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The Effects of Stochastic Variables on the Analysis of Stock Market Prices

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ABSTRACT: *In this paper, stochastic differential equation with some imposed parameters in the model was considered. The problem was solved by adopting Ito's theorem to obtain an analytical solution which was used to generate various discrepancies on various asset prices. The asset values were obtained through the influences of some key stochastic variables which shows as follows:(i) increase in α when μ and σ are fixed increases the value of asset returns (ii) a little increase on time when return rates and stock volatility are fixed also increases the value of assets (iii) an increase in the volatility parameter increases the value of asset pricing (iv) , (v) a measure of α parameter shows the various levels of long term investment plans . Finally, the normality probability plots are not statistically significant and besides do come from a common distribution which has a vital meaning in the assessment of asset values for capital market investments. However, the Tables, graphs and other stock variables were discussed. The governing investment equation is reliable and therefore is found to be adequate.*

KEY WORDS: asset value, normality test, financial market and stochastic analysis.

INTRODUCTION

In several areas of life, models are essential to enhance the understanding of the configuration the system's performance. These models are made up of many parameters and their results where the there is a correlation between the parameters and physical quantities. If the parameters are known, then they are said to be Ordinary Differential Equations (ODEs) and the models are termed deterministic. But, if there is noise, there may be no adequate information on the parameters. Hence, stochastic models which include this noise term in the deterministic model described as Stochastic Differential Equations (SDEs). A comparison of the SDEs and the Deterministic models reveals that a comprehensive insight about real-life experience is provided by the SDEs than the Deterministic models. Therefore, in many applications of human endeavors such as finance, etc, the SDEs play significant roles. For example, [1] has said that the system fulfills the principle of causality, in many applications of SDEs. this implies that the expected condition of the system is determined only by the existing one, implying that, it does not depend on the precedent. This can be noticed or identified in [1]. Though, the results of phenomena in many fields like engineering, physics, economics, etc., may be unnoticed at the point of occurrence. Hence, the SDEs with the principle of causality, give only an estimation of the actual experiences. Thus, other terms like time delay, realized from the previous condition of the system is included in the model to produce an accurate one known as Stochastic Delay Differential Equations (SDDEs). The SDDEs which give a mathematical formulation for this kind of system, produce the two; Deterministic Delay Differential Equations (DDEs) and Stochastic Ordinary Differential Equations (SODEs). The applications of SDDEs can be seen in many areas like [2,3] for physics, [4-6] for Bio-physics, [7-9] for Economic and Finance and [10-12] for Biology. In this study, we consider the example from the stock market. In recent time, SDEs have played an important role in the estimation of financial assets. The solution to the numerous complex pricing problems have been provided by SDEs. Effective market hypothesis is considered as a basic assumption, in these asset pricing models. In conformity to this hypothesis [1];

- All accessible past information is analyzed and already in the present price of the stock and no information about upcoming performance is given.
- There is a quick response of markets to any new information about asset, i.e. asset prices move randomly.

Taking into consideration these two assumptions, changes in the asset price define a Markov process. Although [12] indicated that stock returns depend on past returns. It is seen in [13] that the investor expects the stock price to follow a Black-Scholes diffusion process while the insider understands that the drift and volatility of the stock price process are affected by certain trading periods. They crave to create a pricing model made up of the historical information to analyze the current and to estimate future prices, thereby resulting to the prediction of the market movement and improvement on investments. On the contrary, this could not be achieved with SDEs, because of the efficient market hypothesis. Thus, the inclusion of that information in the SDEs, as a delay term is a more pragmatic mathematical formulation, called SDDE. It is used to evaluate stock prices. Rather than assuming and making use of the SDE as a model but making use of the new model and the Markov properly. It is better to postulate that the future stock price relies also on past or previous condition but not on the existing condition.

To a greater extent, a reasonable number of scholars have looked into the modelling of stock prices and its asset value functions, like [14-17], etc.

Earlier studies have therefore investigated similar problems but did not consider the effects of some stochastic variables in assessing asset values. In particular, some studies, for instance [14] ,[15] and [20] etc.

In this study we considered Non-linear Stochastic Differential Equation (SDE). Apart from correctly posing the models for the assessment of asset values, we also solved in details by the method of Ito's to obtain a solution which gave various conditions of analyzing some stochastic variables via asset values. More so, a statistical normality test was done independently to show some levels of probability distributions. To this end, our novel contribution is unique , efficient and reliable in time varying investments.

MATHEMATICAL PRELIMINARIES

For proper understanding of this paper we therefore present few intricate definitions covering this dynamic area of study:

Definition 1: Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set T, $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 2: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).

Definition 3: Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k \tag{1}$$

where X_k are independent and identically distributed random variables.

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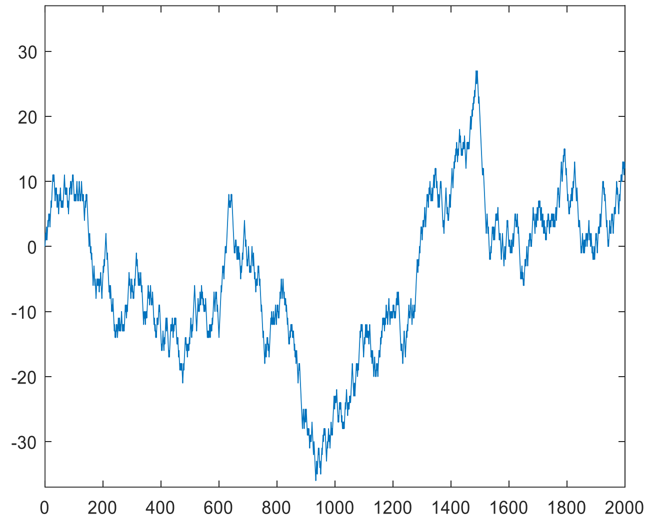

Figure 1: Sample trajectory of random walk

Definition 4: (Differential Equation): is an equation which has functions and their derivatives. In reality the functions is associated to real quantities whereas the derivatives denotes rate of change. Example of differential equation is follows

$$\frac{dS(t)}{dt} = \mu S(t) \tag{2}$$

$$S(0) = S_0 \tag{3}$$

where $S(t)$ represent asset price, μ rate of return, $\frac{dS(t)}{dt}$ is the rate of change of asset price and S_0 is the initial stock price; (2) and (3) can be obtained using variable separable which gives:

$$S(t) = S_0 e^{\mu t} \tag{4}$$

Therefore μ is not known completely which is subject to environmental effects. Therefore (2) can be written as

$$dS(t) = \mu S(t)\,\mathrm{dt} + \sigma S(t)\,\mathrm{dz(t)} \tag{5}$$

Where σ is the volatility, dZ is the Brownian motion or Wieners process which is random term, the stochastic term added to (2) gives (5) which makes it stochastic differential equation

Definition 5: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), ..., W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g as follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T,$$

$$X(0) = x_0,$$

where $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f : [0, \mathrm{T}] \times \mathbb{R}^n$ *and* $g : [0, T] \times \mathbb{R}^n \to \times \mathbb{R}^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S)$$

Where dX, dZ are terms known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$,[22].

Theorem 1.1:(Ito's lemma). Let $f(S, t)$ be a twice continuous differential function on $[0, \infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0\ ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higer order terms } (h.\text{o}, \text{t})\ ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2}(a_t dt + bdz(t))^2$$

$$= \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} b_t^2 dt,$$

$$=\left(\frac{\partial F}{\partial S_t}a_t+\frac{\partial F}{\partial t}dt+\frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2\right)dt+\frac{\partial F}{\partial S_t}b_t dz(t)$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies and hence, the function $F(S,t)$,Ito's lemma gives:

$$dF=\left(\mu S\frac{\partial F}{\partial S}+\frac{\partial F}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 F}{\partial S^2}\right)dt+\sigma S\frac{\partial F}{\partial S}dz(t)$$

MATHEMATICAL FORMULATION OF THE PROBLEM

The some stochastic variables was considered in the assessment of asset values which is assumed to arise in financial market. The entire origin of stock dynamics is found in a complete probability space $(\Omega,\mathrm{F},\wp)$ with a finite time investment horizon $T>0$. Therefore, we have the following stochastic differential equation below;

$$dX_1(t)=\alpha\mu\mathrm{X}_1(t)dt+\beta\sigma X_1(t)dZ^{(1)}(\mathrm{t}) \tag{6}$$

with the following initial conditions:

$$X_1(0)=X_0, t>0 \tag{7}$$

where, $X_1(t)$ are asset prices, The expression dZ , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion where μ is an expected rate of returns on stock, α is measures the rate at which the drift (return) reaches its long term investments plans, β is a constant, σ is the volatility of the stock , dt is the relative change in the price during the period of time.

METHOD OF SOLUTION

The model (1) is stochastic differential equations whose solution are not trivial. We implement the methods of Ito's lemma in solving for $\mathrm{X}_1(t)$ To grab this problem we note that we can forecast the future worth of the asset with sureness.

From (6) Let $f\left(X_1,t\right)=\ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_1}=\frac{1}{X_1},\ \frac{\partial^2 f}{\partial X_1^2}=-\frac{-1}{X_1^2},\ \frac{\partial f}{\partial t}=0 \tag{7}$$

According to Ito's gives

$$df\left(X_1,t\right)=\sigma X_1\frac{\partial f}{\partial X_1}dZ(t)+\left((\mu X_1(t)\frac{\partial f}{\partial X_1}+\frac{1}{2}\sigma^2 X_1^2\frac{\partial^2 f}{\partial X_1^2}+\frac{\partial f}{\partial t}\right)dt \tag{8}$$

Substituting (6) and (7) into (8) gives

$$=\beta\sigma X_1\frac{1}{X_1}dZ(t)+\left((\alpha\mu X_1(t)\frac{1}{X_1}+\frac{1}{2}\sigma^2 X_1^2(-\frac{1}{X_1^2})+0\right)dt$$

$$=\beta\sigma dZ(t)+\left((\alpha\mu-\frac{1}{2}\sigma^2\right)dt$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_1=\int_0^t df\left(X_u,u\right)=\int\left((\alpha\mu-\frac{1}{2}\sigma^2\right)du+\beta\int_0^t\sigma dZ(t)$$

$$\ln X_1-\ln X_0=\left((\alpha\mu u-\frac{1}{2}\sigma^2 u\right)|_0^t+\beta\left(\sigma Zu\right)|_0^t \tag{9}$$

$$\ln\left(\frac{X_1}{X_0}\right)=\left((\alpha\mu-\frac{1}{2}\sigma^2\right)t+\beta\sigma Z(t)$$

Taking the ln of the both sides and applying the initial condition in (7) gives:

$$X_1(t)=X_0\exp\left(\alpha\mu-\frac{1}{2}\sigma^2\right)t+\beta\sigma Z(t) \tag{10}$$

RESULTS AND DISCUSSION

This Section presents result whose solution are in (10), Hence we have the following:

$X_1\left(t\right)=X_0\exp\left(\alpha\mu-\frac{1}{2}\sigma^2\right)t+\beta\sigma dz\left(t\right)$, where, $dz=1, t=1$ and $\beta=0.25$

Table 1: The effects of α on the assessment of asset values through stock returns.

Initial stock prices (X_0)	Returns (μ)	α	Volatility (σ)	*Asset values* $X_1(t)$
5.00	0.5	0.1	0.2	5.2023
	0.5	0.2	0.2	5.4664
	0.5	0.3	0.2	5.7441
	0.5	0.4	0.2	6.0361
6.00	0.9	0.5	0.2	9.2736
	0.9	0.6	0.2	10.1423
	0.9	0.7	0.2	11.0926
	0.9	0.8	0.2	12.1325
5.20	0.95	0.25	0.2	6.5134
	0.95	0.35	0.2	7.1576
	0.95	0.45	0.2	7.8659
	0.95	0.55	0.2	8.6448

Table 1 describes the influence of α on the assessment of asset values. It is obvious that increase in α when μ *and* σ are fixed increases the value of asset returns. This also shows the various levels of long term investment plans. The changes of asset values are very informative to investors.

Table 2: The effects of time t influences over asset values changes .

$$X_1(t) = X_0 \exp\left(\alpha\mu - \frac{1}{2}\sigma^2\right)t + \beta\sigma dz(t), \text{where, } dz = 1 \text{ and } \beta = 2$$

	Time (t)	Returns (μ)	α	Volatility (σ)	*Asset values* $X_1(t)$
5.00	0	0.5	0.1	0.2	0.4
	1	0.5	0.2	0.2	5.8164
	2	0.5	0.3	0.2	11.7883
	3	0.5	0.4	0.2	18.3583
6.00	0	0.9	0.5	0.2	0.4
	1	0.9	0.6	0.2	10.4922
	2	0.9	0.7	0.2	22.4852
	3	0.9	0.8	0.2	36.6476
5.70	0	0.95	0.25	0.2	0.4
	1	0.95	0.35	0.2	8.1910
	2	0.95	0.45	0.2	17.5348
	3	0.95	0.55	0.2	28.6637

Clearly it can be seen the influences of time on asset values; an increase on time when return rates and stock volatility are fixed: 0.1-0.4,0.5-0.8,0.25-0.55 and 0.2 increases the value of assets. This implies that time is a serious factor on asset values; lots of assets appreciates and depreciates in value over time.

Table 3: The effects of stock volatility (σ) on the assessment of asset values changes

$$X_1(t) = X_0 \exp\left(\alpha\mu - \frac{1}{2}\sigma^2\right)t + \beta\sigma dz(t), \text{ where, } dz = 1, t = 1 \text{ and } \beta = 2$$

Initial stock prices (X_0)	Returns (μ)	α	Volatility (σ)	*Asset values* $X_1(t)$
5.00	0.5	0.1	0.1	5.4301
	0.5	0.1	0.2	5.5523
	0.5	0.1	0.3	5.6251
	0.5	0.1	0.4	5.6522
6.00	0.9	0.2	0.5	7.3392
	0.9	0.2	0.6	7.2
	0.9	0.2	0.7	7.0224
	0.9	0.2	0.8	6.8162
5.70	1.0	0.3	0.91	6.9056
	1.0	0.3	0.92	6.8793
	1.0	0.3	0.93	6.8529
	1.0	0.3	0.94	6.8264

Table 3 show-cased the various levels of stock volatility on the assessment of asset values. An increase in the volatility parameter increases the value of asset pricing. This is physically consistent because volatility causes significant changes on the price history of asset over time.

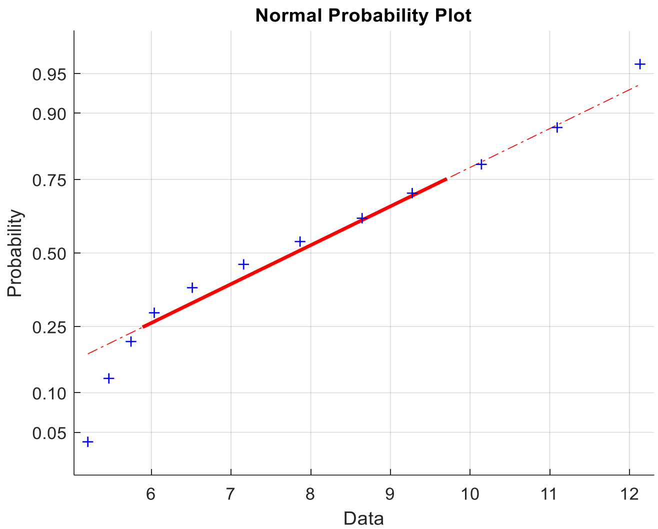

Figure 2: Normal probability plot on the effects of α for assessment of asset values through stock returns

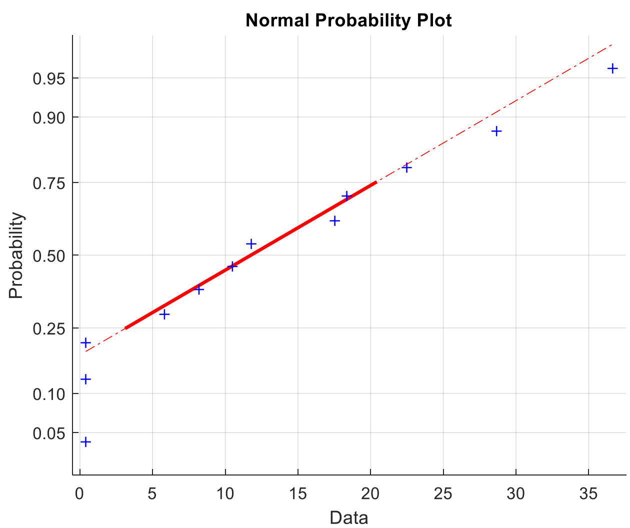

Figure 3: Normal probability plot on the effects of stock volatility over asset values changes

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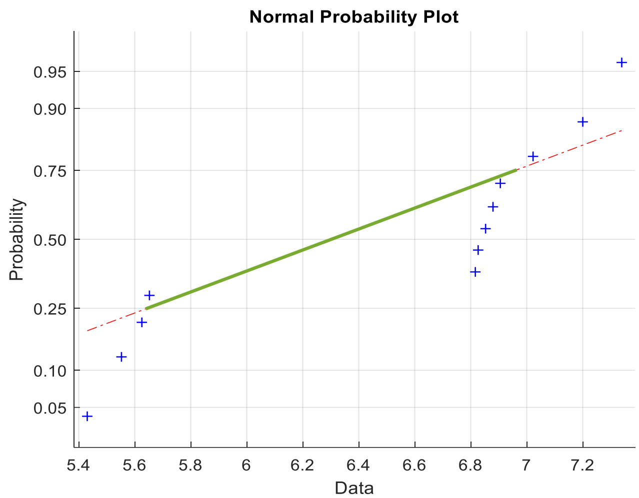

Figure 4: Normal probability plot on the effects of time t influences over asset values changes

The normality probability plots for Figures 2,3 and 4 are not statistically significant and do not come from a come distributions which has finance attributes on the assessment of asset values for capital market investments.

CONCLUSION

This study, considered stochastic analysis of discrepancies on various stochastic variables over asset prices. The asset values were obtained through the influences of some key stochastic variables which shows as follows: increase in α when μ *and* σ are fixed increases the value of asset returns. ,a little increase on time when return rates and stock volatility are fixed: 0.1-0.4,0.5-0.8,0.25-0.55 and 0.2 increases the value of assets., an increase in the volatility parameter increases the value of asset pricing and α parameter shows the various levels of long term investment plans . The normality probability plots are not statistically significant and besides do come from a common distributions which has a vital meaning in the assessment of asset values for capital market investments.

We shall be looking at the theoretical studies of the proposed model in the next study.

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