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A Full and Detailed Proof for the Riemann Hypothesis & the Simple Inductive proof of Goldbach's Conjecture

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ABSTRACT: *As in my previous two papers [2] & [3] about the boundary of the prime gap still cause some misunderstanding, I here in this paper tries to clarify those detailed steps in proving such boundary of the prime gap for a contradiction. Indeed, the general idea of my designed proof is to make all of the feasible case of the Riemann Zeta function with exponents ranged from 1 to s = u + v*I becomes nonsense (where u, v are real numbers with I is imaginary equals to* $(-1)^{1/2}$ *except that u = 0.5 with some real numbers v as the expected zeta roots. Once if we can exclude all other possibilies unless u = 0.5 with some real numbers v in the Riemann Zeta function's exponent "s", then the Riemann Hypothesis will be proved immediately. The truth of the hypothesis further implies that there is a need for the shift from the line x = 0 to the line x = 0.5 as all of the zeta roots lie on it. However, NOT all of the points on x = 0.5 are zeros as we may find from the model equation that has been well established in [2]. One of my application is in the quantum filtering for an elimination of noise in a quantum system but NOT used to filter human beings like the political counter-parts.In general, this author suggests that for all of the proof or disproof to any cases of hypothesis, one may need to point out those logical contradictions [14] among them. Actually, my proposition works very well for the cases in my disproof of Continuum Hypothesis [15] together with the proof in Riemann Hypothesis etc.*

KEYWORDS: detailed proof, Riemann hypothesis, simple inductive proof, Goldbach's Conjecture

INTRODUCTION

In my previous papers [2] & [3], this author has applied the telescopic multiplicative method as an elementary way to find the shortcut to the prime gaps. In the present paper, this author will try to us the same method for computing clearly how one may find exactly the prime gaps. By going a step, if we can exclude (or make nonsene) to all of the feasible index s in the Riemann Zeta function for s = u+v*I where u, v belongs to real number except u = 0.5 with some v belongs to real, then the Riemann Hypothesis will be proved. As it is well known that s = 0.5 + v*I must be true for some v belongs to real, then if all otherwise are roots of the Riemann Zeta function with the assumption that those equations of Riemann Zeta function are correctly satisfied, however, such assumption of all Riemann Zeta forms of equations will lead to some computed prime gaps boundary's contradictions or nonsense. The only exception is 0.5+v*I as the roots for some v belongs to real in the Riemann Zeta function which is known to be true for all of those Riemann Zeta forms of equations. In other words, 0.5 + v*I for some real values of v will satisfy all of the Riemann Zeta forms of equations and will NOT cause prime gap boundary's contradictions as shown in my paper [3]. Hence the Riemann Hypothesis will thus be said to be valid immediately together with a suggestion of a shift from line x = 0 to x = 0.5.

The Boundary for Prime Gaps (or A Proof for the Truthness of Riemann Hypothesis)

To show the following Riemann Zeta equation(s) – ξ(s) may be only true for the case of s = 0.5:

$\prod_{i=1}^{\infty}(z-z_i)^s$ = ξ(s) = $\sum_{n=1}^{\infty} 1/n^s$ = $\prod_{j=1}^{\infty}\left(1-1/prime_j\right)^{-s}$ ------------ (*')

Case I: s = 1 for ξ(s),

On the contrary, suppose we have the following equation for number theory of Riemann Zeta function ξ(1):

$\prod_{i=1}^{\infty}(z-z_i)$ = ξ(1) = $\sum_{n=1}^{\infty} 1/n$ = $\prod_{j=1}^{\infty}\left(1-1/prime_j\right)^{-1}$ ------------ (*)

By definition:

$$P_{j+1} = [\sum_{i=1}^{j+1} 1/i] \,/\, [\prod_{i=1}^{j}(1-1/prime_i)^{-1}\left(1-\frac{1}{Prime_{j+1}}\right)^{-1}]$$

Also, P_{j+1} = $\prod_{i=1}^{j+1}\left(z-z_j\right)$ / [($\prod_{i=1}^{j}(1-1/prime_i)^{-1}$* $\left(1-1/prime_{j+1}\right)^{-1}$)] -----------(*')

$\left(1-1/prime_{j+1}\right)$*$P_{j+1}$ = [$\prod_{i=1}^{j}(z-z_i)$ * (z - z_{j+1})/ ($\prod_{i=1}^{j}(1-1/prime_i)^{-1}$)]

If we first replace the $prime_{j+1}$ & $prime_j$ in (*') by P_{j+1} & P_j , then we have:

$P_{j+1} - 1$ = $\prod_{i=1}^{j}(z-z_i)$ *(z - z_{j+1}) / H_j ------------- (by definition of Harmonic Sum

$H_j = \sum_{i=1}^{j} 1/i$)

P_{j+1} = ($\prod_{i=1}^{j}(z-z_i)$ * (z - z_{j+1})/ $\sum_{i=1}^{j} 1/i$) +1 ----------(**")

But it is well known that $ln(j+1) \leqslant \sum_{i=1}^{j+1} 1/i \leqslant 1 + ln(j+1)$

($\prod_{i=1}^{j}(z-z_i)$ * (z – z_{j+1}) / ($1+ln(j+1)$)) ≤ P_{j+1} ⩽ ($\prod_{i=1}^{j}(z-z_i)$ * (z – z_{j+1}) / $ln(j+1)$)

(Since 1 / [1+ ln(j+1)] ≤ 1 ≤ 1 / ln (j+1), hence the multiplication of 1 / [1+ ln(j+1)] -- a smaller value leads to a smaller value, and 1 / ln (j+1) – a larger value leads to a larger value, to both sides of the inequality respectively will obviously preserve the above inequality. But NOT a normal inequality multiplication method.)

But P_j = [$\prod_{j=1}^{i}\left(z-z_j\right)$ / ($\prod_{j=1}^{i}\left(1-1/p_j\right)^{-1}$)] or P_j = ($\prod_{j=1}^{i}\left(z-z_j\right)$ / $\sum_{i=1}^{j} 1/i$)

----- (by definition of P_j and equation (*)),

i.e. ($\prod_{i=1}^{j}(z-z_i)$ / $1+ln(j)$) ≤ P_j ⩽ ($\prod_{i=1}^{j}(z-z_i)$ / $ln(j)$), thus we have:

or 1 / $1+ln(j)$ ≤ P_j ⩽ $ln(j)$ ----- (*******)

[(1 / $ln(j+1)$)^2 - (1 / $ln(j)$)^2] ⩽ [(1 / $ln(j+1)$)^2 - (1 / $1+ln(j)$)^2] ≤ (P_{j+1})^2 – (P_j)^2 ⩽ [(1 / $ln(j)$)^2 - (1 / $1+ln(j)$)^2] ≤ [(1 / $1+ln(j+1)$)^2 - (1 / $1+ln(j)$)^2]

Or [(1 / $1+ln(j+1)$)^2 - (1 / $ln(j)$)^2] ≤ (P_{j+1})^2 – (P_j)^2 ⩽ [(1 / $ln(j+1)$)^2 - (1 / $1+ln(j)$)^2]

----- (Equation (**)

By applying the Taylor approximation $\frac{2(x-1)}{(x+1)}$ to both $ln(j)$ and $ln(j+1)$ for the above equation (**), we may get:

$0 . \leq .-[(P_{j+1})^2 - (P_j)^2] . \leqslant . 7/144$

Also,
$[(1 / ln(j+1))^2 + (1 / 1 + ln(j+1))^2] \leqslant [(1 / ln(j+1))^2 + (1 / ln(j))^2] \leq (P_{j+1})^2 + (P_j)^2 \leqslant [(1 / ln(j))^2 + (1 / 1 + ln(j))^2] \leq [(1 / 1 + ln(j))^2 + (1 / 1 + ln(j))^2]$ ----- (Equation (**')
By applying the Taylor approximation $\frac{2(x-1)}{(x+1)}$ to both $ln(j)$ and $ln(j+1)$ for the above equation (**), and let x tends to infinity, we may get:

$1/9 \ . \leq . (P_{j+1})^2 + (P_j)^2 \ . \leq . 1/4$

After simplifying, we may get:

$1/81 . \leqslant . (P_{j+1})^2 * (P_j)^2 . \leqslant . 1/36,$

But it is well known that:

$(1 - ½) (1 - 1/3) (1 - 1/5) \ldots (1 - 1/prime_j)(1 - 1/prime_{j+1})*\xi(1) = 1 + \sum_{p/n for p>Prime} \frac{1}{n}$ -------- (*****)

$(1 - (½)^s)(1 - (1/3)^s)(1 - (1/5)^s) \ldots (1 - (1/prime_j)^s)(1 - (1/prime_{j+1})^s)*\xi(s) = 1 + \sum_{p/n for p>Prime} \frac{1}{n^s}$ -------- (*****")

i.e. $(1 - 1/prime_{j+1}) = (1 + \sum_{p/n for p>Prime} \frac{1}{n}) / [\xi(1)*(1 - ½) (1 - 1/3) (1 - 1/5) \ldots (1 - 1/prime_j)]$

But $\sum_{p/n for p>Prime} \frac{1}{n}$ tends to zero as Prime tends to infinity, thus

$(1 - 1/Prime_{j+1}) = 1/[\xi(1)* \sum_{i=1}^{j} 1/i],$ -------- (*****''')
or
$1- 1/P_j^2 = 1/Prime_{j+1}$, or

$1/ [1- 1/P_j^2] = Prime_{j+1}$

i.e. $(P_j^2) / [(P_j^2) - 1] = Prime_{j+1}$

$Prime_{j+1} - Prime_j = \{(P_j^2) / [(P_j^2) - 1]\} - \{(P_{j-1}^2) / [(P_{j-1}^2) - 1]\}$

After simplifying, we may get:
$Prime_{j+1} - Prime_j = [-(P_j^2 - P_{j-1}^2)] / [p_j^2 p_{j-1}^2 - (p_j^2+p_{j-1}^2) +1]$ ----------(***)
Hence, $0 . \leq .$**$Prime_{j+1}$ – $Prime_j$**$. \leq . 63 / 1024$ which is obviously contradicting to the fact that the prime gaps are always positive and non-fractional. Indeed, the contradiction for the fractional valued boundary occurs is mainly due to the initial assumption of the Riemann zeta equation, for s = 1 or $\xi(1)$ in (*).

Case II: s = u+v*I, where u, v belongs to real number for ξ(s),
Now, replace the $\xi(1)$ by $\xi(s)$ with s = u + v*I where {u, v are real numbers and
$I = (-1)^{1/2}$ }/{0.5+y*I) for some y belongs to real}, then for the Harmonic sum H_j , it tends to cot(j) and $\sum_{i=1}^{j} 1/i^s = r*cot[j - tan^{-1}(v/u)] = r*\{[u*cot(j) + v] / [u - v*cot(x)]\}$, [1], where r = |s| = (|u|² +

$|v|^2)^{1/2}$ and u = |u| or replaced i by k as the difference between the harmonic series and the $\sum_{i=1}^{j} 1/i^s$ is just by a rotation plus a zoom in or out.

$P^*_{j+1} = (\prod_{i=1}^{j}(z - z_i)^s * (z - z_{j+1})^s / \sum_{i=1}^{j+1} 1/i^s)$

$= (\prod_{i=1}^{j}(z - z_i)^s * (z - z_{j+1})^s) / (r^*\{[u^*\cot(j+1) + v] / [u - v^*\cot(j+1)]\})$

$= [u - v^*\cot(j+1)] * [(\prod_{i=1}^{j}(z - z_i)^s * (z - z_{j+1})^s] / \{r^*[u^*\cot (j+1) + v]\}$

i.e. If the s changes from 1 to s = u + v*i, then there is a factor of [u – v*cot(j+1)] / {r *[u*cot(j+1)+v]}.

$P^*_{j+1} = [u - v^*\cot(j+1)] / \{r *[u^*\cot(j+1)+v]\} * P_{j+1}$

Hence by rearrange those terms, we have:

$P^*_{j+1} = [P_{j+1}]^*([u - v^*\cot(j+1)] / \{r *[u^*\cot(j+1)+v]\})$

i.e. the difference between s = 1 or P_{j+1}, and s = u + v*I or P^*_{j+1} is only by the factor of ([u – v*cot(j+1)] / {r *[u*cot(j+1)+v]}). Or to transform the P_{j+1} into P^*_{j+1}, we have to multiple the factor ([u – v*cot(j+1)] / {r *[u*cot(j+1)+v]}).

Let W = ([u – v*cot(j+1)] / {r *[u*cot(j+1)+v]}) ------- (*"*)

Next, from (*****"), we may get:

$[1- (1/Prime_{j+1})^S] = 1 / [\xi(s)^*[(1- (1/2)^S)(1- (1/3)^S)...(1- (1/Prime_j)^S]$

But by the assumption, $\sum_{i=1}^{j} 1/i^s = [(1- (1/2)^S)(1- (1/3)^S)...(1- (1/Prime_j)^S] = P_j^*$, hence

$[1- (1/Prime_{j+1})^S] = 1 / [P_j^*]^2$,

Rearrange gives, $Prime_{j+1} = 1 / (1 - 1/[P_j^*]^2)^{1/s} = [(P_j^*)^2 / (P_j^*)^2 - 1]^{1/s}$

$Prime_{j+1} - Prime_j = [(P_j^*)^2 / (P_j^*)^2 - 1]^{1/s} - [(P_{j-1}^*)^2 / (P_{j-1}^*)^2 - 1]^{1/s}$

$Prime_{j+1} - Prime_j = \{ [(P_j^*)^2 - 1] + 1 / [(P_j^*)^2 - 1]\}^{1/s} - \{ [(P_{j-1}^*)^2 - 1] + 1 / [(P_{j-1}^*)^2 - 1]\}^{1/s}$

$= [1 + (1/s)^*(1/[(P_j^*)^2 - 1])] - [1 + (1/s)^*(1/[(P_{j-1}^*)^2 - 1])]$

$= (1/s)^*[(P_{j-1}^*)^2 - (P_j^*)^2] / \{[(P_j^*)^2 - 1]^*[(P_{j-1}^*)^2 - 1]\}$

$= [(1/s)^*(-1/j)^*(2P_{j-1}^* + 1/j)] / \{[(P_j^*)^2(P_{j-1}^*)^2 - [(P_j^*)^2+ (P_{j-1}^*)^2] + 1\}$

i.e. $(\overline{s} / |s|^2)^*(-1/j)^*(2P_{j-1}^* + 1/j)] / \{[(P_j^*)^2(P_{j-1}^*)^2 - [(P_j^*)^2+ (P_{j-1}^*)^2] + 1\}$,

Alternatively, we may consider:

$(Prime_{j+1})^s - (Prime_j)^s = \{ [(P_j^*)^2 - 1] + 1 / [(P_j^*)^2 - 1]\} - \{ [(P_{j-1}^*)^2 - 1] + 1 / [(P_{j-1}^*)^2 - 1]\}$

As $(Prime_{j+1})^s = [1 + (Prime_{j+1} - 1)]^s$

$= 1 + s^*(Prime_{j+1} - 1)$,

thus $(Prime_{j+1})^s - (Prime_j)^s = s^*(Prime_{j+1} - Prime_j)$

$= [(P_{j-1}^*)^2 - (P_j^*)^2] / \{[(P_j^*)^2 - 1]^*[(P_{j-1}^*)^2 - 1]\}$,

$Prime_{j+1} - Prime_j = [(1/s)^*(-1/j)^*(2P_{j-1}^* + 1/j)] / \{[(P_j^*)^2(P_{j-1}^*)^2 - [(P_j^*)^2+ (P_{j-1}^*)^2] + 1\}$

But $P^*_{j+1} = [P_{j+1}]^*W$ & s = u+vi,

$Prime_{j+1} - Prime_j = \{[-(\overline{s} / |s|^2)^*(1/j)] (2W^*P_{j-1} + (1/j)])\} / [W^4(P_j)^2(P_{j-1})^2 - W^2[(P_j)^2 + (P_{j-1})^2] + 1]$,

or two methods produce the same expression result for the prime gap.

In terms of s = u +v*I, we may get:

$Prime_{j+1} - Prime_j = [(1/s)^*(-1/j)^*(2P_{j-1}^* + 1/j)] / \{[(P_j^*)^2(P_{j-1}^*)^2 - [(P_j^*)^2+ (P_{j-1}^*)^2] + 1\}$

-------- (******)

In fact, when the denumerator attains its minimum, then the prime gap gets its maximum value, hence, we differentiate the above implict equations by Chain Rule and set the result equals to zero for the minimum values ,

$$\frac{\partial f(w)}{\partial k} = \frac{\partial f(w)}{\partial w} * \frac{\partial w}{\partial k}$$

Then let f(W) = $W^4*P_j^2*P_{j-1}^2 - W^2*[P_j^2 + P_{j-1}^2] + 1$, $\frac{\partial f(w)}{\partial w}$, this gives us:

$$= \{4W^3*[(P_j^2)(P_{j-1}^2)] - 2W*[P_j^2 + P_{j-1}^2]\}*\frac{\partial w}{\partial k}$$

Also, we need to approximate W in (*"*) by taylor series and then differentiate W with respect to k (or replaced by x), we have:

$$W = ([u - v*\cot(x+1)] / \{r*[u*\cot(x+1)+v]\})\text{-------}(*\text{"}*\text{"})$$

$$= ([u - v*2*(x) / (x+2)] / \{r*[u*2*(x) / (x+2)] +v\})$$

$$\frac{\partial w}{\partial x} = \frac{\left[-\frac{4v}{(x+2)^2}\right]\left\{r\left[\frac{2ux}{x+2}\right]+v\right\}-\left[\frac{ux-2vx+2u}{x+2}\right]\left\{[0]+r\left[\frac{4u}{(x+2)^2}\right]\right\}}{\left\{r\left[\frac{2ux}{x+2}\right]+v\right\}^2}$$

Thus, $\frac{\partial f(w)}{\partial x} = \{4W^3*[(P_j)(P_{j-1})] - 2W*[P_j + P_{j-1}]\}*\left(\frac{\left[-\frac{4v}{(x+2)^2}\right]\left\{r\left[\frac{2ux}{x+2}\right]+v\right\}-\left[\frac{ux-2vx+2u}{x+2}\right]\left\{[0]+r\left[\frac{4u}{(x+2)^2}\right]\right\}}{\left\{r\left[\frac{2ux}{x+2}\right]+v\right\}^2}\right)$

$= 0$

W = 0 or $W^2 = [P_j^2 + P_{j-1}^2]/\{2*[(P_j^2)(P_{j-1}^2)]\} = 0$ or

$8uvrx + 4v^2(x+2) - 4ur(ux - 2vx + 2u) = 0$

v = 1 / [cot(x) / ln(x)] or $x = -\frac{2(u^2r-v^2)}{u^2r-4uvr-v^2}$, $W^2 = \frac{9(2x^5-5x^4+4x^3-x^2-18x+9)}{2(x^2-3)^2((x-1)^2-3)^2}$

by putting $x = -\frac{2(u^2r-v^2)}{u^2r-4uvr-v^2}$, we get $W = \frac{\left[\frac{u}{2}+\frac{v^2}{2ur}\right]}{r\left[\frac{u^2}{2v}+v-\frac{v}{2r}\right]}$ which is an optimum value.

For $W^2 = \frac{9(2x^5-5x^4+4x^3-x^2-18x+9)}{2(x^2-3)^2((x-1)^2-3)^2} = 0$: x = ½ or x = ½ - I$\sqrt{11}$/2 or x = ½ + I$\sqrt{11}$/2 or x = ½ - I$\sqrt{13}$/2 or x = ½ + I$\sqrt{13}$/2

w = (u – 1.83048v) / [r*(1.83048u + v)] or w = [u – (0.00221+/- 1.00142i)v] / [r*((0.00221+/- 1.00142i)u + v] or w = [u – (0.00124+/- 1.00079i)v] / [r*((0.00124+/- 1.00079i)u + v]

(N.B. All of the above solved equation and computational values were calculated by the Canada's Maple Soft 2022 with a lisenced Student version)

$P^*_{j+1} = [P_{j+1}]*W = [P_{j+1}]*\frac{\left[\frac{u}{2}+\frac{v^2}{2ur}\right]}{r\left[\frac{u^2}{2v}+v-\frac{v}{2r}\right]}$ and $(1 / 1 + ln(j)) \leq P_j \leqslant (1 / ln(j))$ by (*******)

Prime $_{j+1}$ – Prime $_j$ = $\{[-1 / (u+vi)*j]*(2W*P_{j-1} + (1/j)])\} / [W^4(P_j)^2(P_{j-1})^2 - W^2[(P_j)^2 + (P_{j-1})^2] + 1]$,

By taking limit j tends to infinity and $(1 / 1 + ln(j)) \leq P_j \leqslant (1 / ln(j))$, $[W^4(P_j)^2(P_{j-1})^2 - W^2[(P_j)^2 + (P_{j-1})^2] + 1]$ tends to 1 and $2W^* P_{j-1}$ tends to zero,

thus we have:

Prime $_{j+1}$ – Prime $_j$ = $\{[-1 / (u+vi)*j]* (1/j)]\}$

Hence, the computed prime gap is obviously in a complex number format but the result should theoretically be an integer. Hence, a contradiction occurs due to the assumpution that ξ(s) satisfies all of the Riemann Zeta equations (*****).

(N.B. Certainly, one may define the complex number prime by using the Gaussian integer or the algebraic (integer) ring. But this author wants to remind that for a complex number to be a Gaussian prime if and only if it fulfills a certain conditions as listed in [13]. But now, both of u & v are any real numbers that may not belong to such kind of the algebraic ring. Indeed, if one is researching about the computational number theory together with the Gaussian prime gaps, then it may be possible that there are some feasible u and v that belong to the real number with $u^2 + v^2$ is an ordinary prime.

However, by definition, a Gaussian integer is when both of the real and imaginary parts are integers which contradict to the fact that

s = {u + v*I/0.5+y*I when u, v & y belong to real numbers}. Unless it is our world's recognization to relax or extend the condition of the Gaussian integer by considering both of the real and imaginary parts are real numbers, otherwise my proof in case II is still validated. If relaxed, then the Gaussian Integers ring may induce a principal ideal domain and in some conditions if an element z is irreducible, the ideal (z) is maximal that may establish the corresponding field. By fixing the field, we may fix the prime together with a suitable elliptic curve defined over the field. If one can select a particular prime, one may get a faster modular reduction for the elliptic curve computations. The outcome may be we can solve the Fermat's Last Theorem. In practice, the above depictions may lead to another deeper and finer research which may be left to those who feel interested instead of my present one. In the meantime, under the normal Gaussian integer definition – u & v must be integers, this author proposes that ξ(s) will still cause a contradictary complex value in the prime gap for the aforementioned set of s. Hence, the above s for ξ will be excluded.

Case III: s = 0.5 +/- [4*cot(ln(x)) / $(x+1)^2$]*I [3], = 0.5 +/- 2*I when x tends to infinity with Taylor approximation cot(x) = (1/x) – (x/3) & ln(x) = 2(x-1)/(x+1).

From (******), there is something interesting as we may still try to find the minimum value of the prime gap for if $W^2 = 0$. Then,

([u – v*cot(j+1)] / {r *[u*cot(j+1)+v]}) = 0

and

v = 1/ [cot(k)/ln(k)] = $[\cot(k)/\ln(k)]^{-1}$ = -[cot(k)/ln(k)]

which is just the conjugate pair of v = [cot(k)/ln(k)]. Hence, it is consistent with my computational result for Riemann Zeta function's non-trivial zeros that obtained in my paper [3].

Then we may integrate the above equation to get the primivate, set up the differential equation of the proposed primivate and solve it by either laplace or fourier transform. Indeed, the delta impluses of the answer implies that we may already have establish the practical quantum filter together with the convolution for the next generation of commerical quantum signal processing etc which belongs to the fields of engineering for business in our everyday usage. Actually, what the delta function or the Laplace transform does is to analyse the signal (or filter those impulses). (This author may continue the research and written thesis whenever time and conditions are fulfilled & available as this is a kind of research chain for knowledge). This fact may be the **limitation** of the present paper but in terms of science or pure mathematics fields, my proof for the non-trivial zeros of Riemann Zeta function or the truthness of Riemann Hypothesis is full and complete.

At the same time, $\text{Prime}_{j+1} - \text{Prime}_j$, (******) is reduced to $(1/j)^2 = (P_j - P_{j-1})^2$ as W tends to zero. When j tends to infinity, the minimum prime gap also tends to zero which is consistent with the present well-known proved result and there is NO contradiction occurred as all otherwise complex numbered exponents will always lead to one.

Hence, we may conclude that

s = 0.5 +/- [4*cot(ln(x)) / $(x+1)^2$]*I is the only non-trivial zeros of Riemann Zeta function.

i.e. $\prod_{i=1}^{\infty}(z-z_i)^{0.5+yi}$ = ξ(0.5 +y*I) = $\sum_{n=1}^{\infty} 1/n^{0.5+yi} = \prod_{j=1}^{\infty}\left(1-1/prime_j\right)^{-(0.5+yi)}$ = 0--------(*'')

Remaks: The addition of two primes (odd) is always even unless one of the prime equals to 2 (even). The prime gap may be expressed as the addition and difference between $p_{j+1} = \sum_{i=1}^{j+1} 1/i$ and $p_j = \sum_{i=1}^{j} 1/i$ for ξ(1). Thus, in advancing a step, replace the P_j & P_{j-1} by $Prime_j$ & $Prime_{j-1}$
$X_{j+1} – X_j = \{-[(Prime_j)^2 – (Prime_{j-1})^2]\} / [(Prime_j)^2 (Prime_{j-1})^2 - ((Prime_j)^2 + (Prime_{j-1})^2) + 1]$

I.e. $(0)*2n \le X_{j+1} – X_j \le (k)*2n$ for any one of the primes NOT equal to 2 or some of the elements in the set of even numbers 2n for some constant k.
Or $(0)*(2n+1) \le X_{j+1} – X_j \le (k')*(2n+1)$ for one of the addition primes equals to 2 or some of the elements in the set of odd numbers 2n+1 for some constant k'.

My suggestion is to prove the above two statements span for all elements in the set of even numbers or odd numbers by mathematical induction individually. However, the focus of the present paper is in the field of prime gap while the proof of the Goldbach's conjecture will be shown in the next section (part) by induction.

Indeed, by the Jurkat-Richard theorem, we may have the fact that every integer greater than 6 is a sum of distinct primes [6]. This fact may imply the Chen's theorem which states every sufficient large even number can be written as the sum of either two primes or a prime with a semiprime [7]. Actually, Chen's theorem is a weakened form of the Goldbach's conjecture.

Therefore, the difference (in boundary) of a prime gap is a fractional number which contraicting to the fact that prime gap must be a positive integer as my computational result is $0 \le$ **$Prime_{j+1}$ – $Prime_j$** $\le 63/1024$. Moreover, it is also well known that $\sum_{i=1}^{\infty} \frac{1}{prime_i}$ diverges, [8], for all prime $prime_i$s to infinity which is obviously a contradiction. Thus, the both contradictions (diverges & fractional prime gap difference) are indeed originated from the assumption that s = 1 in the equation (*) or ξ(1)) plus all otherwise will give you a contradictary in prime gap difference with complex number as the final answer, these contradictions lead to the conclusion that the equation (*) may be only true for the 0.5 + I*y as shown in my previous aforementioned proof in the previoust section. In practice, we may consider the fractional boundary of the prime gap $(1/j)^2$(which tends to be a smaller positive value as j tends to infinity) a form of constitency for s = 0.5 +/- I*y for some y belons to real. But at the same time, for otherwise of all complex power exponents, the prime gap will give you a contradictary bounding prime gaps in complex number. Therefore, there may be a type of new kind philosophy as I named it as "recursive contradiction" or "contradiction entanglement" etc. Moreover, for ξ(s), s = u + v*I where {u, v are real numbers and $I = (-1)^{1/2}$ }/{0.5+yI) for some y},
Such kind of all contradictary results are due to the fact that there are something wrong in the assumptions of the equations $\prod_{i=1}^{\infty}(z - z_i)^s$ = ξ(s) = $\sum_{n=1}^{\infty} 1/n^s = \prod_{j=1}^{\infty}\left(1 - 1/prime_j\right)^{-s}$ are true for (s = 1 & {u, v are real numbers and $I = (-1)^{1/2}$ }/{0.5+y*I).

To conclude in this section, this author's practical proof of Riemann Hypothesis by excluding all of the feasible cases such as s = 1 and s = u+v*I for some real unumbers u and v. The only left answer is just the 0.5 +/- $[4*\cot(\ln(x)) / (x+1)^2]*I$ which have been shown as the root of Riemann Zeta fuction in [3]. My proved fact will then eleminate the contradictary results found in the boundary of prime gap values as equation (*****) will give a trivial zeros. Or there will be NO more boundary gap contradiction when s = 0.5 +/- $[4*\cot(\ln(x)) / (x+1)^2]*I$. Then, the Riemann Hypothesis is proved. **At the same time, the above proving method may be viewed as a kind of quantum filtering to a**

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quantum system's noise but NOT for filtering the human political counter-parts. Or otherwise, such (or all) kind(s) of filtering will become Harry Potter's JEG ER VOLDEMORT.

In brief, one may solve the Riemann Hypothesis by considering the following three cases:
Case I: s = 1 or ξ(1) which leads to the contradiction that the prime gap is a fractional number and should be excluded;

Case II: s = {u + v*I / 0.5+y*I for some y belongs to real} that leads to the contradiction that the prime gap is a complex-value and should be excluded;

Case III: s = 0.5 + y*I for some y belongs to real that makes NO contradictary results and should be the most favourable non-trivial zeros to the Riemann Zeta function or the Riemann Hypothesis is thus proved to be correct in such sense.

An Inductive Proof for Goldbach's Conjecture
My proof for the Goldbach's conjecture (for any natural number N, it can be expressed as the sum of two primes) as follow:

1 = 1
2 = 2
3 = 1 + 2
4 = 2 + 2 / 1 + 3
5 = 3 + 2 / 2 + 3
6 = 1 + 5 / (2 + 2) + 2 / 3 + 3
7 = 5 + 2 / 3 + 4 / 2 + 2 + 3
8 = 3 + 5 / 3 + 3 + 2 / 1 + 7 / 4 + 4
9 = 2 + 7 / 2 + 2 + 5
10 = 3 + 7 / 5 + 5
11 = 2 + 2 + 7 / 3 + 3 + 5
12 = 5 + 7 / 1 + 11 / 6 + 6
13 = 2 + 11 / 5 + 5 + 3
14 = 3 + 11 / 7 + 7 / 1 + 13 / 7 + 7
15 = 2 + 13 / 11 + 2 + 2
16 = 3 + 13 / 11 + 5 / 8 + 8

For the even numbers, which can be written in the form of 2*n
Suppose it is true for n = k, i.e. 2*k = (k-1)+(k+1) where k-1 & k+1 are both primes,
Now consider 2*(k+1) = k + (k+2)
= [(k+1)-1] + [(k+1)+1]
= (m – 1) + (m + 1) where m = k+1
which is in the form of n = 2k = (k-1) + (k+1), hence it is true for n = 2k+2,
For the odd numbers, which can be written in the form of 2*n+1.
Suppose it is true for n = 2k+1, i.e. 2*k+1 = [(k-1)-1]+[(k-1)+1]+3 where k-2 & k & 3 are three primes,
Now consider 2*(k+1)+1 = 2k+3
= (k-1) + (k+1) + 3
= [((k-1)+1)-1] + [((k-1)+1)+1] + 3

= (p – 1) + (p + 1) where p = (k-1)+1 = k
which is in the form of n = 2k+1 = [(k-1)-1] + [(k-1)+1], hence it is true for n = 2k+3.

Hence, for any even valued natural number N, it can be expressed as the sum of two primes inductively (Strong Goldbach Conjecture) [4]. For any odd valued natural number N, it can be expressed as the sum of three primes inductively (Weak Goldbach Conjecture) [5]. To conclude, we may say that Goldbach is inductively true for all natural numbers N no matter even or odd numbers. However, there are still some problems for the mathematical induction in a philosophical way such as the Hume's problem [9].

Indeed, by the Jurkat-Richard theorem, we may have the fact that every integer greater than 6 is a sum of distinct primes [7]. This fact may imply the Chen's theorem which states every sufficient large even number can be written as the sum of either two primes or a prime with a semiprime [8]. Actually, Chen's theorem is a weakened form of the Goldbach's conjecture. This author wants to remark that if one request a full combinatorial inductive proof to Goldbach's conjecture, please refer to the paper titled "A Combinatorial Proof of Goldbach's Conjecture" by Hisanobu Shinya.

CONCLUSION

In a nutshell, this author have already proved the truthness of Rienamm Hypothesis mainly by the multiplicative telescopic method together with the differences in prime gaps which will give the complex number as the boundary and hence a contrdiction whenever s NOT equals to 0.5 +/- $[4*\cot(\ln(x)) / (x+1)^2]*I$ for Riemann Zeta function $\xi(s)$. When s = 1 for $\xi(1)$, the boundary of the prime gap become a fractional number which is certainly a contradiction. Finally, for the case of s = 0.5 +/- $[4*\cot(\ln(x)) / (x+1)^2]*I$, my proof for $\xi(s)$ just have given $v = [\cot(x)/\ln(x)]^{-1}$ together with the result of minimum prime gaps approaching to zero. This facts are consistent with my previous paper[3] and the present well-known number theoretists' results. Thus, both of them induce no contradictions.

In reality, from (*****""), $Prime_{j+1} = (P_j^2) / [(P_j^2)- 1]$ will give some stranger results such as irrational values in the summation to x where x = (+/-) $\sqrt{3}$ (but we can only sum to an integer or the (stranger) prime number model equation

$1 + [3x^2 / (x^2 – 3)^2 - 1]$ that attains its minimum at x = (+/-) $\sqrt{3}$ etc, as the assumption of the equation (*****"") for finding a minimum prime equals to -8 or |-8| = 8. However, we may still have a well defined Harmonic series in the long extended summation format but NOT defined in the summation's expression symbol format. To be precise, we may change/turn the (strange) summation into an integration or the $\int_1^{\sqrt{3}} \left(\frac{1}{x}\right) dx$ = $\ln|(\sqrt{3})|$ for the minimum value of the P_j or minimum prime equals to $1 + 1/[\ln|(\sqrt{3})|^2-1]$. Practically, the prime model equation in the present case may be $(\int_1^{x} \left(\frac{1}{k}\right) dk)^2 / (\int_1^{x} \left(\frac{1}{k}\right) dk)^2 – 1)$ in the integral format or $1+\frac{\partial[\ln(\ln(k)^2)-1]}{(2\partial \ln(k))}$ in differential equation format. In the sense of Java computing, we may write a program segment about the recursive logarithm, calculate the complexity of the above PDE's algorithm (or my proposed prime model equation in the present case), by Big O notation etc. In order to obtain the expected true prime number model equation, we may need to apply the spatial point pattern method to investigate the prime number finding process relationship through the R and JASP statistical programming etc or in [11].

In practice, as this author have already found the model equation of the Riemann non-trivial zeros in [3], one may then apply the fact that the model equation of the prime number is just the fourier transform [10] of the Riemann Zeros' model equation (i.e. 0.5 +/- i*($\frac{4}{(x+1)^2} cot(ln(x))$)) or the Fourier type duality relation between the prime numbers and zeros of the Riemann Zeta function. Hence, the model equation of the prime number is:
1.25331 δ(w) + (4 i sqrt(2 π) δ(w) cot(log(x)))/[(x + 1)^2] (by Fourier Transform Calculator in [12]).

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