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Transient Solution of a Two Homogeneous Servers Markovian Queueing System with Environmental, Catastrophic and Restoration Effects

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ABSTRACT: *In this paper we consider a finite capacity Markovian queueing system with two identical servers under two environmental conditions. Change in environmental conditions also affects the state of the queueing system. Further the system is also suffered by randomly occurring disasters, which destroys all the present customers of the system in both the environmental conditions. Then a repair process is started and after the successful repair the system is ready for working. Here, repair time is known as the restoration time. We modeled this queueing system and obtained the transient state solution by using probability generating function technique.*

KEYWORDS: catastrophes, environment, restoration, finite capacity, probability generating function.

INTRODUCTION

In this paper we consider simple Markovian queues that are subjected to environmental and catastrophic failures. We have considered two environmental conditions and transition from environmental conditions E to F behave like the occurrence of catastrophe. This paper is the generalization of our previous work [Bura Gulab et. al (2016)] in which we do not consider the repair process. Previously we assumed that the system is ready immediately for new arrival after the occurrence of catastrophes but in real life the system will take some time to recover, that time is known as restoration time. In this paper we consider that the system will be ready for new arrival after some restoration time. Many authors have utilized the concept of restoration in catastrophic queues see for e.g. [Bura Gulab and Ram Niwas (2015), Goel (1979), Jain and Kanethia (2006), Jain and Kumar (2007), Jain and Bura Gulab (2010)]. B. Krishna Kumar et al. (2000) obtained the time dependent solution of the catastrophic queues and after that a number of authors have generalized their ideas. Liu and Liu [2023] studied the transient probabilities of an M/PH/1 queue model with catastrophes which is regarded as a generalization of an M/M/1 queue model with catastrophes.

Queueing models with environmental change, catastrophes and restoration may be suitable to be applied in many practical situations in biological sciences and agricultural sciences etc. This model finds their applications in biological phenomenon like the birth and death of many creatures such as cockroaches, mosquitoes, ants etc. is depending upon the changes in temperature (environment). The catastrophes may also occur with these creatures in both the environmental states i.e., spray etc which make them zero instantaneously, and after restoration time the system again working normal.

In the next section, we present the assumptions and definitions of the model. In section 3 and 4, we have modeled and analysed this queueing system and obtained the time dependent solution by using probability generating function technique. In section 5, further a particular case is also derived and discussed.

2 Assumptions and Definitions

a) The customers arrive according to a Poisson process one by one at the service facility. We consider a non-homogeneous arrival pattern, i.e. there may exist two arrival rates, namely λ_1 and 0 of which only one is working at a time.
b) The service time of two identical servers is assumed to be exponentially distributed. further, corresponding to arrival rate λ_1 the Poisson service rate is μ_1 for both the servers and corresponding to the arrival rate 0 the service rate is μ_2 for both the servers.
c) The system state when operating with arrival rate λ_1 and service rate μ_1 is classified as E whereas when working with arrival rate 0 and service rate μ_2 is designated as F.
d) The Poisson transition rates from environmental states F to E and E to F are denoted by α and β respectively.
e) The catastrophe may occur according to a Poisson process with rate ζ. The effect of each occurrence makes the queue instantly empty.
f) Restoration times are i.i.d random variables follows exponential distribution with parameter η. The customers are not allowed to occur during the restoration time.
g) The discipline of the queue is first-come-first-served.
h) The capacity of the system is fixed M i.e.,if at any time instant there are M customers in the queue then the new arrival will not be allowed to join the queue.

Define,

P_{00} (t)= represents the joint probability that the system is empty and in state E at time t without the occurrence of catastrophe.

Q_{00} (t)= represents the joint probability that the system is empty and in state F at time t without the occurrence of catastrophe.

P_{000} (t)= represents the joint probability that the system is empty and in state E at time t with the occurrence of catastrophe.

Q_{000} (t)= represents the joint probability that the system is empty and in state F at time t with the occurrence of catastrophe.

Where

$P_0 = P_{00} + P_{000}$ and $Q_0 = Q_{00} + Q_{000}$

$P_n(t)$ =Joint probability that the system is in state E and there are n customers are in the system at time t.

$Q_n(t)$ = Joint probability that the system is in state F and there are n customers are in the system at time t.

$R_n(t)$ = Probability of n customers in the system at time t.

$$R_n(t) = P_n(t) + Q_n(t) \tag{1}$$

Let us reckon time t from an instant when there are zero customers in the queue and the system is in the environmental state E so that the initial conditions associated with $P_n(t)$ and $Q_n(t)$ becomes,

$$P_n(0) = \begin{cases} 1; & n = 0 \\ 0; & otherwise \end{cases}$$
$$Q_n(0) = 0; \quad \forall n \tag{2}$$

3 Equations Governing the Queueing System:

$$\frac{d}{dt}P_{00}(t) = -(\lambda_1 + \beta + \zeta)P_{00}(t) + \mu_1 P_1(t) + \alpha Q_{00}(t) + \eta P_{000}(t) \quad n = 0 \tag{3}$$

$$\frac{d}{dt}P_{000}(t) = -(\lambda_1 + \beta + \zeta + \eta)P_{000}(t) + \alpha Q_{000}(t) + \zeta \sum_{n=0}^{M} P_n(t) \tag{4}$$

$$\frac{d}{dt}P_1(t) = -(\lambda_1 + \mu_1 + \beta + \zeta)P_1(t) + 2\mu_1 P_2(t) + \alpha Q_1(t) + \lambda_1 P_0(t); n = 1 \tag{5}$$

$$\frac{d}{dt}P_n(t) = -(\lambda_1 + 2\mu_1 + \beta + \zeta)P_n(t) + 2\mu_1 P_{n+1}(t) + \lambda_1 P_{n-1}(t) + \alpha Q_n(t) \tag{6}$$

$$\frac{d}{dt}P_M(t) = -(2\mu_1 + \beta + \zeta)P_M(t) + \lambda_1 P_{M-1}(t) + \alpha Q_M(t); n = M \tag{7}$$

$$\frac{d}{dt}Q_{00}(t) = -(\alpha + \zeta)Q_{00}(t) + \mu_2 Q_1(t) + \beta P_{00}(t) + \eta Q_{000}(t) \tag{8}$$

$$\frac{d}{dt}Q_{000}(t) = -(\alpha + \zeta + \eta)Q_{000}(t) + \beta P_{000}(t) + \zeta \sum_{n=o}^{M} Q_n(t) \tag{9}$$

$$\frac{d}{dt}Q_1(t) = -(\mu_2 + \alpha + \zeta)Q_1(t) + 2\mu_2 Q_2(t) + \beta P_1(t) \tag{10}$$

$$\frac{d}{dt}Q_n(t) = -(2\mu_2 + \alpha + \zeta)Q_n(t) + 2\mu_2 Q_{n+1}(t) + \beta P_n(t); 0 < n < M \quad (11)$$

$$\frac{d}{dt}Q_M(t) = -(2\mu_2 + \alpha + \zeta)Q_M(t) + \beta P_M(t); n = M \quad (12)$$

4 Transient Analysis of the Model:

Let, the Laplace Transform of $f(t)$ be

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \quad (13)$$

Laplace transform of equations (3)-(12) with initial conditions given in (2), gives,

$$(s + \lambda_1 + \beta + \zeta)\bar{P}_{00}(s) - 1 = \mu_1 \bar{P}_1(s) + \alpha \bar{Q}_{00}(s) + \eta \bar{P}_{000}(s);\ n = 0 \quad (14)$$

$$(s + \lambda_1 + \beta + \zeta + \eta)\bar{P}_{000}(s) = \alpha \bar{Q}_{000}(s) + \zeta \sum_0^M \bar{P}_n(s) \quad (15)$$

$$(s + \lambda_1 + \mu_1 + \beta + \zeta)\bar{P}_1(s) = 2\mu_1 \bar{P}_2(s) + \alpha \bar{Q}_1(s) + \lambda_1 \bar{P}_0(s); n = 1 \quad (16)$$

$$(s + \lambda_1 + 2\mu_1 + \beta + \zeta)\bar{P}_n(s) = 2\mu_1 \bar{P}_{n+1}(s) + \lambda_1 \bar{P}_{n-1}(s) + \alpha \bar{Q}_n(s); \quad (17)$$

$$(s + 2\mu_1 + \beta + \zeta)\bar{P}_M(s) = \lambda_1 \bar{P}_{M-1}(s) + \alpha \bar{Q}_M(s) \quad (18)$$

$$(s + \alpha + \zeta)\bar{Q}_0(s) = \mu_2 \bar{Q}_1(s) + \beta \bar{P}_0(s) + \zeta \sum_{n=0}^M \bar{Q}_n(s) \quad (19)$$

$$(s + \alpha + \zeta + \eta)\bar{Q}_{000}(s) = \beta \bar{P}_{000}(s) + \zeta \sum_{n=o}^M \bar{Q}_n(s) \quad (20)$$

$$(s + \alpha + \mu_2 + \zeta)\bar{Q}_1(s) = 2\mu_2 \bar{Q}_2(s) + \beta \bar{P}_1(s) \quad (21)$$

$$(s + 2\mu_2 + \alpha + \zeta)\bar{Q}_n(s) = 2\mu_2 \bar{Q}_{n+1}(s) + \beta \bar{P}_n(s); \quad (22)$$

$$(s + 2\mu_2 + \alpha + \zeta)\bar{Q}_M(s) = \beta \bar{P}_M(s); \quad (23)$$

Define, the probability generating function by ,

$$P(z,s) = \sum_{n=0}^M \bar{P}_n(s) z^n \quad (24)$$

$$Q(z,s) = \sum_{n=0}^M \bar{Q}_n(s) z^n \quad (25)$$

$$R(z,s) = \sum_{n=0}^M \bar{R}_n(s) z^n \quad (26)$$

where

$$R(z,s) = P(z,s) + Q(z,s) \quad (27)$$

and

$$\bar{R}_n(s) = \bar{P}_n(s) + \bar{Q}_n(s) \tag{28}$$

Multiplying equations (14)-(18) by z^n, Summing over the respective range of n and using equations (24)-(26),we have

$$(s+\lambda_1+\beta+\zeta+2\mu_1)\sum_{n=0}^{M}\bar{P}_n(s)z^n - \frac{2\mu_1}{z}\left[\sum_{n=0}^{M-1}\bar{P}_{n+1}(s)z^{n+1} - \bar{P}_0(s)\bar{P}_0(s) - \bar{P}_1(s) + \bar{P}_1(s)\right]$$
$$-\lambda_1 z\left[\sum_{n=1}^{M}\bar{P}_{n-1}(s)z^{n-1} + z^M\bar{P}_M(s) - z^M\bar{P}_M(s)\right] - \alpha\sum_{n=0}^{M}\bar{Q}_n(s)z^n - 2\mu_1\bar{P}_0(s)$$
$$-\lambda_1\bar{P}_M(s)z^M - 1 - \zeta\sum_{n=0}^{M}\bar{P}_n(s) - \mu_1 z\bar{P}_1(s) - \mu_1\bar{P}_1(s) = 0$$

$$(s+\lambda_1+\beta+\zeta+2\mu_1)P(z,s) - \tfrac{2\mu_1}{z}P(z,s) + 2\mu_1\bar{P}_1(s) - \lambda_1 zP(z,s) + \tfrac{2\mu_1}{z}(1-z)\bar{P}_0(s)$$
$$-\lambda_1(1-z)\bar{P}_M(s)z^M - \alpha Q(z,s) - \lambda_1 z^M(1-z)\bar{P}_M(s) - 1 - \zeta\sum_{n=0}^{M}\bar{P}_n(s) - \mu_1 z\bar{P}_1(s) - \mu_1\bar{P}_1(s)$$
$$= 0$$

$$[(s+\lambda_1+\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1]P(z,s) - \alpha zQ(z,s) + 2\mu_1(1-z)\bar{P}_0(s)$$
$$-\lambda_1(1-z)\bar{P}_M(s)z^{M+1} - z - z\zeta\sum_{n=0}^{M}\bar{P}_n(s) + z\mu_1(1-z)\bar{P}_1(s) = 0 \tag{29}$$

Similarly, from equations (19)-(23) on using equations (24)-(26), we have

$$(s+2\mu_2+\alpha+\zeta)\sum_{n=0}^{M}\bar{Q}_n(s)z^n - \frac{2\mu_2}{z}[\sum_{n=0}^{M-1}\bar{Q}_n(s) - \bar{Q}_0(s) + \bar{Q}_0(s) + \bar{Q}_1(s)$$
$$-\bar{Q}_1(s)] - \beta\sum_{n=0}^{M}\bar{P}_n(s)z^n - \zeta\sum_{n=0}^{M}\bar{Q}_n(s) - 2\mu_2\bar{Q}_0(s) - \mu_2\bar{Q}_1(s) - \mu_2 z\bar{Q}_1(s) = 0$$

$$(s+2\mu_2+\alpha+\zeta)Q(z,s) - \frac{2\mu_2}{z}Q(z,s) + 2\mu_2\bar{Q}_1(s) + \frac{2\mu_2}{z}\bar{Q}_0(s) - \beta P(z,s)$$
$$-\zeta\textstyle\sum_{n=0}^{M}\bar{Q}_n(s) - 2\mu_2\bar{Q}_0(s) - \mu_2\bar{Q}_1(s) - \mu_2\bar{Q}_1(s)z = 0$$

$$[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)]Q(z,s) + \beta zP(z,s) + \zeta z\sum_{n=0}^{M}\bar{Q}_n(s)$$
$$-2\mu_2\bar{Q}_0(s)(1-z) - z\mu_2(1-z)\bar{Q}_1(s) = 0 \tag{30}$$

From equation (30), we have

$$P(z,s) = \alpha z Q(z,s) - 2\mu_1(1-z)\bar{P}_0(s) + \lambda_1(1-z)\bar{P}_M(s)z^{M+1} + \zeta z + \sum_{n=0}^{M} \bar{P}_n(s) - \mu_1 z\bar{P}_1(s)(1-z)/[z(s+\lambda_1+2\mu_1+\beta+\zeta) - \lambda_1 z^2 - 2\mu_1] \tag{31}$$

From equation (29), we have

$$Q(z,s) = \frac{2\mu_2(1-z)\bar{Q}_0(s) - \beta z P(z,s) - \zeta z \sum_{n=0}^{M} \bar{Q}_n(s) + z\mu_2(1-z)\bar{Q}_1(s)}{[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)]} \tag{32}$$

Putting the value of $Q(z,s)$ in (31), we get

$$P(z,s) = \alpha z\left[\frac{2\mu_2(1-z)\bar{Q}_0(s) - \beta z P(z,s) - \zeta z \sum_{n=0}^{M} \bar{Q}_n(s) + z\mu_2(1-z)\bar{Q}_1(s)}{2\mu_2 - z(s+2\mu_2+\alpha+\zeta)}\right]$$
$$-2\mu_1(1-z)\bar{P}_0(s) + \lambda_1(1-z)\bar{P}_M(s)z^{M+1} + z + \zeta z\sum_{n=0}^{M}\bar{P}_n(s)$$
$$-\mu_1 z\bar{P}_1(s)/[z(s+\lambda_1+2\mu_1+\beta+\zeta) - \lambda_1 z^2 - 2\mu_1]$$

$$P(z,s) = 2\alpha z\mu_2(1-z)\bar{Q}_0(s) - \alpha\zeta z^2\sum_{n=0}^{M}\bar{Q}_n(s) - 2\mu_1(1-z)\bar{P}_0(s)[2\mu_2$$
$$-z(s+2\mu_2+\alpha+\zeta)] + \lambda_1(1-z)\bar{P}_M(s)z^{M+1}[2\mu_2 - z(s+2\mu_2$$
$$+\alpha+\zeta)] + z[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)] + \zeta z\sum_{n=0}^{M}\bar{P}_n(s)\,[2\mu_2$$
$$-z(s+2\mu_2+\alpha+\zeta)] - \mu_1(1-z)z\bar{P}_1(s)[2\mu_2$$
$$-z(s+2\mu_2+\alpha+\zeta)] + \mu_2(1-z)z^2\alpha\bar{Q}_1(s)/[z(s+\lambda_1+2\mu_1$$
$$+\beta+\zeta) - \lambda_1 z^2 - 2\mu_1][2\mu_2 - z(s+2\mu_2+\alpha+\zeta) + \beta z^2\alpha] \tag{33}$$

Similarly, putting the value of $P(z,s)$ in equation (32), we get

$$\beta z[\alpha z Q(z,s) - 2\mu_1(1-z)\bar{P}_0(s) + \lambda_1(1-z)\bar{P}_M(s)z^{M+1} + z + z\zeta\sum_{n=0}^{M}\bar{P}_n(s)$$
$$-\mu_1 z(1-z)\bar{P}_1(s)/(s+\lambda_1+\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1]$$
$$+[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)]Q(z,s) + \zeta z\sum_{n=0}^{M}\bar{Q}_n(s)$$
$$-2\mu_2\bar{Q}_0(s)(1-z) - 2\mu_2\bar{Q}_1(s)(1-z) = 0$$

$$\{\beta\alpha z^2 + [2\mu_2 - z(s+2\mu_2+\alpha+\zeta)][(s+\lambda_1+\beta+\zeta+2\mu_1)z - \lambda_1 z^2$$
$$-2\mu_1]\}Q(z,s) = 2\mu_1(1-z)\beta z\bar{P}_0(s) + 2\mu_2(1-z)[(s+\lambda_1+\beta+\zeta+2\mu_1)z$$

$$-\lambda_1 z^2 - 2\mu_1]\bar{Q}_0(s) - \beta\lambda_1(1-z)\bar{P}_M(s)z^{M+2} - \beta z^2 - \beta z^2\zeta\sum_{n=0}^{M}\bar{P}_n(s)$$
$$+\zeta z[\lambda_1 z^2 + 2\mu_1 - (s+\lambda_1+\beta+\zeta+2\mu_1)z]\sum_{n=0}^{M}\bar{Q}_n(s) + z\mu_2\bar{Q}_1(s)(1$$
$$-z)[(s+\lambda_1+\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1] + \mu_1 z^2(1-z)\beta\bar{P}_1(s) = 0$$

$$\begin{aligned}Q(z,s) = {} & 2\mu_1(1-z)\beta z\bar{P}_0(s) + 2\mu_2(1-z)[(s+\lambda_1+\beta+\zeta+2\mu_1)z \\ & -\lambda_1 z^2 - 2\mu_1]\bar{Q}_0(s) - \beta\lambda_1(1-z)\bar{P}_M(s)z^{M+2} - \beta z^2 \\ & -\beta z^2\zeta\sum_{n=0}^{M}\bar{P}_n(s) + \zeta z[\lambda_1 z^2 + 2\mu_1 - (s+\lambda_1+\beta+\zeta \\ & +2\mu_1)z]\sum_{n=0}^{M}\bar{Q}_n(s) + \mu_1 z^2(1-z)\beta\bar{P}_1(s) + z\mu_2(1-z)[(s+\lambda_1 \\ & +\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1]\bar{Q}_1(s)/\beta\alpha z^2 + [2\mu_2 - z(s \\ & +2\mu_2\alpha+\zeta)][(s+\lambda_1+\beta+\zeta+2\mu_1)z - \lambda_1 z^2 - 2\mu_1]\end{aligned} \tag{34}$$

Now from equation (27), we have

$$\begin{aligned}R(z,s) = {} & [\alpha z + \mu_2(s+\lambda_1+\beta+\zeta+\mu_1)z - \lambda_1 z^2 - \mu_1]2\mu_2(1-z)\bar{Q}_0(s) \\ & +\zeta z\sum_{n=0}^{M}\bar{Q}_n(s)\,[\lambda_1 z^2 - z(s+\lambda_1+\mu_1+\beta+\zeta) + 2\mu_1 - \alpha z] + (1 \\ & -z)\bar{P}_0(s)2\mu_1[z(s+2\mu_2+\alpha+\zeta) - 2\mu_2 + z\beta] + (1-z)\bar{P}_M(s)\lambda_1[z^{M+1}2\mu_2 \\ & -z(s+2\mu_2+\alpha+\zeta) - \beta z^{M+2}] + \zeta z\sum_{n=0}^{M}\bar{P}_n(s)\,[2\mu_2 - z(s+2\mu_2+\alpha+\zeta) \\ & -\beta z] + z[2\mu_2 - z(s+2\mu_2+\alpha+\zeta)] - \beta z^2 + \bar{P}_1(s)z\mu_1(1-z)[z\beta - [2\mu_2 \\ & -z(s+2\mu_2+\alpha+\zeta)]] + z\mu_2(1-z)\bar{Q}_n(s)[z\alpha + z(s+2\mu_1+\lambda_1+\zeta+\beta) \\ & -2\mu_1 - \lambda_1 z^2]/{-z^2 s^2} + s[\lambda_1 z^3 - z^2(\lambda_1+2\mu_1+2\mu_2+\alpha+\beta+2\zeta) \\ & +z(2\mu_1+2\mu_2)] - z^2\zeta(\beta+\alpha+\zeta) + (1-z)[-z^2\lambda_1(2\mu_2+\alpha+\zeta) \\ & +z[2\alpha\mu_1 + 2\mu_2(2\mu_1+\lambda_1+\beta+\zeta)] - 4\mu_2\mu_1]\end{aligned} \tag{35}$$

The unknown quantities in equation (35) are determined as follows:
Setting z=1, in equations (33) and (34) respectively,we have

$$P(1,s) = \sum_{n=0}^{M}\bar{P}_n(s) = \frac{(s+\alpha+\zeta)}{s(s+\beta+\alpha+\zeta)} \tag{36}$$
$$Q(1,s) = \sum_{n=0}^{M}\bar{Q}_n(s) =$$

$$\frac{\beta}{s(s+\beta+\alpha+\zeta)} \tag{37}$$

Equation (35) is a polynomial in z and exists for all values of z, including the three zeros of the denominator. Hence, the quantities of interest i.e. $\bar{P}_0(s)$, $\bar{Q}_0(s)$ and $\bar{R}_M(s)$ are obtained by setting the numerator equal to zero and substituting the three zeros a_1, a_2, a_3 (say) of the denominator (at which the numerator must vanish).
The various state probabilities of the number of customers in the form of Laplace transform can be picked up from equation (35) as the coefficient of different powers of z.

5 Particular Cases:

Case 1: Now taking $\alpha \to \infty$,$\beta \to \infty$ and setting μ_1=μ_2=μ (say)in relation (35), we have

$$r(z,s) = 2\mu(1-z)\bar{R}_0(s) - \lambda_1 z^{M+1}(1-z)\bar{P}_M(s) + z(1-z)\mu\bar{R}_1(s) - \frac{\zeta z}{s}/\lambda_1 z^2 \\ -z(s+\lambda_1+2\mu+\zeta)+2\mu \tag{38}$$

where

$$\bar{R}_0(s) = \bar{P}_0(s) + \bar{Q}_0(s)$$

$$r(z,s) = \lim_{\beta\to 0}[\lim_{\alpha\to\infty} R(z,s)]$$

Equation (38) is a polynomial in z and exists for all values of z, including the two zeros a_1 and a_2 (say)of the denominator. Hence R_0 and P_M can be obtained by setting the numerator equal to zero.

Case I1: **If $\xi = 0$ (i.e., no catastrophe is allowed in the system),** then from equation (38), we have

$$r(z) = \frac{\mu R_0 - \lambda_1 z^{M+1} P_M}{\mu - \lambda_1 z} \tag{39}$$

The condition, $\lim_{z\to 1} r(z) = 1$ gives

$$\mu R_0 - \lambda_1 P_M = \mu - \lambda_1 \tag{40}$$

As r(z) is analytic, the numerator and denominator of equation (39) must vanish simultaneously for z= μ/λ_1, which is a zero of its denominator. Equating the numerator of equation (39) to zero for z= μ/λ_1 we have

$$R_0 = \rho^{-M} P_M, \qquad \rho = \lambda_1/\mu < 1 \tag{41}$$

Which is a well known result of the M/M/1 queue with finite waiting space M.

CONCLUSION

In the present paper, we consider a two homogeneous servers Markovian queueing system with environmental change, catastrophe and restoration effects. We have modeled and analysed the queueing system and obtained a time dependent solution by using probability generating function technique. A particular case of the model is also derived and discussed. In real time queueing problems with environmental change after suffering from random catastrophes needs some time to regain its normal position, that time is called the restoration time. This generalization of the queueing model makes the system more practical and has numerous applications in wide variety of areas particularly in agriculture and bio-sciences etc.

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