynomial as basis function in deriving Multistep methods, such as [1, 3].

Some of these methods are in the form of predictor – corrector method and this method has a major setback in which it depends on additional information for its starting point. Due to this set back, then researchers came up with another method that will not depend on any additional information for the starting value. The method that will be self-starting, hence came the introduction of Block Linear Multistep method. The Block method has the properties of Runge – Kutta method which is self- starting unlike the predictor – corrector method.

This paper examines the paper of [10] which proposes the Block Linear Multistep method in solving equation (1) using power series as basis function by interpolating at only one grid point and collocating at several off-grid points on the interval $0\ \left(\frac{1}{12}\right)\frac{1}{2}$. However, this paper presents the derivation of Block Linear Multistep method to solve equation (1) by using Chebyshev polynomial as basis function. Interpolation was done at only one point and collocation was done at several off-grid points in the interval of $0\ \left(\frac{1}{14}\right)\frac{1}{2}$. The Chebyshev polynomial was used as basis function because of the fact that it is the most accurate polynomial among other monomials in the interval [-1, 1] [9].

METHODOLOGY

The derivation of continuous Block Liner Multistep method for solving first order ordinary differential equations is hereby presented. To achieve this, Chebyshev polynomial is hereby adopted as basis function. Chebyshev polynomial have been used extensively to develop Linear Multistep method in solving differential equations of different degrees [4,5,6].

The Chebyshev polynomial, according to [9] is defined as

$$T_r\,(x) = cos\left[r\,cos^{-1}\left\{\frac{2x-(b+a)}{b-a}\right\}\right] \cong \sum_{n=0}^{r} C_n^r\,x^n \qquad (2)$$

Satisfying the recurrence relation

$$T_{r+1}(x) = 2\left(\frac{2x-(b+a)}{b-a}\right)T_r(x) - T_{r-1}(x) \text{ for } r \geq 1 \qquad (3)$$

From equation (3),

$$T_0(x) = 1; \qquad T_1(x) = \frac{2x-(b+a)}{b-a} \qquad (4)$$

In this presentation, let

$$y_{(x)} = \sum_{r=0}^{k} a_r\,T_r\,(x) \qquad (5)$$

where $T_r(x)$ is the Chebyshev polynomial which can be generated recursively by

$$T_{r+1}(x) = 2x\,T_r(x) - T_{r-1}(x); r = 1\,(1)k \qquad (6)$$

where

$$T_0(x) = 1\,, \qquad T_1(x) = x$$

Here let $k = 8$ in (5) based on the number of interpolation and collocation points, that is

$$y_{(x)} = a_0\,T_0\,(x) + a_1\,T_1(x) + a_2\,T_2\,(x) + a_3\,T_3(x) + a_4\,T_4(x) + a_5\,T_5\,(x) + a_6\,T_6\,(x) + a_7\,T_7\,(x) + a_8\,T_8\,(x) \qquad (7)$$

substituting values of $T_r(x)$ where $r = 0\,(1)8$ then equation (7) becomes

$$y_{(x)} = a_0 + a_1 x + a_2(2x^2 - 1) + a_3(4x^3 - 3x) + a_4(8x^4 - 8x^2 + 1) + a_5(16x^5 - 20x^3 + 5x) + a_6(32x^6 - 48x^4 + 18x^2 - 1) + a_7(64x^7 - 112x^5 + 56x^3 - 7x) + a_8(128x^8 - 256x^6 + 160x^4 - 32x^2 + 1) \quad (8)$$

By letting $T_r(x)$ $in\ equation\ (5)\quad be\ \ T_r\left(\frac{2x-b-a}{b-a}\right)$ yields

$$y_{(x)} = \sum_{r=0}^{8} a_r\, T_r\left(\frac{2x-b-a}{b-a}\right) \quad (9)$$

Evaluating $\frac{2x-b-a}{b-a}$ in the interval $0 \leq x \leq \frac{1}{2}$, hence $\frac{2x-b-a}{b-a}$ becomes $\frac{4x-4kh-h}{h}$. On substituting x with $\frac{4x-4kh-h}{h}$ in equation (8) leads to

$$\begin{aligned} y(x) = {} & a_0 + a_1\left[\frac{4x-4kh-h}{h}\right] + a_2\left[2\left(\frac{4x-4kh-h}{h}\right)^2 - 1\right] + a_3\left[4\left(\frac{4x-4kh-h}{h}\right)^3 - 3\left(\frac{4x-4kh-h}{h}\right)\right] + a_4\left[8\left(\frac{4x-4kh-h}{h}\right)^4 - 8\left(\frac{4x-4kh-h}{h}\right)^2 + 1\right] \\ & + a_5\left[16\left(\frac{4x-4kh-h}{h}\right)^5 - 20\left(\frac{4x-4kh-h}{h}\right)^3 + 5\left(\frac{4x-4kh-h}{h}\right)\right] \\ & + a_6\left[32\left(\frac{4x-4kh-h}{h}\right)^6 - 48\left(\frac{4x-4kh-h}{h}\right)^4 + 18\left(\frac{4x-4kh-h}{h}\right)^2 - 1\right] \\ & + a_7\left[64\left(\frac{4x-4kh-h}{h}\right)^7 - 112\left(\frac{4x-4kh-h}{h}\right)^5 + 56\left(\frac{4x-4kh-h}{h}\right)^3 - 7\left(\frac{4x-4kh-h}{h}\right)\right] \\ & + a_8\left[128\left(\frac{4x-4kh-h}{h}\right)^8 - 256\left(\frac{4x-4kh-h}{h}\right)^6 + 160\left(\frac{4x-4kh-h}{h}\right)^4 - 32\left(\frac{4x-4kh-h}{h}\right)^2 + 1\right] \end{aligned} \quad (10)$$

Differentiating equation (10) once yields.

$$y'(x) = \frac{4a_1}{h} + a_2\left[\frac{16}{h}\left(\frac{4x-4kh-h}{h}\right)\right] + a_3\left[\frac{48}{h}\left(\frac{4x-4kh-h}{h}\right)^2 - \frac{12}{h}\right]$$

$$+ a_4\left[\frac{128}{h}\left(\frac{4x-4kh-h}{h}\right)^3 - \frac{64}{h}\left(\frac{4x-4\,kh-h}{h}\right)\right]$$

$$+a_5\left[\frac{320}{h}\left(\frac{4x-4kh-h}{h}\right)^4 - \frac{240}{8}\left(\frac{4x-4kh-h}{h}\right)^2 + \frac{20}{h}\right]$$

$$+ a_6\left[\frac{768}{h}\left(\frac{4x-4kh-h}{h}\right)^5 - \frac{768}{h}\left(\frac{4x-4kh-h}{h}\right)^3 + \frac{144}{h}\left(\frac{4x-4kh-h}{h}\right)\right]$$

$$+ a_7\left[\frac{1792}{h}\left(\frac{4x-4kh-h}{h}\right)^6 - \frac{2240}{h}\left(\frac{4x-4kh-h}{h}\right)^4 + \frac{672}{h}\left(\frac{4x-4kh-h}{h}\right)^2 - \frac{28}{h}\right]$$

$$+ a_8\left[\frac{4096}{h}\left(\frac{4x-4kh-h}{h}\right)^7 - \frac{6144}{h}\left(\frac{4x-4kh-h}{h}\right)^5 + \frac{2560}{h}\left(\frac{4x-4kh-h}{h}\right)^3 - \frac{256}{h}\left(\frac{4x-4kh-h}{h}\right)\right] \quad (11)$$

Interpolating equation (10) at $x = x_k$ and collocating equation (11) in the interval $x_k \le x \le x_{k+n}$ with $n = \frac{1}{2}$ in the regular interval of $0\left(\frac{1}{14}\right)\frac{1}{2}$ yield the set of algebraic equations below

$$a_0 - a_1 + a_2 - a_3 + a_4 - a_5 + a_6 - a_7 + a_8 = y_{n.}$$

$$4a_1 - 16\,a_2 + 36a_3 - 64a_4 + 100a_5 - 144a_6 + 196a_7 - 256a_8 = hf_n$$

$$3294172a_1 - 9411920a_2 + 10285884a_3 - 768320a_4 - 15771140a_5 +28188720a_6 - 24901604a_7 + 3070720a_8 = 823543hf_{n+\frac{1}{14}}.$$

$$3294172a_1 - 5647152a_2 - 2621892a_3 + 14290752a_4 - 10941700a_5$$

$$-10181808a_6 + 25500188a_7 - 114040432a_8 = 823543hf_{n+\frac{2}{14}}.$$

$$3294172a_1 - 1882384a_2 - 9075780a_3 + 7222208a_4 + 12546940a_5$$
$$-15135120a_6 - 12520676a_7 + 24268544a_8 = 823543hf_{n+\frac{3}{14}}.$$

$$3294172a_1 + 1882384a_2 - 9075780a_3 - 7222208a_4 + 12546940a_5$$
$$+15135120a_6 + 12520676a_7 - 24268544a_8 = 823543hf_{n+\frac{4}{12}}.$$

$$3294172a_1 + 5647152a_2 - 2621892a_3 - 14290752a_4 - 10941700a_5$$
$$+10181808a_6 + 25500188a_7 + 114040432a_8 = 823543hf_{n+\frac{5}{14}}.$$

$$3294172a_1 + 9411920a_2 + 10285884a_3 + 768320a_4 - 15771140a_5$$
$$-28188720a_6 - 24901604a_7 - 3070720a_8 = 823543hf_{n+\frac{6}{14}}.$$

$$4a_1 + 16a_2 + 36a_3 + 64a_4 + 100a_5 + 144a_6 + 196a_7 + 256a_8 = hf_{n+\frac{7}{14}}.$$

Solving the system of linear equation above using Maple software yields the following values of $a_i, i = 0(1)8$

$$a_0 = y_n + \frac{4293403}{226492416}hf_n + \frac{32700101}{377487360}hf_{n+\frac{1}{14}} + \frac{190267}{125829120}hf_{n+\frac{1}{7}}$$
$$+\frac{94976651}{1132462080}hf_{n+\frac{3}{14}} + \frac{988967}{377487360}hf_{n+\frac{2}{7}} + \frac{4626629}{125829120}hf_{n+\frac{5}{14}}$$
$$+\frac{19110833}{1132462080}hf_{n+\frac{3}{7}} + \frac{1047251}{377487360}hf_{n+\frac{1}{2}}$$

$$a_1 = \frac{104621}{23592960}hf_n + \frac{182035}{4718592}hf_{n+\frac{1}{14}} + \frac{87661}{2621440}hf_{n+\frac{1}{7}} + \frac{229075}{4718592}hf_{n+\frac{3}{14}}$$
$$+\frac{229057}{4718592}hf_{n+\frac{2}{7}} + \frac{87661}{2621440}hf_{n+\frac{5}{14}} + \frac{182035}{4718592}hf_{n+\frac{3}{7}} + \frac{104621}{23592960}hf_{n+\frac{1}{2}}$$

$$a_2 = -\frac{59683}{15728640}hf_n - \frac{503867}{15728640}hf_{n+\frac{1}{14}} - \frac{14063}{15728640}hf_{n+\frac{1}{7}} - \frac{92267}{3145728}hf_{n+\frac{3}{14}} + \frac{92267}{3145728}hf_{n+\frac{2}{7}} + \frac{14063}{15728640}hf_{n+\frac{5}{14}} + \frac{503867}{15728640}hf_{n+\frac{3}{7}} + \frac{59683}{15728640}hf_{n+\frac{1}{2}}$$

$$a_3 = \frac{253477}{70778880}hf_n + \frac{249851}{14155776}hf_{n+\frac{1}{14}} - \frac{27881}{2621440}hf_{n+\frac{1}{7}} - \frac{149989}{14155776}hf_{n+\frac{3}{14}} - \frac{149989}{14155776}hf_{n+\frac{2}{7}} - \frac{27881}{2621440}hf_{n+\frac{5}{14}} + \frac{249851}{14155776}hf_{n+\frac{3}{7}} + \frac{253477}{70778880}hf_{n+\frac{1}{2}}.$$

$$a_4 = -\frac{252791}{94371840}hf_n + \frac{798847}{94371840}hf_{n+\frac{1}{14}} + \frac{754943}{31457280}hf_{n+\frac{1}{7}} - \frac{206143}{18874368}hf_{n+\frac{3}{14}} + \frac{206143}{18874368}hf_{n+\frac{2}{7}} + \frac{754943}{31457280}hf_{n+\frac{5}{14}} + \frac{798847}{94371840}hf_{n+\frac{3}{7}} + \frac{252791}{94371840}hf_{n+\frac{1}{2}}$$

$$a_5 = \frac{16807}{7864320}hf_n - \frac{2401}{2621440}hf_{n+\frac{1}{14}} - \frac{26411}{2621440}hf_{n+\frac{1}{7}} + \frac{69629}{7864320}hf_{n+\frac{3}{14}} + \frac{69629}{7864320}hf_{n+\frac{2}{7}} - \frac{26411}{2621440}hf_{n+\frac{5}{14}} - \frac{2401}{2621440}hf_{n+\frac{3}{7}} + \frac{16807}{7864320}hf_{n+\frac{1}{2}}.$$

$$a_6 = -\frac{184877}{141557760}hf_n + \frac{487403}{141557760}hf_{n+\frac{1}{14}} + \frac{16807}{15728640}hf_{n+\frac{1}{7}} - \frac{319333}{28311552}hf_{n+\frac{3}{14}} + \frac{319333}{28311552}hf_{n+\frac{2}{7}} - \frac{16807}{15728640}hf_{n+\frac{5}{14}} + \frac{487403}{141557760}hf_{n+\frac{3}{7}} + \frac{184877}{141557760}hf_{n+\frac{1}{2}}.$$

$$a_7 = \frac{16807}{23592960} hf_n - \frac{16807}{4718592} hf_{n+\frac{1}{14}} + \frac{16807}{2621440} hf_{n+\frac{1}{7}} - \frac{16807}{4718592} hf_{n+\frac{3}{14}} - \frac{16807}{4718592} hf_{n+\frac{2}{7}} + \frac{16807}{2621440} hf_{n+\frac{5}{14}} - \frac{16807}{4718592} hf_{n+\frac{3}{7}} + \frac{16807}{23592960} hf_{n+\frac{1}{2}}.$$

$$a_8 = \frac{117649}{377487360} hf_n + \frac{823543}{377487360} hf_{n+\frac{1}{14}} - \frac{823543}{125829120} hf_{n+\frac{1}{7}} + \frac{823543}{75497472} hf_{n+\frac{3}{14}} - \frac{823543}{75497472} hf_{n+\frac{2}{7}} + \frac{823543}{125829120} hf_{n+\frac{5}{14}} - \frac{823543}{377487360} hf_{n+\frac{3}{7}} + \frac{117649}{377487360} hf_{n+\frac{1}{2}}.$$

Substituting these values of $a's$ into equation (10) and letting $P = \frac{4x-4kh-h}{h}$ yields the continuous scheme

$$y_{(x)} = y_n + \frac{hf_n}{8847360}\{186381 - 5400P + 2700P^2 + 101528P^3 - 76146P^4 - 403368P^5 + 336140P^6 + 403368P^7 - 352947P^8\} +$$

$$\frac{hf_{n+\frac{1}{14}}}{8847360}\{963781 + 52920P - 37044P^2 - 978040P^3 - 1026942P^4 - 7779240P^5 + 3966452P^6 + 3630312P^7 - 7411887P^8\} +$$

$$\frac{hf_{n+\frac{1}{7}}}{8847360}\{166257 - 264600P + 308700P^2 + 4582872P^3 - 8020026P^4 - 7779240P^5 + 15126300P^6 + 3630312P^7 - 7411887P^8\} +$$

$$\frac{hf_{n+\frac{3}{14}}}{8847360}\{1101177 + 1323000P + 4630500P^2 - 3706360P^3 + 19458390P^4 + 4782792P^5 - 27899620P^6 - 2016840P^7 + 12353145P^8\} +$$

$$\frac{hf_{n+\frac{2}{7}}}{8847360}\{-335993+1323000P+4630500P^2-3706360P^3-19458390P^4+4782792P^5+27899620P^6-2016840P^7-12353145P^8\}+$$

$$\frac{hf_{n+\frac{5}{14}}}{8847360}\{172431-264600P-308700P^2+4582872P^3+8020026P^4-7779240P^5+15126300P^6+3630312P^7+7411887P^8\}+$$

$$\frac{hf_{n+\frac{3}{7}}}{8847360}\{-48069+52920P+37044P^2-978040P^3-1026942P^4+3399816P^5+3966452P^6-2016840P^7-2470629P^8\}+$$

$$\frac{hf_{n+\frac{1}{2}}}{8847360}\{5875-5400P-2700P^2+101528P^3+76146P^4-403368P^5-336140P^6+403368P^7+352947P^8\} \quad (12)$$

Equation (12) is the continuous scheme.

Evaluating equation (12) at various point in the interval $0\left(\frac{1}{14}\right)\frac{1}{2}$ yields the block discrete off grid multistep method.

The discrete equations are

$$y_{n+\frac{1}{14}}=y_n+\frac{751}{34560}hf_n+\frac{139849}{1693440}hf_{n+\frac{1}{14}}-\frac{4511}{62720}hf_{n+\frac{1}{7}}+\frac{123133}{1693440}hf_{n+\frac{3}{14}}-\frac{88547}{1693440}hf_{n+\frac{2}{7}}+\frac{1537}{62720}hf_{n+\frac{5}{14}}-\frac{11351}{1693440}hf_{n+\frac{3}{14}}+\frac{275}{338688}hf_{n+\frac{1}{2}}$$

$$y_{n+\frac{1}{7}}=y_n+\frac{41}{1960}hf_n+\frac{733}{6615}hf_{n+\frac{1}{14}}-\frac{71}{5880}hf_{n+\frac{1}{7}}+\frac{34}{735}hf_{n+\frac{3}{14}}-\frac{1927}{5292}hf_{n+\frac{2}{7}}+\frac{39}{2205}hf_{n+\frac{5}{14}}-\frac{29}{5880}hf_{n+\frac{3}{7}}+\frac{4}{6615}hf_{n+\frac{1}{2}}$$

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$$y_{n+\frac{3}{14}} = y_n + \frac{265}{12544} hf_n + \frac{1359}{12544} hf_{n+\frac{1}{14}} + \frac{1377}{62720} hf_{n+\frac{1}{7}} - \frac{5927}{62720} hf_{n+\frac{3}{14}} - \frac{3033}{62720} hf_{n+\frac{2}{7}} + \frac{1377}{62720} hf_{n+\frac{5}{14}} - \frac{373}{62720} hf_{n+\frac{3}{7}} + \frac{9}{12544} hf_{n+\frac{1}{2}}$$

$$y_{n+\frac{2}{7}} = y_n + \frac{139}{6615} hf_n + \frac{724}{6615} hf_{n+\frac{1}{14}} + \frac{108}{6615} hf_{n+\frac{1}{7}} + \frac{892}{6615} hf_{n+\frac{3}{14}} - \frac{53}{6615} hf_{n+\frac{2}{7}} + \frac{108}{6615} hf_{n+\frac{5}{14}} - \frac{32}{6615} hf_{n+\frac{3}{7}} + \frac{4}{6615} hf_{n+\frac{1}{2}}$$

$$y_{n+\frac{5}{14}} = y_n + \frac{265}{12544} hf_n + \frac{36725}{338688} hf_{n+\frac{1}{14}} + \frac{775}{37632} hf_{n+\frac{1}{7}} + \frac{4625}{37632} hf_{n+\frac{3}{14}} + \frac{13625}{338688} hf_{n+\frac{2}{7}} + \frac{1895}{37632} hf_{n+\frac{5}{14}} - \frac{275}{37632} hf_{n+\frac{3}{7}} + \frac{275}{378688} hf_{n+\frac{1}{2}}$$

$$y_{n+\frac{3}{7}} = y_n + \frac{41}{1960} hf_n + \frac{27}{254} hf_{n+\frac{1}{14}} + \frac{27}{1960} hf_{n+\frac{1}{7}} + \frac{34}{245} hf_{n+\frac{3}{14}} - \frac{27}{1960} hf_{n+\frac{2}{7}} + \frac{438717}{1003520} hf_{n+\frac{5}{14}} + \frac{41}{1960} hf_{n+\frac{3}{7}} + 0hf_{n+\frac{1}{2}}$$

$$y_{n+\frac{1}{2}} = y_n + \frac{751}{34560} hf_n + \frac{3577}{34560} hf_{n+\frac{1}{14}} + \frac{49}{1280} hf_{n+\frac{1}{7}} + \frac{2989}{34560} hf_{n+\frac{3}{14}} + \frac{2989}{34560} hf_{n+\frac{2}{7}} + \frac{49}{1280} hf_{n+\frac{5}{14}} + \frac{3577}{34560} hf_{n+\frac{3}{7}} + \frac{751}{34560} hf_{n+\frac{1}{2}} \qquad (13)$$

The derived method can be written in the form of

$$y(x) = \propto_0 y_n + h\sum_{j=0}^{\frac{1}{2}} \beta_j f_{n+j}$$

which is of the form

$$L\{\,y\,(x);\;h\,\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{14}} \\ y_{n+\frac{2}{14}} \\ y_{n+\frac{3}{14}} \\ y_{n+\frac{4}{14}} \\ y_{n+\frac{5}{14}} \\ y_{n+\frac{6}{14}} \\ y_{n+\frac{7}{14}} \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} [\,y_n\,] -$$

$$h \begin{bmatrix} \frac{751}{34560} & \frac{139849}{1693440} & \frac{-4511}{62720} & \frac{123133}{1693440} & \frac{-88547}{1693440} & \frac{1537}{62720} & \frac{-11351}{1693440} & \frac{275}{338688} \\ \frac{41}{1960} & \frac{733}{6615} & \frac{-71}{5880} & \frac{34}{735} & \frac{-1927}{5292} & \frac{39}{2205} & \frac{-29}{5880} & \frac{4}{6615} \\ \frac{265}{12544} & \frac{1359}{12544} & \frac{1377}{62720} & \frac{5927}{62720} & \frac{-3033}{62720} & \frac{1377}{62720} & \frac{-373}{62720} & \frac{9}{12544} \\ \frac{139}{6615} & \frac{724}{6615} & \frac{108}{6615} & \frac{892}{6615} & \frac{-53}{6615} & \frac{108}{6615} & \frac{-32}{6615} & \frac{4}{6615} \\ \frac{265}{12544} & \frac{36725}{338688} & \frac{775}{37632} & \frac{4625}{37632} & \frac{13625}{338688} & \frac{1895}{37632} & \frac{-275}{37632} & \frac{275}{338688} \\ \frac{41}{1960} & \frac{27}{245} & \frac{27}{1960} & \frac{34}{245} & \frac{27}{1960} & \frac{438717}{1003520} & \frac{41}{1660} & 0 \\ \frac{751}{34560} & \frac{3577}{34560} & \frac{49}{1280} & \frac{2989}{34560} & \frac{2989}{34560} & \frac{49}{1280} & \frac{3577}{34560} & \frac{751}{34560} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{14}} \\ f_{n+\frac{2}{14}} \\ f_{n+\frac{3}{14}} \\ f_{n+\frac{4}{14}} \\ f_{n+\frac{5}{14}} \\ f_{n+\frac{6}{14}} \\ f_{n+\frac{7}{14}} \end{bmatrix} = 0 \qquad (14)$$

Also the above can be written in block form

$$A^{(0)}\, Y_m = ey_n + h^{\mu}\, df\,(y_n) + h^{\mu}\, bF\,(Y_m)$$

In which μ is the order of the differential equation and

$$A^{(0)} = \begin{pmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{pmatrix}, \quad e = \begin{pmatrix} 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1 \end{pmatrix},$$

$$b = \begin{pmatrix} \frac{139849}{1693440} & \frac{-4511}{62720} & \frac{123133}{1693440} & \frac{-88547}{1693440} & \frac{1537}{62720} & \frac{-11351}{1693440} & \frac{275}{338688} \\ \frac{733}{6615} & \frac{-71}{5880} & \frac{34}{735} & \frac{-1927}{5292} & \frac{39}{2205} & \frac{-29}{5880} & \frac{4}{6615} \\ \frac{1359}{12544} & \frac{1377}{62720} & \frac{5927}{62720} & \frac{-3033}{62720} & \frac{1377}{62720} & \frac{-373}{62720} & \frac{9}{12544} \\ \frac{724}{6615} & \frac{108}{6615} & \frac{892}{6615} & \frac{-53}{6615} & \frac{108}{6615} & \frac{-32}{6615} & \frac{4}{6615} \\ \frac{36725}{338688} & \frac{775}{37632} & \frac{4625}{37632} & \frac{13625}{338688} & \frac{1895}{37632} & \frac{-275}{37632} & \frac{275}{338688} \\ \frac{27}{245} & \frac{27}{1960} & \frac{34}{245} & \frac{27}{1960} & \frac{438717}{1003520} & \frac{41}{1660} & 0 \\ \frac{3577}{34560} & \frac{49}{1280} & \frac{2989}{34560} & \frac{2989}{34560} & \frac{49}{1280} & \frac{3577}{34560} & \frac{751}{34560} \end{pmatrix}$$

$$d = \left[\frac{751}{34560}, \frac{41}{1960}, \frac{265}{12544}, \frac{139}{6615}, \frac{265}{12544}, \frac{41}{1960}, \frac{751}{34560}\right]^T$$

and

$$F(Y_m) = \left[y_{n+\frac{1}{14}},\, y_{n+\frac{2}{14}},\, y_{n+\frac{3}{14}},\, y_{n+\frac{4}{14}},\, y_{n+\frac{5}{14}},\, y_{n+\frac{6}{14}},\, y_{n+\frac{7}{14}}\right]^T$$

ANALYSIS OF THE DERIVED METHOD.

In this section, the analysis of the derived method shall be critically examined. Such analysis includes the order and error constant of the method, zero stability and region of absolute stability of the method.

For any Linear Multistep method to converge, it must be consistence and zero stable. Let the linear operator $L\{\, y\,(x);\; h\,\}$ associated with the block scheme derived be defined as

$$L\{\, y\,(x);\; h\,\} = A^{(0)}\, Y_m - e y_n - h^{\mu} d f\,(y_n) - h^{\mu} bF\,(\,Y_m) \qquad (15)$$

Expanding equation (15) using Taylor series and comparing the coefficients of h gives

$$L\{\, y\,(x);\; h\,\} = C_0\, y(x) + C_1\, h\, y'(x) + C_2\, h^2\, y''(x) + \ldots + C_P\, h^P\, y^P(x)$$

$$+ C_{P+1}\, h^{P+1}\, y^{P+1}\,(x) + \cdots \qquad (16)$$

Definition. The linear operator L and associated derived continuous linear multistep method (14) are said to be of order $P \; if \; C_0 = C_1 = C_2 = \cdots = C_P = 0 \; and \; C_{P+1} \neq 0.$ C_{P+1} is called the error constant and the local truncation error is given by

$$t_{n+k} = C_{P+1}\, h^{(P+1)}\, y^{(P+1)}(x_n) + 0(h^{P+2}) \quad [10, 11,13].$$

Expanding (16) in Taylor series expansion gives

$$\begin{pmatrix}
\sum_{j=0}^{\infty}\frac{\left(\frac{1}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{751h}{34560}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\,y_n^{j+1}\left\{\frac{139849}{1693440}\left(\frac{1}{14}\right)^j + \frac{-4511}{62720}\left(\frac{2}{14}\right)^j + \frac{123133}{1693440}\left(\frac{3}{14}\right)^j + \cdots + \frac{275}{338688}\left(\frac{7}{14}\right)^j\right\} \\
\sum_{j=0}^{\infty}\frac{\left(\frac{2}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{41h}{1960}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{733}{6615}\left(\frac{1}{14}\right)^j - \frac{71}{5880}\left(\frac{2}{14}\right)^j + \frac{34}{735}\left(\frac{3}{14}\right)^j + \cdots + \frac{4}{6615}\left(\frac{7}{14}\right)^j\right\} \\
\sum_{j=0}^{\infty}\frac{\left(\frac{3}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{265\,h}{12544}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\,y_n^{j+1}\left\{\frac{1359}{12544}\left(\frac{1}{14}\right)^j - \frac{1377}{62720}\left(\frac{2}{14}\right)^j + \frac{5927}{62720}\left(\frac{3}{14}\right)^j + \cdots + \frac{9}{12544}\left(\frac{7}{14}\right)^j\right\} \\
\sum_{j=0}^{\infty}\frac{\left(\frac{4}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{139h}{6615}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{724}{6615}\left(\frac{1}{14}\right)^j + \frac{108}{6615}\left(\frac{2}{14}\right)^j + \frac{892}{6615}\left(\frac{3}{14}\right)^j + \cdots + \frac{4}{6615}\left(\frac{7}{14}\right)^j\right\} \\
\sum_{j=0}^{\infty}\frac{\left(\frac{5}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{265\,h}{12544}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\,y_n^{j+1}\left\{\frac{36725}{338688}\left(\frac{1}{14}\right)^j + \frac{775}{37632}\left(\frac{2}{14}\right)^j + \frac{4625}{37632}\left(\frac{3}{14}\right)^j + \cdots + \frac{275}{338688}\left(\frac{7}{14}\right)^j\right\} \\
\sum_{j=0}^{\infty}\frac{\left(\frac{6}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{41h}{1960}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{27}{245}\left(\frac{1}{14}\right)^j + \frac{27}{1960}\left(\frac{2}{14}\right)^j + \frac{34}{245}\left(\frac{3}{14}\right)^j + \cdots + \frac{41}{1660}\left(\frac{6}{14}\right)^j\right\} \\
\sum_{j=0}^{\infty}\frac{\left(\frac{7}{14}\right)^j}{j!}\,y_n^j - y_n - \frac{751\,h}{34560}\,y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}\left\{\frac{3577}{34560}\left(\frac{1}{14}\right)^j + \frac{49}{1280}\left(\frac{2}{14}\right)^j + \frac{2989}{34560}\left(\frac{3}{14}\right)^j + \cdots + \frac{751}{34560}\left(\frac{7}{14}\right)^j\right\}
\end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

Equating coefficients of the Taylor series expansion in equation (17) to zero yield

$$C_0 = C_1 = C_2 = C_3 = \cdots = C_8 = 0$$

The derived method yields a uniform order 8 with error constant

$$C_9 = [-1.56\,(-05), 3.24\,(-07), 1.39\,(-07), 4.21\,(-08), 4.67\,(-08), (-2.14\,(-08), 1.46\,(-08)]$$

Since the derived method is of order > 1, then it is consistent and has ability to converge.

ZERO STABILITY

Definition: The derived block method is said to be zero stable if the roots Z_s , $s = 1, 2 \ldots k$ of the characteristic polynomial $\rho(z)$ defined by $\rho(z)$ = det $(z\, A^0 - E)$ satisfies $|z_s| \leq 1$ and every root satisfying $|z_s| \leq 1$ has multiplicity not exceeding the order of the differential equation. Moreover, as $h \to 0, \rho(z) = z^{r-\mu}\,(z-1)^{\mu}$ where μ is the order of the differential equation, r is the order of the matrix A^0 and E [7].

For our method

$$\rho(z) \quad = \left| z\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \right|,$$

Therefore

$$P(z) = z^6\,(z-1) = 0$$

$\Rightarrow \quad z^6 = 0 \quad 0r \;\; z - 1 = 0$

$\therefore \quad z_1 = z_2 = z_3 = z_4 = z_5 = z_6 = 0 \;\; and \; z_7 = 1$

Hence the derived method is zero stable.

REGION OF ABSOLUTE STABILITY

Definition: The block integrator (3.37) is said to be absolutely stable if for a given h, all the roots z_s of the characteristic polynomial $\pi\left(z, \overline{h}\right) = \rho(z) + \overline{h}\,\sigma(z) = 0 \; satisfies \;\; z_s < 1,$

$$s = 1\,(1) n \;\; where \;\; \overline{h} = \lambda\, h \; and \; \lambda = \frac{\partial f}{\partial y}$$

Here we adopt the boundary locus method for the region of absolute stability of the derived block method by substituting the test equation $y' = -\lambda y$ into the derived block method leads to the recurrence equation

$$y^{i+1} = M(z)y^i, \; z = \lambda h, \quad i = 1(1)\mu - 1$$

and the stability function is given by

$$M(z) = B_2 + Z\, A_2\, (1 - ZA_1\,)^{-B_1}$$

where the stability polynomial of our method is

$$\rho(\lambda, z) = \det\,(\lambda I - M(z))$$

where A_1 , A_2, B_1, and B_2 are obtained from the coefficient of collocation points and interpolation points respectively.

Therefore the region of absolute stability of our method can be found using

$$\overline{h}\,(r) = -\left(\frac{A^{(0)}\,Y_m\,(r) - ey_n\,(r)}{dfy_n(r) - bF\,Y_m\,(r)}\right) \quad (18)$$

Rewriting equation (18) in trigonometric ratio nothing that $r = e^{i\theta}$, then the equation becomes

$$\overline{h}\left(e^{i\,\theta}\right) = -\left\{\frac{A^{(0)}\,Y_m\left(e^{i\theta}\right) - ey_n\left(e^{i\theta}\right)}{dfy_n\left(e^{i\theta}\right) - bF\,Y_m\left(e^{i\theta}\right)}\right\} \quad (19)$$

Simplifying (3.40) using MATLAB and plotting the graph gives the figure bello

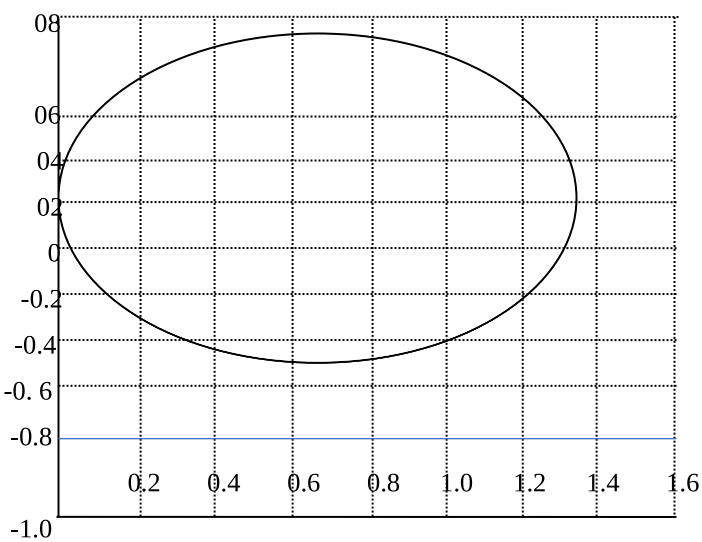

Figure 1: *Region of absolute stability of our method.*

NUMERICAL EXPERIMENTATION

Here the derived Block method shall be applied to some differential equations and compare the output with the analytical solution so as to see the level of convergency of the new method. Although the method presented by [10] is of order 6, while this new method is of order 8, the new method will perform better that [10] method because the higher the order of a Linear Multistep method, the better the result is.

Aat this point, the following numerical experiment shall be considered in which the output shall be compared with the analytical solution so as to see the level of convergence of the approximate solution to the computed output.

Illustration I:

Solve the differential equation

$y'(x) + ky = a \sin bx \ ; y(0) = 1$ where $a, b, and\ k$ are arbitrary constant

The analytical solution is

$$y(x) = \left(\frac{a}{k^2 + b^2}\right)(k \sin bx - b \cos bx\ ) + \left(\frac{k^2 + b^2 + ab}{k^2 + b^2}\right) e^{-kx}$$

By letting $a = b = k = 1$, then

$$y(x) = \frac{1}{2}(\sin x - \cos x + 3e^{-x})$$

Table I below shows the output to Illustration I.

TABLE 1: The Numerical Computation Results to Illustration I

x	y- Exact	y- Computed	Absolute Error
0.0	1.000000000000000	0.999838879857878	1.611201421E – 4
0.1	0.909670752738341	0.909670752739023	6.820 E – 13
0.2	0.837397506093883	0.837397506095809	1.929 E – 12
0.3	0.781319189790444	0.781319189790761	3.170 E – 13
0.4	0.739658743206342	0.739658743206977	6.350 E – 13
0.5	0.710717477925865	0.710717477925542	3.230 E – 13
0.6	0.692870883383718	0.692870883383921	2.030 E – 13
0.7	0.684565705663716	0.684565705663911	2.010 E – 13
0.8	0.684318136952011	0.684318136952205	1.940 E – 13
0.9	0.690712960289308	0.690712960287092	2.216 E – 12
1.0	0.702403501227042	0.702403501220104	6.938 E – 12

Illustration II

The differential equation

$$v(t) = -(av + bt),$$

where a and b are arbitrary constants represents equation of motion of a particle moving in a resisting medium. Determine the motion of the particle at any point in time given that $V(0) = u$. The exact solution is

$$v(t) = \frac{b-abt}{a^2} + \left(\frac{a^2u-b}{a^2}\right)e^{-at}.$$

Let $a = 2, b = u = 1$, then

$$v(t) = \frac{1}{4}(1 - 2t + 3e^{-2t})$$

Table 2 below shows the numerical computation to Illustration II

TABLE 2: The Numerical Output to Illustration II

t	v(t) - Exact	v(t) - Computed	Absolute Error
0.0	1.000000000000000	0.999989764532567	1.023546743 E – 5
0.1	0.814048064808486	0.814048064780915	2.7571 E – 11
0.2	0.652740034526729	0.652740034526882	1.53 E – 13
0.3	0.511608727070520	0.511608727070192	3.28 E – 13
0.4	0.386996723087916	0.386996723089046	1.13 E – 12
0.5	0.275909580878582	0.275909580878012	5.70 E – 13
0.6	0.175895658934151	0.175895658934021	1.30 E – 13
0.7	0.084947722956205	0.084947722956003	2.02 E – 13
0.8	0.001422388495992	0.001422388495012	9.80 E – 13
0.9	-0.076025833833810	-0.076025833833051	7.59 E – 13
1.0	- 0.148498537572540	- 0.14849853757210	4.40 E – 13

Illustration III

In an alternating current circuit containing resistance R, an inductance L, the equation of current passing through the circuit is represented as

$$Ri + L\frac{di}{dt} = E_0 \sin \omega t; \quad i(0) = 0$$

Determine the amount of current in the circuit. The analytical solution is

$$i(t) = \left(\frac{E_0}{R^2 + \omega^2 L^2}\right)(R \sin \omega t - \omega L \cos \omega t) + \left(\frac{E_0 \omega L}{R^2 + \omega^2 L^2}\right) e^{-\frac{Rt}{L}}$$

Where R, L, E_0, ω are arbitrary constants. Let R, L, E_0, ω respectively be $1,1, 2, 2$, then

$$i(t) = \frac{2}{5}(\sin 2t - 2\cos 2t + 2\, e^{-t})$$

Table 3 below shows the numerical computation to Illustration III

TABLE 3: The Numerical Computation Results to Illustration III

t	i(t) - Exact	i(t) - Computed	Absolute Error
0.0	0.000000000000000	0.000000000753683	7.53683 E – 10
0.1	0.019284404473799	0.019284404473054	7.450 E – 13
0.2	0.073903144183538	0.073903144183502	3.600 E – 14
0.3	0.158243070975646	0.158243070975011	6.350 E – 13
0.4	0.265833105710588	0.265833105721091	1.0503 E – 11
0.5	0.389571076998754	0.389571076998901	1.470 E – 13
0.6	0.521978739680773	0.521978739680024	7.490 E – 13
0.7	0.655474420708319	0.655474420701078	7.241 E – 12
0.8	0.782652230351410	0.782652230351036	3.740 E – 13
0.9	0.896556455898227	0.896556455890812	7.415 E – 12
1.0	0.990939992905141	0.990939992905003	1.380 E – 13

Illustration IV:

The equation of motion of an object is as given as

$$m\frac{dv}{dt} = mk(1 - e^{-t}) - mcv$$

Where m, k and c are arbitrary constants. Determine the speed of the object at any point in time such that $v(0) = 0$. The analytical solution is

$$v(t) = k\left(\frac{1}{c} - \frac{e^{-t}}{c-1} + \frac{e^{-ct}}{c(c-1)}\right)$$

Let $c = k = m = 2$, then the analytic solution is

$$v(t) = 1 - 2e^{-t} + e^{-2t}$$

Table IV below shows the comparison between analytical solution and the computed solution to Illustration IV

TABLE 4: The Result of Experimentation to Illustration IV

t	v(t) - Exact	v(t) - Computed	Absolute Error
0.0	0.000000000000000	0.000000000000914	9.140 E – 13
0.1	-0.391611083073314	-0.39161108307102	2.294 E – 12
0.2	-0.772485470284700	-0.772485470284004	6.960 E – 13
0.3	-1.150905979057980	-1.150905979056107	1.873 E – 12
0.4	-1.534320431165319	-1.534320431160663	4.656 E – 12
0.5	-1.929563100228814	-1.929563100209017	1.9797 E – 11
0.6	-2.343043388868816	-2.343043388861013	7.803 E – 12
0.7	-2.780908450999347	-2.780908450990109	9.238 E – 12
0.8	-3.249185338990281	-3.249185338981126	9.155 E – 12
0.9	-3.753907334092313	-3.753907334090914	1.399 E – 12
1.0	-4.301228373681478	-4.301228373680216	1.262 E - 12

Illustration V

The differential equation

$$a\frac{dC}{dt} = b + dm - Cm$$

represents the concentration C of impurities in an oil purifier, determine the concentration C at any point in time given that $C(0) = c_0$ and $a, b, d, and\ m$ are arbitrary constants. The true solution is

$$C(t) = \left(\frac{b+dm}{m}\right)\left(1 - e^{-\frac{mt}{a}}\right) + c_0 e^{-\frac{mt}{a}}$$

Let $a = b = d = m = 1\ and\ c_0 = 5$, then

$$C(t) = 2 + 3e^{-t}$$

Table V below shows the numerical computation to Illustration V showing the level of convergency of the computed result with the analytical solution.

TABLE 5: Output of Experimentation to Illustration V

t	c(t) - Exact	c(t) - Computed	Absolute Error
0.0	5.000000000000000	4.999979361798292	2.063820171 E – 5
0.1	4.714512254107879	4.714512254100815	7.0640 E – 12
0.2	4.456192259233946	4.456192259231028	2.9180 E – 12
0.3	4.222454662045154	4.222454662040924	4.2300 E – 12
0.4	4.010960138106918	4.010960138104002	2.9160 E – 12
0.5	3.819591979137900	3.819591979921073	7.83173 E – 10
0.6	3.646434908282079	3.646434908291064	8.9850 E – 12
0.7	3.489755911374228	3.489755911370912	3.3160 E – 12
0.8	3.347986892351665	3.347986892359121	8.2470 E – 12
0.9	3.219708979221797	3.219708979220228	1.5690 E – 12
1.0	3.103638323514327	3.103638323519005	4.6780 E – 12

DISCUSSION OF RESULTS AND CONCLUSION.

A critical examination of table of results from Tables 1 to 5 above that shows the numerical computation to illustrations I to V in which the analytical solution is hereby compared with the approximate solution. It shows clearly that the approximate solution converges favorably well to the analytical solution in each of the illustration. The error margin is highly infinitesimal and this shows that the method derived performs very well when compare its output with the analytical solution.

At this juncture, it is hereby concluded that the method is suitable to handle any first order ordinary differential equation and hereby equally recommended for use.

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