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Analytical Methods of Calculating the Lifetime Distribution for Some Models of Standby Redundant Systems

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ABSTRACT: *In present day technology, reliability of equipment's is increased by employing the method of standby systems, that is, the introduction of extra units. The purpose of the supplementary units is to take over operation if the basic units break down. Moreover, to increase the effectiveness of standby systems, units that have failed are repaired. In this paper the reliability function and the mean lifetime are obtained for the cases of loaded, nonleaded and lightly loaded systems with and without renewal.Moreover, the effectiveness of repair is calculated for some distributions for different numbers of main units.*

KEYWORDS: reliability, mean lifetime, renewable system, preventive maintenance, loaded, nonloded and lightly loaded system.

INTRODUCTION

Formulation of The Problem

We confine ourselves to the case of a system composed of n main unites and spare unit under the following conditions:

i. As soon as one of the main units fails, the standby unit takes up the load.
ii. The failed unit is sent immediately for repair.
iii. The repair completely restores all the original properties of the unit that failed.
iv. The repair time is a random variable with an arbitrary distribution G(t).
v. The period of failure free operation of the units is random and distributed according to the law.

$F_i(t) = 1 - \exp(-a_i t), a_i > 0$ and i= 1,2,……,n

for the i-th main unite and according to the law

$B(t) = 1 - \exp(-bt), b > 0$ for the reserve unit.

The Laplace transform of the reliability function and the mean lifetime for six models of standby redundant systems (the loaded, the nonleaded and lightly loaded system with and without renewal) are obtained.

Moreover, I present the effect of repair and also the effect of the choice of the function G (.) on the mean lifetime.

SOME MODELS FOR LIFETIME DISTRIBUTION OF STANDBY REDUNDANT SYSTEMS

We investigate the mean lifetime for some models of standby redundant systems.
We shall say that our system breaks down if either two units or more fail at the same time or if a second failure occurs while the first failed unit is still being repaired.
Denote by R (t) the probability of failure-free operation during the period (0,t). Let us introduce the Laplace transforms:

$g(s)=\int_0^\infty e^{-st}\, dG(t)$, $r(s) = -\int_0^\infty e^{-st}\, dR(t)$

Model 1. Loaded Standby System without Renewal

In this case, the event of failure-free operation of the system during the period (0,t) is decomposable into two mutually independent events:

i - No failure occurs prior to time t with probability equal to.

$$\prod_{i=1}^{n+1} \bar{F}_i(t) \quad , \quad \text{Where } \bar{F}_i(t) = 1-F_i(t)$$

(Note that for simplicity B (t) is replaced by $F_{n+1}(t)$)

ii- $\bar{A}$ failure occurs at time z (< t) and instantaneously the standby assumes the load of the failed unit and then the system works to time t without failure. The probability of this event is equal to

$$\sum_{j=1}^{n+1} \prod_{i=1}^{n+1} \bar{F}_i(t) \int_0^t dF_j(z) = \sum_{j=1}^{n+1} \prod_{i=1}^{n+1} \bar{F}_i(t)\, F_j(t) \ , \quad i \neq j$$

Hence

$$R(t) = \sum_{j=1}^{n+1} \prod_{i=1}^{n+1} \bar{F}_i(t) - n\prod_{i=1}^{n+1} \bar{F}_i(t) \ , \quad i \neq j \qquad (1.1)$$

Therefore

$$R(t) = e^{-\bar{A}t}\left[\sum_{i=1}^{n+1} e^{a_i t} - n\right] \qquad (1.2)$$

Where $\bar{A} = \sum_{i=1}^{n+1} a_i$.

By Using Laplace transform, equation (1.2) is converted to:

$$r(s) = \sum_{i=1}^{n+1} \frac{\bar{A}-a_i}{(\bar{A}-a_i)+s} - \frac{n\bar{A}}{\bar{A}+s} \qquad (1.3)$$

and hence the mean lifetime will be:

$$T_1 = -\left[\frac{dr(s)}{ds}\right]_{s=0} = 1/\bar{A}\left[\sum_{i=1}^{n+1} \frac{\bar{A}}{\bar{A}-a_i} - n\right] \qquad (1.4)$$

Model 2: Loaded Standby System with Renewal

The event we are Interested in (flawless operation of The system during time from 0 to t) Is the union of three mutually independent events.

I. No failure occurs during the period (0,t) with Probability equal to $\prod_{i=1}^{n+1} F_i(t)$,

II. A failure occurs at time z (< t) , the remaining units operate to time t without failure and the repair time is completed after t .The probability of this event is equal to

$$\sum_{j=1}^{n+1} \prod_{i=1}^{n+1} \bar{F}_i(t) \int_0^t \bar{G}(t-z)\,dF_j(z)$$

III. The first failure occurs at time z (< t), the repair of this unit is completed also prior to time t, during the repair period the remaining units were functional. From the repair to time t , the system functioned normally. The Probability of this event is:

$$\sum_{j=1}^{n+1} \int_{z=0}^{t} \int_{y=0}^{t-z} \prod_{i=1}^{n+1} \bar{F}_i(z)\bar{F}_i(y)R(t-y-z)\,dG(y)\,dF_j(z) \ , \ i \neq j$$

Hence

R (t) = $\prod_{i=1}^{n+1} \bar{F_i}(t) + \sum_{j=1}^{n+1} \prod_{i=1}^{n+1} \bar{F_i}(t) \int_0^t \bar{G}(t-z)\, dF_j(z) +$
$\sum_{j=1}^{n+1} \int_0^t \int_0^{t-z} \prod_{i=1}^{n+1} \bar{F_i}(z)\bar{F_i}(y) R(t-y-z) dG(y) dF_j(z)\, , i \neq j$

Therefore:

$$R(t) = e^{-\bar{A}t} + e^{-\bar{A}t} \sum_{i=1}^{n+1} a_i\, e^{a_i t} \int_0^t \bar{G}(t-z) e^{-a_i z} dz + \sum_{j=1}^{n+1} a_i \int_0^t \int_0^{t-z} e^{-\bar{A}y-(\bar{A}-a_i)z} R(t-y-z) dG(y) dZ \qquad (1.5)$$

After some manipulations, the Laplace transform of (1.5) will be:

$$r(s) = \frac{\sum_{i=1}^{n+1} a_i(\bar{A}-a_i)(\bar{A}-a_i+s)^{-1} g^*(\bar{A}-a_i+s)}{(\bar{A}+s) - \sum_{i=1}^{n+1} a_i\, g(\bar{A}-a_i+s)}$$

where

g (s) = 1-g* (s) .

Thus, the mean lifetime will be

$$\acute{T}_1 = \frac{1+\sum_{i=1}^{n+1} a_i(\bar{A}-a_i)^{-1} g^*(\bar{A}-a_i)}{\bar{A}-\sum_{i=1}^{n+1} a_i g(\bar{A}-a_i)} \qquad (1.\ 6)$$

Model 3. Nonloaded Standby System without Renewal

The event of flawless operation of the system during the interval (0,t) is decomposable into two mutually independent events:

i- No breakdown occurs during the time (0,t) with probability equal to: $\prod_{i=1}^{n} \bar{F_i}(t)$.
ii- At moment z (< t) a failure occurs. The remaining elements operate flawlessly up to time t. The probability of this event is equal to: $\sum_{j=1}^{n} \prod_{i=1}^{n} \bar{F}_i(t) \int_0^t \bar{F}_i(t-z) dF_j(z)$, i ≠ j .

Hence

$$R(t) = \prod_{i=1}^{n} \bar{F}_i(t) + \sum_{j=1}^{n} \prod_{i=1}^{n} \bar{F}_i(t) \int_0^t \bar{F}_j(t-z) dF_j(z)\, , i \neq j$$

therefore:

$$R(t) = e^{-At}(1+At) \qquad (1.7)$$

where

$$A = \sum_{i=1}^{n} a_i\ .$$

The Laplace transform of (1.7) will be:

$$r(s) = \left[\frac{A}{A+S}\right]^2$$

and hence the mean lifetime is given by.

$$T_2 = \frac{2}{A} \quad . \qquad (1.8)$$

From equations (1.4) and (1.8) It can be shown that the mean lifetime for the nonloaded system is greater than that of the loaded system.

Model 4: Nonloaded Standby System with Renewal

In this case, the event of failure-free operation through the time from 0 to t is decomposable into three mutually independent events.

i. No failure occurs prior to time t, with probability equal to:

$$\prod_{i=1}^{n} \bar{F}_i(t)$$

ii. A failure occurs at time z (<t), the remaining units work flawlessly to time t. The repair of the failed unit is completed after t. The probability of this event is equal to:

$\sum_{j=1}^{n} \prod_{i=1}^{n} \bar{F}_i(t) \int_0^t \bar{F}_j(t-z)\bar{G}(t-z)dF_j(z)\,, i \neq j$

iii - At moment z (< t) a breakdown occurs, the repair of the broken unit is completed prior to time t, during the repair period, the system works flawlessly. From the repair to time t, the system functioned normally. The probability of this event is equal to:

$\sum_{j=1}^{n} \int_0^t \int_0^{t-z} \prod_{i=1}^{n} \bar{F}_i(z) \prod_{i=1}^{n} \bar{F}_i(y) R(t-y-z)\, dG(y)dF_j(z).$

Hence

$$R(t) = \prod_{i=1}^{n} \bar{F}_i(t) + \sum_{j=1}^{n}\prod_{i=1}^{n} \bar{F}_i(t) \int_0^t \bar{F}_j(t-z)\bar{G}(t-z)dF_j(z) + \sum_{j=1}^{n} \int_0^t \int_0^{t-z} \prod_{i=1}^{n} \bar{F}_i(z) \prod_{i=1}^{n} \bar{F}_i(y)R(t-y-z)dG(y)dF_j(z)\,.$$

Therefore

$$R(t) = e^{-At} + Ae^{-At}\int_0^t \bar{G}(t-z)dz + A\int_0^t \int_0^{t-z} e^{-A(z+y)}R(t-y-z)dG(y)dz\ . \quad (1.9)$$

Application of the properties of Laplace transforms converts (1.9) into the equation:

$$r(s)\frac{A^2\ g^*(A+S)}{},$$

and hence the mean lifetime will be:

$$T_2^{'} = \frac{1+\ g^*(A)}{Ag^*\ (A)}\ . \quad (1.10)$$

Model 5: <u>Lightly Loaded Standby System without Renewal</u>

In this case, the event of failure free operation during the period (0,t) is composed from three mutually independent events.

i- Neither one of the main units nor the standby unit fails prior to time t, with probability equal to

$$\prod_{i=}^{n} \bar{F}_i\ (t)\ \bar{B}\ (t)\ .$$

ii- The standby unit fails at time z (<t) while the main units operate up to time t. The probability of this event is equal to

$$\prod_{i=1}^{n} \bar{F}_i(t) \int_0^t dB(z) = \prod_{i=1}^{n} \bar{F}_i(t)B(t)\,.$$

iii- One of the main units fails at time z(<t) and then the system works up to time t without failure. The probability of this event is equal to:

$$\sum_{j=1}^{n} \prod_{i=1}^{n} \bar{F}_i\ (t) \int_0^t \bar{B}\ (z)\bar{F}_j(t-z)dF_j(z)\ .$$

Hence

$$R(t) = \prod_{i=1}^{n} \bar{F}_i\ (T) + \sum_{j=1}^{n} \prod_{i=1}^{n} \bar{F}_i\ (t) \int_0^t \bar{B}\ (z)\bar{F}_j(t-z)dF_j(z)\ .$$

Therefore

$$R(t) = e^{-At}\left[1 + \frac{A}{b}(1-e^{-bt})\right]\ . \quad (1.11)$$

The Laplace transform of (1.11) will be

$$r(s)\frac{A\ \ (A+b\ )}{(\ A+S\ )\ \ (A+b+S)}$$

and hence the mean lifetime Is given by:

$$T_3\frac{2\ A+b}{A\ \ (A+b)}\ . \quad (1.12)$$

It can be easily seen from equations (1.8) and (1.12) that the mean lifetime for the lightly loaded system is less than that of the nonloaded system.

Model 6: Lightly Loaded Standby System with Renewal

The event we are interested in (flawless operation of the system during the time from 0 to t) is the union of five mutually independent events:

i- No failure occurs during the period (0,t) with probability:

$$\prod_{i=1}^{n} \bar{F}_i(t)\bar{B}(t)\ .$$

ii- One of the main units fails at time z (< t). The repair time of the failed unit is completed after t. The system works up to time t without failure. The probability of this event is equal to:

$$\sum_{j=1}^{n}\prod_{i=1}^{n} \bar{F}_i(t)\int_0^t \bar{B}(z)\bar{F}_j(t-z)\bar{G}(t-z)dF_j,$$

iii- The spare unit fails at time z(<t). The repair time is completed after t and then main units operate flawlessly up to time t. The probability of this event is:

$$\prod_{i=1}^{n} \bar{F}_i\,(\mathrm{t})\ \int_0^t \bar{G}(t-z)\mathrm{dB(z)}$$

iv- At time z(<t) one of the main units breaks down The repair time is completed prior to time t, during the repair period the system operates flawlessly. From the repair time to t the system functioned normally. The probability of this event is

$$\sum_{j=1}^{n}\int_{z=0}^{t}\int_{y=0}^{t-z}\prod_{i=1}^{n} \bar{F}_i(z)\,\bar{B}(z)(y)R(t-y-z)dG(y)dF_j(z).$$

v- The spare unit fails at time z(<t). The repair time Is completed prior to t, during the repair time the main units were functional. From time of repair to time t, the system functioned normally. The probability of this event is equal to

$$\int_0^t\int_0^{t-z}\prod_{i=1}^{n} \bar{F}_i\,(z)\bar{F}_i(y)R(t-y-z)dG(y)dB(z).$$

Hence

$$R(t)=\prod_{i=1}^{n} \bar{F}_i(t)\,\bar{B}(t)+\sum_{j=1}^{n}\prod_{i=1}^{n} \bar{F}_i\,(t)\int_0^t \bar{B}(z)\bar{F}_j\,(t-z)\bar{G}(t-z)dF_j(z)$$
$$+\prod_{i=1}^{n} \bar{F}_i(t)\int_0^t \bar{G}(t-z)\,dB(z)+\sum_{j=1}^{n}\int_0^t\int_0^{t-z}\prod_{i=1}^{n} \bar{F}_i\,(z)\bar{B}(z)\prod_{i\neq 1}^{n} \bar{F}_i\,(y)R(t-$$
$$y-z)+\int_0^t\int_0^{t-z}\prod_{i=1}^{n} \bar{F}_i(z)\bar{F}_i(y)R(t-y-z)dG(y)dB(z)$$

Therefore

$$R(t)=e^{-(A+b)t}+(A+b)e^{-At}\int_0^t e^{-bz}\bar{G}(t-z)dZ+$$
$$(A+b)\int_0^t\int_0^{t-z} e^{-(A+b)z-Ay}R(t-y-z)dG(y)dz. \quad (1.13)$$

After some manipulations, the Laplace transform of (1.13)
Will be:

$$r(s)=\frac{A\ \ (A+b\ )g^{*}(A+S)}{(\,A+S\,)\,[\,S+(A+b)\ g^{*}\ (A+S)]} \quad (1.14)$$

Hence the mean lifetime is given by:

$$T_3^{'}=\frac{A+\ (A+b)g^{*}\ (\,A\,)}{A\,(A+b)\ g^{*}\ (A)} \quad (1.15)$$

We note that the results concerning Laplace transforms and the mean lifetime for the cases of nonloaded with and without renewal and for lightly loaded without renewal can be obtained from (1.14) and (1.15); (for nonloaded systems. b=0 and for systems without renewal g(.)=0). Moreover, the results in [Belyaev, Yu.K. and Gnedenko, B.V.,Kovalenko, I.N.,1962] [Gnedenko,B.V. 1969] can be obtained as special cases from the present results by putting $a_i = a$ for i = 1, 2, …, n.

THE EFFECTIVENESS OF REPAIR ON THE MEAN LIFETIME FOR SOME MODELS OF STANDBY REDUNDANT SYSTEMS:

In the cases that are of most practical interest, the mean duration of repairs is considerably less than the mean time of the flawless operation of the system. We discuss the effect of repair on the mean lifetime and also the effect of the choice of the function G(.).To derive our results we need the following limit theorems.

Suppose that the function G(t) depends on a certain Parameter v and for any ε > o as $v \to \infty$

$$1 - G_v(\varepsilon) \to 0$$

It can be seen that the following relation immediately follows from (1.15):

$$g_v(A) \to 1 \quad \text{as} \quad v \to \infty$$

The converse is also true, If for any g> 0 and as $v \to \infty$ we have the relation $g_v(s) \to 1$, then for any t > 0 as $v \to \infty$, $G_v(t) \to 1$. Let us assume that T_v is the length of time between two successive failures and that the repair time is a random variable with a distribution function $G_v(t)$, and put:

$$a_v = \frac{(A+b)}{A} g^*_v(\text{A})$$

We can prove the two following theorems by using the Method in [Gnedenko,B.V. 1969].

<u>Theorem 1:</u>

If as $v \to \infty$, $G_v(\varepsilon) \to 1$ for any ε > 0 (conditions from (i) to (v) are assumed to be satisfied), then the distribution of the random variable T_v / a_v converges to the distribution $1-e^{-At}$ as $v \to \infty$.

<u>Theorem 2:</u>

If in addition to the above-mentioned conditions, the Following are satisfied.

$$m_1(v) = \int_0^\infty t dG_v(t) = 1/v \quad , m_2(v) = \int_0^\infty t^2 dG_v(t) < +\infty \quad \text{and}$$

Then the mean lifetime of a system with standby relief is asymptotically equal to the mean lifetime of the system under the assumption that

$$G_v(t) = 1 - e^{-vt}$$

To estimate the effect of repair on the operational effectiveness of a system it is natural to consider the ratio of the mean lifetime of a system with repair to that without repair. The effectiveness of repair is given by the use of (1.12) and (1.15)

$$e_v = \frac{A + (A+b)g*(A)}{(2A+b)g*(A)}$$

Let us now determine what effect the choice of the function $G_v(t)$ has on the value of e_v. In this case, we shall naturally take all $G_v(t)$ participating in the comparison to have the same expectation, which is assumed to be equal to $1/v$. For this purpose, consider the following distribution functions.

I. $$G_v(t) = \begin{cases} 0 & t \le 0 \\ \frac{1}{2} & 0 < t \le 2/v \\ 1 & t > 2/v \end{cases}$$

II. $$G_v(t) = \begin{cases} 0 & t \le 0 \\ 1 - e^{-vt} & t > 0 \end{cases}$$

III. $$G_v(t) = \begin{cases} 0 & t \le 0 \\ (v/2)\, t & 0 < t \le 2/v \\ 1 & t > 2/v \end{cases}$$

IV. $$G_v(t) = \begin{cases} 0 & t \le 0 \\ \frac{1}{2}(3v)^3 \int_0^t z^2 e^{-3vz} dz & t > 0 \end{cases}$$

V. $$G_v(t) = \begin{cases} 0 & t \le 0 \\ v^2\, t^2/2 & 0 < t \le 1/v \\ 2vt - \frac{v^2 t^2}{2} - 1 & 1/v < t \le 2/v \end{cases}$$

VI. $$G_v(t) = \begin{cases} 0 & t \le 1/v \\ 1 & t > 1/v \end{cases}$$

We confine ourselves to the case of the nonloaded standby system and in the following tables we give all calculations dealing with the effectiveness of repair for the enumerated distributions for n=1, 2, 3 and 4 when $a_i = a$, that is, A=na, (n is the number of main units and (a) is the failure rate of the unit).

The tables give an amazingly small spread of the effectiveness of repair for such utterly different distributions of repair times that we have chosen. The Somewhat greater effectiveness for the first two distributions is due to the fact that they have an appreciable possibility of repair within short periods of time. The fact that the last distribution requires one and the same time for any repair somewhat reduces the effectiveness. The fact that the figures given in each table are so close to each other follows from theorem (2). Finally, it can be seen that the effectiveness of repair decreases when the number of main units Increases.

Table I

n=1 , A=a

$G_v(t)$	e_v	v/a			
		1	2	3	4
I	$1+\frac{1+e^{-2a/v}}{2(1-e^{-2a/v})}$	1.66	2.08	3.04	6.02
II	$1+v/2a$	1.50	2.00	3.00	6.00
III	$1+\frac{v(1-e^{-2a/v})}{2(2a-v(e^{-2a/v}))}$	1.38	1.86	2.85	5.84
IV	$1+\frac{(3v)^3}{2\left((a+3v)^3-((3v)^3)\right)}$	1.36	1.85	2.84	5.84
V	$1+\frac{v^2(1-e^{-a/v})^2}{2(a^2-v^2(1-e^{-a/v})^2)}$	1.33	1.81	2.80	5.84
VI	$1+\frac{e^{-a/v}}{2(1-e^{-a/v})}$	1.29	1.77	2.76	5.75

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$G_v(t)$	e_v	v/a			
		1	2	3	4
I	$1+\dfrac{1+e^{-4a/v}}{2(1-e^{-4a/v})}$	1.52	1.66	2.08	3.53
I I	$1+\dfrac{v}{4a}$	1.25	1.50	2.00	3.50
I I I	$1+\dfrac{v(1-e^{-4a/v})}{2(4a-v(1-e^{-4a/v}))}$	1.16	1.38	1.86	3.34
I V	$1+\dfrac{(3v)^3}{2((2a+3v)^3-(3v)^3)}$	1.14	1.36	1.85	3.34
V	$1+\dfrac{v^2(1-e^{-2a/v})^2}{2(4a^2-v^2(1-e^{-2a/v})^2)}$	1.11	1.33	1.81	3.30
V I	$1+\dfrac{e^{-2a/v}}{2(1-e^{-2a/v})}$	1.08	1.29	1.77	3.26

Table I I

n=2 , A=2a

Table I I I

n = 3 , A = 3a

$G_v(t)$	e_v	v/a			
		1	2	3	4
I	$1+\dfrac{1+e^{-6a/v}}{2(1-e^{-6a/v})}$	1.50	1.55	1.79	2.72
I I	$1+v/6a$	1.17	1.33	1.67	2.67
I I I	$1+\dfrac{v(1-6a/v)}{2(6a-v(1-e^{-6a/v}))}$	1.10	1.23	1.54	2.52
I V	$1+\dfrac{(3v)^3}{2((3a+3v)^3-(3v)^3)}$	1.07	1.21	1.52	2.51
V	$1+\dfrac{v^2(1-e^{-3a/v})^2}{2(9a^2-v^2(1-e^{-3a/v})^2)}$	1.05	1.18	1.49	2.47
V I	$1+\dfrac{e^{-3a/v}}{2(1-e^{-3a/v})}$	1.03	1.14	1.45	2.43

Table I V
n = 4 , A = 4a

$G_v(t)$	e_v	v/a			
		1	2	3	4
I	$1+\frac{1+e^{-8a/v}}{2(1-e^{-8a/v})}$	1.50	1.52	1.66	2.32
I I	$1+\frac{v}{8a}$	1.13	1.25	1.50	2.25
I I I	$1+\frac{v(1-e^{-8a/v})}{2(8a-v(e^{-8a/v}))}$	1.07	1.16	1.38	2.10
I V	$1+\frac{(3v)^3}{2((4a+3v)^3-(3v)^3)}$	1.04	1.14	1.36	2.09
V	$1+\frac{v^2(1-e^{-4a/v})^2}{2(16a^2-v^2(e^{-4a/v})^2)}$	1.03	1.11	1.33	2.06
V I	$1+\frac{e^{-4a/v}}{2(1-e^{-4a/v})}$	1.01	1.08	1.29	2.02

COMPLIANCE WITH ETHICAL STANDARDS

CONFLICT OF INTEREST

The author deciare that they have no competing interests.

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AVALIABILITY OF DATA AND MATERIALS

The data sets collected and / or analyzed during the current study are available from the corresponding author on request. The corresponding author had full access to all the data in the study and takes responsibility for the integrity of the data and the accuracy of the data analysis.

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