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Reilly's Law and Logistic Type Equation in a Model for Studying Group Selection and Intercity Relations

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ABSTRACT: *This paper presents a model based on logistic type equations, but taking Reilly's Law into account. It consists of differential equations, each of which describes the dynamics of a population located at the center of a territory whose resources it exploits. Similar differential equations, but without elements that take into account the Reilly's Law, have already been presented for studying the evolution of altruism from binary altruism (between two individuals) to Benthamian altruism (toward the group). The model presented here aims to study the outcome of competition for resources at least partially shared by populations concentrated at different points. It shows that a group's degree of cohesion is crucial to its survival if it competes with other groups for resources, even without the use of violence. It also shows that cooperation can replace competition between different populations with mutual benefits (even if the resulting equilibrium can be rather precarious). Finally, the differential equations used, while containing elements similar to those found in Newton's gravitational law (as well as Reilly's), predict the existence, alongside the force of attraction between economic actors, of its opposite (that is a sort of social antigravity).*

KEYWORDS: Logistic-Reilly Hybrid Model, Altruism and Selfishness, Benthamian altruism, Group Selection, Synergies and Social Antigravity.

INTRODUCTION

Many authors in the last two centuries attempted to model geographic relations (notably Von Thunen, 1826, Christaller (1933), Lösch (1940)). In this context, a singular contribution was that made by Reilly (1929, 1931) with his Law of Retail gravitation. This law refers to a scenario in which the influence of a town over the territory decreases as the distance increases. This law bears a close analogy to Newton's law of universal gravitation, particularly with regard to its inverse relation to the square of distance. This relation, although it is derived through a heuristic approach, is not obvious considering that the Newtonian law of universal gravitation is applied to three-dimensional realities while geographic actors play their role essentially in two dimensions.In this paper Reilly's Law will be used in the construction of an evolutive model suitable for the study of dynamics among populations.

The model presented here is an evolution of a model inspired by the logistic equation (Malthus, 1789, Lotka, 1925; Volterra, 1926) and introduced in Paolilli (2011). The new model can be applied to a scenario in which the dynamics of cooperation and competition among populations that, concentrated at different points in geographic space, compete for common resources for reasons of relative proximity. It will be shown that the degree of cohesion of a group is crucial to its survival in case it competes, even without violence, with other groups for resources. It will be also shown that cooperation can replace competition among different populations with mutual benefit, thus permitting better global performances (even if the resulting equilibrium can be rather precarious). Finally, the differential equations used, while containing elements similar to those found in Newton's gravitational law (as well as Reilly's), predict the existence, alongside the force of attraction between economic actors, of its opposite (social antigravity).

Paolilli's approach to the origin of altruism and group selection

The aptitudes to cooperation (and altruism) among humans have been the focus of extensive debate. The debate concerned both the nature of the altruistic attitude itself, that is, its real motivations, and the reasons for its existence (origin and affirmation).

About the nature of altruism, Khalil [2003] distinguishes three different theoretical approaches: the egoistic, the egocentric and the alter-centric approach. According to the egoistic approach altruistic behaviors are determined by the expectation of future gains (Axelrod, 1984; Bergstrom and Stark, 1993; Taylor, 1987). In the egocentric approach the altruistic behavior is determined by the fact that in the agent's utility function there is also the utility of the individuals that the agent wants to benefit (Hochman and Rodgers, 1969; Becker, 1976; Dawkins, 1976).

Finally, according to the alter-centric approach individuals tend to show prosocial behaviors (Bowles and Gintis, 2002) due to genetic factors. Also regarding the origin and establishment of altruism, and more generally of cooperative behaviors, there are three main theories: kin selection (for the mathematical aspects of this theory see Eberhard, 1975), reciprocal altruism (Trivers, 1971), that can be linked to the egoistic conception of altruism, and group selection (among others, Wynne-Edwards, 1962; Sober, 1991) Also noteworthy is Price's equation (1970), which can be related to both kin selection and group selection.

However, an alternative scenario (Paolilli, 2009) has been proposed in which the above mentioned theories (especially reciprocal altruism, but also group selection) are not strictly necessary to explain the emergence of altruism. In fact, it has been shown that the emergence of altruism is possible in the presence of a context in which the more empathy grows between agents, the more intense and numerous the relationships aimed at production or exchange become. In such a context, cooperation enhances system performance to such an extent that those who engage in uncooperative behavior naturally tend to be marginalized. In addition, the scholar notes that the existence of groups requires the presence of cooperative attitudes (2011) and therefore group selection can only intervene after the emergence of cooperative attitudes.

Within Paolilli's approach a binary form of altruism, that is between individuals (face-to-face), can therefore emerge earlier than group selection, and then evolve toward different types of altruism, such as Benthamian altruism (Paolilli, 2011, 2015). These types of altruism can thus promote group formation and population concentration and, consequently, feelings and control mechanisms, such as a sense of justice and a tendency to punish selfish behavior, as well as feelings such as envy and behaviors such as gossiping. The latter feelings and behaviors are seemingly dissimilar to altruism, but functional for the survival and affirmation of a group, especially when it competes with other groups. According to this approach the development of control mechanisms are therefore the result of the interactions between the Benthamian altruism and selfishness, which maintains its presence "at least in binary relations and at any rate cannot disappear from the human genre since all humans are vehicles of reproduction and selection" (Paolilli, 2011).

The differential equation presented by Paolilli (2011) to study the dynamics of population concentrations is:

$$dp / dt = k\, p\, (S\, A^{\alpha} - p) \tag{1}$$

It is discussed in the cited paper and here its variables and parameters are briefly explained. A concentrated population p varies according to a coefficient *k*, its size *p* and the still available resources $(S\, A^{\alpha} - p)$. $S\, A^{\alpha}$ is a function of the Cobb-Douglas type: in it *A* is the surface of the area exploited by the population assembled at its center. The exploited surface is circular, given that it is hypothesized the homogeneity of the territory and equivalent transport costs in any direction, and it is expressed in terms of *places*, where the *place* is the unit of surface which is necessary for the survival of an individual, lacking technology, when the population is evenly distributed over the territory[1].

S measures the effect which the synergies permitted by the concentration of population have on the productivity of the territory. Its value is declared as greater than *1*, due to the fact that $S = 1$ indicates the absence of synergies. The value of α depends on the technological level. Its value is less than *1* because, as the exploitation area grows, its net marginal productivity decreases due the increase of the distance.
A population dispersed on an area *A* can assemble at the center of it only if $S\, A^{\alpha} > A$ and, being $\alpha < 1$, this can happen only for $S > 1$. The exploitable area *A* is not a datum. Its value, according to Paolilli (2011), is:

$$A = \alpha\, S^{1/(1-\alpha)} \tag{2}$$

Therefore (1) is rewritten as follows:

$$Dp / dt = k\, p\, [S\, (\alpha\, S^{1/(1-\alpha)})^{\alpha} - p] \tag{3}$$

[1] It should be noted that a human population *p* could perhaps be better represented by average reproductive units (or families) rather than individuals, but this choice would not change the structure of the proposed equations and therefore in what follows we will adopt the definition of *p* as a set of individuals.

where the equilibrium values are: $p = 0$; $p = S\,(\alpha\, S^{1/(1-\alpha)})^{\alpha}$.

The former is a trivial equilibrium point.

Regarding the second equilibrium point, it should be noted that it does not express the maximum value of population concentration, but the value that this can reach by maximizing per capita income. In fact, in the cited paper, implicitly following Sismondi's approach to demography (see Forges Davanzati and Paolilli, 1999, referring to Simonde de Sismondi, 1819) the relation (2) is obtained by equaling the first derivative of $S\,A^{\alpha}$ to 1 with respect to A. In this way (2) furnishes the value of A that allows the best productivity per capita to a concentrated population p. Regarding the reasons why a society may follow a path aimed at per capita income maximization or, conversely, a Malthusian type of population maximization, see Forges Davanzati and Paolilli (1999), Diamond (2005), Paolilli (1998)[2]. However, in this paper the Malthusian approach will be followed and therefore, to obtain the equilibrium value of p, it will be assumed that the concentrated population will not only grow to maximize per capita income, but will continue to grow until it reaches the survival limit. This assumption, on the other hand, permits to simplify the calculations.

A model for studying dynamics among populations that compete and/or cooperate

In this section, a model composed of differential equations obtained from a reworking of (1), but also taking into account Reilly's law, is introduced in order to study what happens when the development of populations concentrated at different points in geographic space leads them to have competitive and/or cooperative relationships with each other.

As mentioned above, a Malthusian viewpoint will be taken, assuming that the population of a group will grow until they reach the survival limit. The term A will be replaced by the population p, thus establishing a direct relationship between the number of available *places* (dependent variable, expressed by $S\,p^{\alpha}$) and the size of the concentrated and cooperating population (independent variable). The differential equation describing the concentrated population dynamics of a group in the absence of other groups is:

$$dp\,/\,dt = k\,p\,[(S\,p^{\alpha} - p)\,/\,(S\,p^{\alpha})] \qquad (4)$$

The term at the denominator into the square brackets is not strictly necessary: it does not change the equilibrium points of the differential equation, but it allows the equation to better describe the population dynamics (see Paolilli, 2015).

The two equilibrium points (E_1 and E_2) for p are: : $p = 0$; $p = S^{1/(1-\alpha)}$.

For the first (trivial) equilibrium point, what was stated in the previous paragraph is valid.

Regarding the second equilibrium point, as mentioned in the previous paragraph, it should be noted that the condition $S = 1$ implies the absence of group formation.

[2] In other papers (see, for example, Paolilli, 2005) whose focus was not population concentration but the study of a population dispersed over a territory, the Malthusian approach was followed. A concentrated population inevitably establishes norms and therefore a Sismondian scenario, with birth control, is more likely. It is also reasonable that new inhabitants will not urbanize when marginal income falls below unity (the subsistence level), if in the absence of redistribution policies.

It is interesting to note that values less than 1 could indicate the existence of antisocial behaviors that would function as a kind of a (social) antigravity such that individuals would be driven to isolate themselves as much as possible: the population density should be lower than that found in the absence of synergies ($S = 1$), as if to indicate the need for "buffer territories" between the areas of exploitation of individuals. In this way, for example, if the value of the equilibrium point E_2 is x, the surface requested by an individual will be $1 / x$ (*places*).

When the exploitation areas of two groups (for simplicity it will be considered a scenario with only two groups) start to intersect, each group will be hindered, in the exploitation of the possibly shared area, by the other. On the other hand, it is also possible that the two groups for some activities can cooperate, by means of trade, thus increasing the potentialities of both the groups, particularly if differently specialized. Each group, therefore, will experience two opposing influences from the other: one competitive (for resources), the other synergistic. The maximum potential population (say it P_{iMAX}) of each group can be therefore so calculated:

$$p_{iMAX} = [S_i\, p_i^{\,\alpha i} + (S_{ij}\, p_j^{\beta j} - \; h_{ij}\;\, p_j^{\gamma j}) / d^{\,\delta}] \tag{5}$$

where $i = 1, 2$ and $j = 2, 1$.

According to (5) the potential population is not only $S_i\; p_i^{\,\alpha}$, but also that made possible by synergies with the other group, expressed by $S_{ij}\;\, p_j^{\beta j}$, where p_j is elevated to β_j because we assume that the influence of the population p_j on the population p_i depends on its technological level. Note that the values of S_{ij} and S_{ji} may be different because the exchanges may be unequal. In (5) it has been taken into account also the fact that the other group can compete for the resources: - $h_{ij}\;\; p^{\gamma j}$. Note that it is reasonable to assume that the exponent γ_j is greater than the exponent β_j both because competitive rather than cooperative behaviors are more likely to be more effective and to allow the model not to exhibit values of one population tending to infinity as the other increases.

It is assumed that both influences, the positive one due to synergies and the negative one due to competition, are inversely proportional to the distance d (elevated to δ) between the two populations. It should be considered that in the modern world distance is of less importance than in the past and therefore the value of δ is likely to be decreasing (however in our simulation $\delta = 2$).The model describing the dynamics of two populations is the following:

$$dp_i / dt = k_i\, p_i\, [(p_{iMAX} - p_i) / p_{iMAX}] \tag{6}$$

with $i = 1, 2$ and $j = 2, 1$.
Also in this case it was considered opportune, for the purpose of a better description of the dynamics of the system, to divide the residual potential population $p_{iMAX} - p_i$ by the potential population p_{iMAX} .
It can be noted that this model, which is inspired by the well-known logistic equation, is therefore also related to Reilly's Law, and thus is a hybrid model.

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The equilibrium points of (6), besides the trivial point $0, 0$, in the case of the presence of only one group coincide with the second point of (4).
However, the point of interest for us is what both populations survive for. Unfortunately, due to the non-integer exponents, it has been determined only graphically (and verified by means of numerical simulations).

In what follows, some of the many possible examples of application of the model are presented in order to study some of the main conditions (parameter values and initial values of variables) that may determine the outcome of cooperation and/or competition between two interacting groups.

In Figure 1 all the parameters related to p_1 and p_2 have the same values and the two populations are very close ($d = 3$). The graph shows that the equilibrium point E_4, where both populations survive, is unstable. In fact, only if the initial populations are equal, do they converge to it. In other words the initial values of p_1 and p_2 must be on the line which goes from the origin of the axes to E_4 and beyond. In all other cases only one population survives (the system converges to E_2 or E_3). Moreover, even a very small difference in the value of a parameter (e.g., of S_1 and S_2) pushes the system toward the survival of only one population.

Figure 2 shows that growing the distance d, E_4 becomes stable. The same, however, happens if the cooperation between the two groups grows: in Figure 3 the distance is the same as in Figure 1, but the cooperation among the groups is higher ($S_{12} = S_{21} = 3{,}5$).

Figure 4 shows a scenario in which there is a difference in the values of the synergies into the groups. For a little difference between S_1 and S_2 ($S_1 > S_2$) the equilibrium points become three: E_4 disappears and only one group can survive.

Figure 5 presents a scenario in which cooperation between the two groups are significantly greater than competition ($S_{12} > h_{12}$ and $S_{21} > h_{21}$). In this scenario E_4, in addition to being stable, involves higher p_1 and p_2 values than if only one group survives.

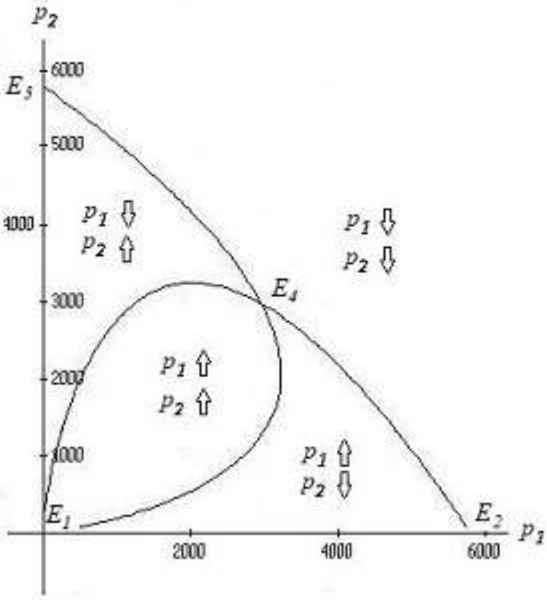

Figure 1. Parameter values: $S_1 = S_2 = 2$; $S_{12} = S_{21} = 2$; $h_{12} = h_{21} = 2$; $d = 3$; $\alpha_1 = \alpha_2 = 0.92$; $\beta_1 = \beta_2 = 0.8$; $\gamma_1 = \gamma_2 = 0.9$; $\delta = 2$. The equilibrium point E_4 , in which both populations survive, is unstable.

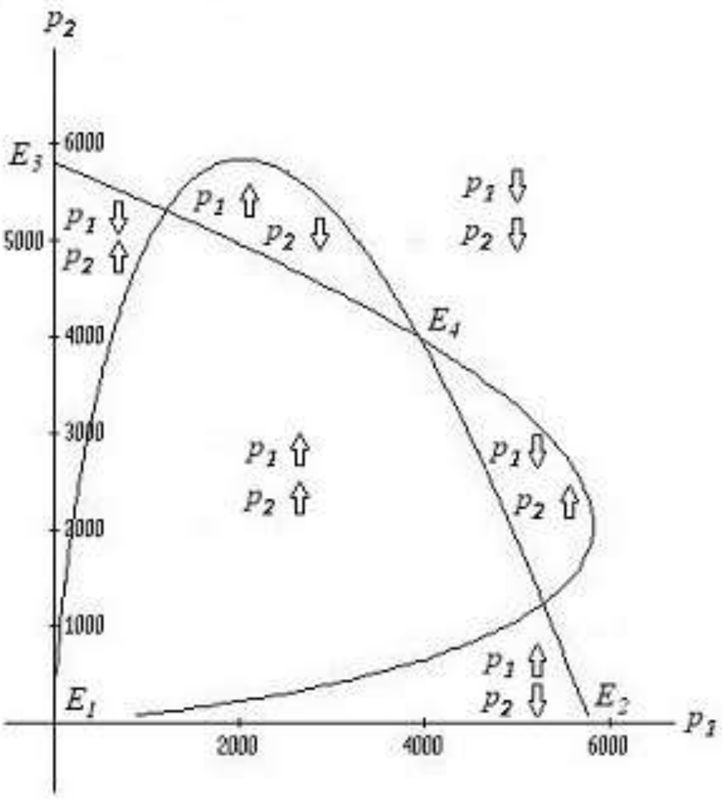

Figure 2. Parameter values: the same as in Figure 1, except $d = 4$ (greater than in Figure 1). The equilibrium point E_4 is stable.

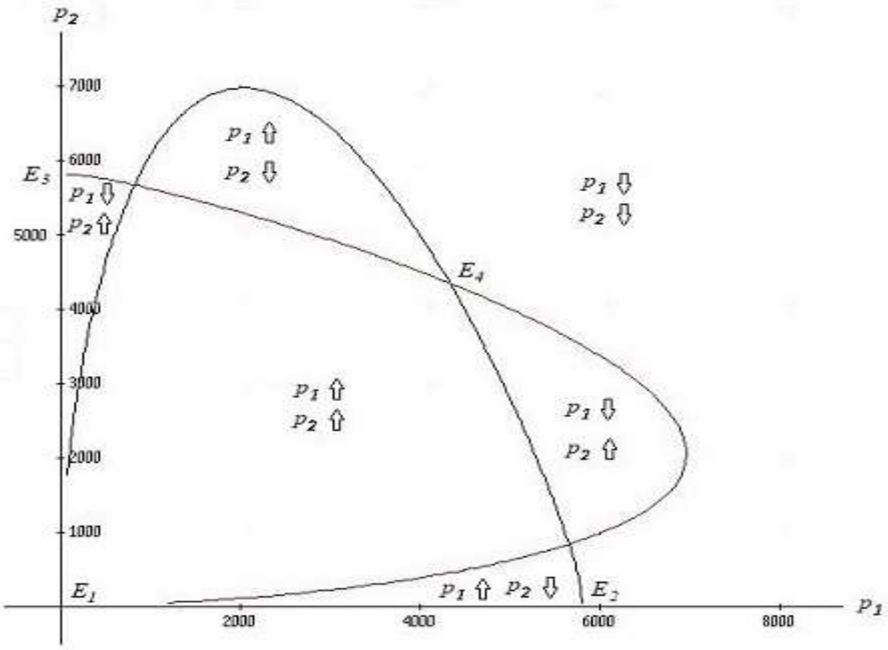

Figure 3. Parameter values: the same as in Figure 1, except $S_{12} = S_{21} = 3.5$. Due to the higher values of S_{12} and S_{21}, E_4 is stable as in Figure 2.

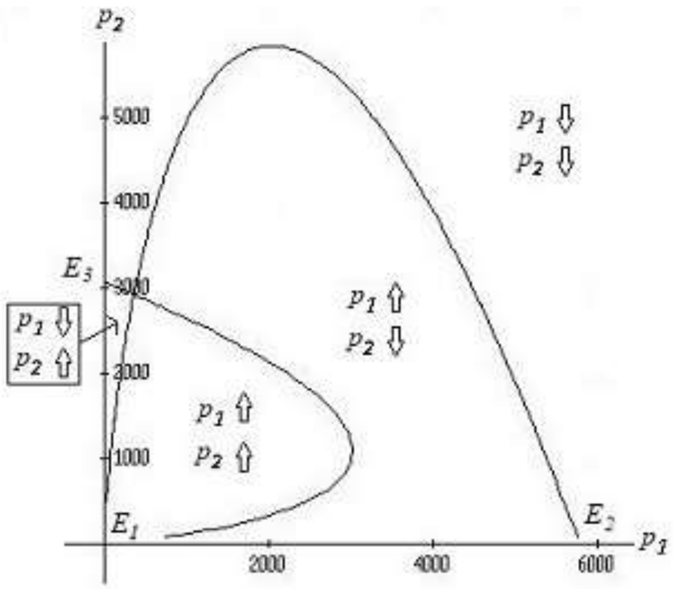

Figure 4. Parameter values: the same as in Figure 1, except: $d = 4$ and $S_2 = 1.9$. The graph shows that even small differences between the values of the S_i can be fatal for the group with the least cooperative individuals.

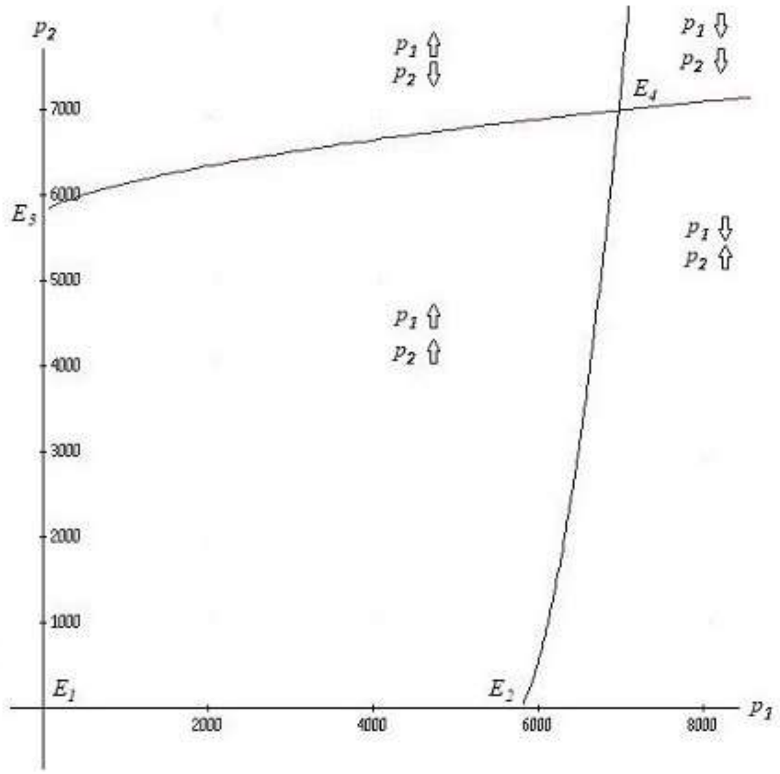

Figure 5. Parameter values: the same as in Figure 1, except $h_{12} = h_{21} = 0.5$. Because of the low values of h_{12} and h_{21}, at the E_4 equilibrium point p_1 and p_2 are each greater than they would be if alone.

CONCLUSION

This paper has introduced a model based on differential equations related to the logistic equation and the Reilly Law. The model presented is the result of a reworking of a differential equation already used to describe population concentration and to study the emergence of Benthamian altruism and thus group selection. Some examples of its use were presented, showing that cooperation and competition between populations concentrated in different points on the earth's surface can coexist and be beneficial for all the populations, even if the resulting equilibrium can be rather precarious. Moreover, it has been observed that the differential equations used to build the model, presenting an analogy with the Newtonian gravitational law, show that the role of synergies seems to recall the role of the constant of gravity, but presenting the possibility to reflect not only a gravitational effect, but even a sort of social antigravity.

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