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A Study on Modeling and Simulation of different Friction Models in Multibody Dynamics

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Abstract: *Friction can be called as resistance to the relative motion between different contacting surfaces. It is a highly complex phenomenon, which occurs in almost all mechanical systems. The paper discusses various numerical studies on different friction models found in the literature. A comprehensive study on different friction models on the dynamics simulation of a multibody system is shown in the paper. To perform a comparison between friction models, they are divided into two groups: Static and dynamic. From the first model proposed by Coulomb to the recent advances by Brown and McPhee, the paper gives more insight into the major improvements in the field of friction. These different models are applied on benchmark problems with continuous contacts, including rolling contact, and later compared based on parameters like accuracy and efficiency. The results are represented through Force-time, velocity-time, and position-time graphs. Although some of the models show different localized effects, most of them present similar tendencies in terms of the resulting friction force. This study shows the effect of different parameters on resulting friction force value. It also examines the model on computational parameters and quantifies the effects for the selection of the appropriate friction model.*

Keywords: Multibody Dynamics; Friction Model; Numerical Simulation

INTRODUCTION

The When two rough surfaces in contact move against each other, a force is exerted between them that resists this motion, called friction force. Friction exists in almost every mechanical system and can have a major influence on static and dynamic performance. The effect of energy dissipation due to friction can change the state of a system slowly if the nature of contact is continuous, or rapidly if the contact is for a very short period like that of an impact. Hence, friction makes it difficult to control a system accurately. Many researchers and scientists have put forward their studies on this topic to solve and model the friction as accurately as possible [1-4].

The distinction between static and kinetic friction has been mentioned in the friction literature for over 200 years, at least since the work of Euler [5]. Euler, who is famous for his work in the application of general laws of mechanics, also conducted experiments to study frictional resistance. The experiment involved a body sliding down an inclined plane at an angle where sliding starts. He observed that there exists no angle for which the body slides slowly. Either the body slides fast or it would not slide at all. He concluded that as soon as sliding initiates, friction force drops significantly. Euler was the first who distinguished between static and kinetic friction. The understanding of static and kinetic friction was further developed by Coulomb [6]. He experimented on finding the different parameters influencing the friction force. He investigated the influence of body material, surface area, normal pressure, time of repose, sliding velocity, etc. The model proposed by Coulomb states that the kinetic friction force or dry friction is independent of the sliding velocity and directly proportional to the normal force. However, the model failed to define the static friction force at zero relative velocity. The discontinuity at origin is unrealistic and unacceptable for the purpose of simulation. The sudden change in the friction-velocity graph at zero was later improved by a non-linear friction force characteristic mentioned by Stribeck (1903). The static friction, which is friction at zero velocity must be higher than the kinetic friction, and should gradually decrease with the increase in relative velocity. This is the so-called Stribeck effect [7-11]. The Stribeck friction is nothing but an addition to the Coulomb friction, which represents stiction between the two surfaces caused by various factors. The values of parameters associated with the Stribeck friction like Stribeck velocity are identified through the experimental setup as no model has put forward the way to calculate them. Even with the Stribeck effect, the problem of discontinuity at zero velocity was not solved. Brown and McPhee worked on a new friction model, which not only made the curve continuous but also differentiable [12]. The model is the best alternative to the Stribeck friction as it does not require extra parameters and is only velocity-dependent.

Friction models that are discussed above solely dependent on the velocity. They only describe the steady-state relation between friction force and relative velocity. There are many examples of mechanical multibody systems, which operate at a near-zero velocity or cross-zero velocity. The static models are unable to describe the friction accurately enough. For these situations, a dynamic model is necessary which introduces extra variables to consider aspects such as pre-sliding displacement or frictional lag. Dahl, in his friction model, introduced an extra state, which can be regarded as the average deflection of the asperities [13,14]. This new state allowed capturing the pre-sliding displacement, hence also making the curve continuous at zero velocity. Since the first dynamic friction model was introduced, many researchers have proposed different dynamic friction models, with additional frictional phenomena [15,16]. One of them being the LuGre model. The LuGre model is an integrated dynamic friction model [17]. It is also referred to as the "integrated dynamic model". The name comes from the abbreviation of the Lund Institute of Technology and INPG Grenoble, the two universities hosting the cooperating scientists.

The selection of the friction force model depends on several factors, such as the complexity of the problem to be solved, the computational efficiency, available computational parameters. As stated above, the models are mainly divided between "static" and "dynamic" friction models. The dynamic models may capture more frictional characteristics due to the inclusion of extra state variables but may end up taking a lot of time to evaluate. Despite the fact that the more basic and simpler models can only take into account kinetic friction, they may be updated to consider the above-stated aspects such as static friction, Stribeck effect, pre-sliding displacement, or frictional lag.

The above-mentioned models are applied to different benchmark problems involving surface contacts and rolling contacts. The friction models are compared and efficiency has been evaluated by noting down the computer processing time consumed during the dynamics simulation of systems. Accuracy however is difficult to evaluate, as there exist no experimental data to compare with.

LITERATURE

Charles-Augustin de Coulomb was the first physicist to study the nature of friction. He developed the first-ever friction model in 1785 by stating that friction force always opposes the relative motion between two contacting surfaces [6]. The magnitude of the Coulomb friction is proportional to the normal contact force. This model can be mathematically described as follows:

$$\mathbf{F_c} = \mu_k \|\mathbf{F_n}\| \tag{1}$$

In which $\mathbf{F_n}$ is the normal contact force, μ_k denotes the coefficient of kinetic friction, $\mathbf{F_c}$ denotes Coulomb friction. Stiction, also known as static friction is not capable of starting a movement. It always opposes motion. Hence, friction force cannot be described as a function of the velocity only. It should be a multi-valued function whose value depends on the external forces acting on the object. It is mathematically expressed as follows:

$$\mathbf{F} = \begin{cases} \mathbf{F_c}\operatorname{sgn}(\mathbf{v}_t) & \text{if } \|\mathbf{v}_t\| \neq 0 \\ \min(\|\mathbf{F}_e\|, \mathbf{F_c})\operatorname{sgn}(\mathbf{F}_e) & \text{if } \|\mathbf{v}_t\| = 0 \end{cases} \tag{2}$$

Where $\mathbf{v}_t$ denotes the relative tangential velocity at the contact, sgn denotes sign function whose value can be -1, 1, or 0 depending on the argument, $\mathbf{F}_e$ denotes the friction force required to reduce the relative tangential velocity to zero from time t_n to t_{n+1}. $\mathbf{F}_e$ is a function of external forces or can be directly equal to external forces for a simple subsystem. It can be seen that the friction force depends on the direction of the relative velocity. Velocity, in turn, depends on friction force for its value and direction.

Forward feedthrough can be seen which can cause a numerical error during the simulation. Hence, Equation 2 is changed and expressed as follows:

$$\mathbf{F} = \begin{cases} \mathbf{F_c}\,\mathrm{sgn}(\mathbf{F}_e) & \text{if } \|\mathbf{F}_e\| > \mathbf{F_c} \\ \mathbf{F}_e & \text{if } \|\mathbf{F}_e\| \leq \mathbf{F_c} \end{cases} \tag{3}$$

Coulomb model is the simplest and widely used friction model, as it only requires one parameter, i.e. kinetic friction coefficient. However, many problems arise when implemented in the numerical simulation of different multibody systems. It is mainly because of the discontinuity and non-differentiability of the graph at origin as shown in Figure 1.

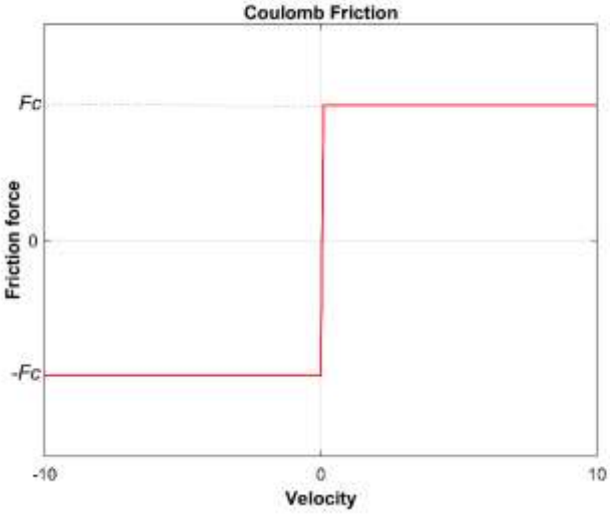

Figure 1. Representation of classis Coulomb Friction model.

Most of the fluids provide resistance against the motion of a solid object moving through them. In the 19th century, this resistance was thought of as friction, just as friction between two solid surfaces. This resistance was later termed viscous friction. Viscous friction is mathematically expressed as follows:

$$\mathbf{F_V} = \mu_V \mathbf{v}_t \tag{4}$$

Where μ_V is the coefficient of viscosity. Viscous friction is often added with the Coulomb friction when lubrication in the contact is in the play. However, the examples discussed in this paper only consist of dry contacts. Hence, viscous friction is neglected everywhere.

Although Coulomb's law presents a suitable approximation for the friction model, it is well established by Euler that the magnitude of friction at zero relative velocity is more than the kinetic friction. The friction value should gradually decrease from static to kinetic friction. This is achieved by the Stribeck curve. The Stribeck effect makes it possible to decrease friction continuously with the increase of relative velocity for a certain velocity regime.

Stribeck friction can be modeled in a number of ways [7-11]. The following is the most utilized mathematical model in the literature:

$$\mathbf{F} = \left(\mathbf{F}_c + (\mathbf{F}_s - \mathbf{F}_c\,)\boldsymbol{e}^{-\left(\frac{|\mathbf{v}_t|}{\mathrm{v}_s}\right)^{\delta\sigma}}\right)\mathrm{sgn}(\mathbf{v}_t) + \mu_v \mathbf{v}_t \tag{5}$$

Where,

$$\mathbf{F}_s = \mu_s \|\mathbf{F}_n\| \tag{6}$$

In which μ_S denotes the static coefficient of friction, $\mathbf{F_S}$ is the magnitude of static friction, v_S represents the Stribeck velocity, $\delta\sigma$ is an exponent which relies on the geometry of the contacting surfaces, often considered to be equal to 2, and μ_V is the viscous friction coefficient. The value of the viscous coefficient is considered for a lubricated contact. However, in this paper, only dry contact is addressed and applied, which involves neglecting the viscous friction coefficient.

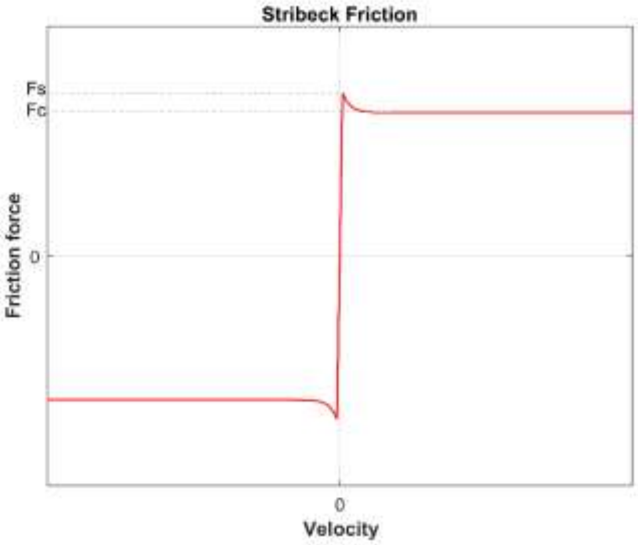

Figure 2. Representation of Stribeck Friction model.

Figure 2 shows the shape for the Stribeck curve, which shows that discontinuity still exists at the origin. As discussed above, this discontinuity creates several numerical issues during a dynamic simulation. Therefore, several researchers have proposed different approaches to prevent numerical instability. One of them was the model proposed by Brown and McPhee.

Recently, Brown and McPhee proposed a new friction model [12], which is suitable to be applied in real-time simulations and optimization problems, since it is continuously differentiable and only velocity-dependent. Neglecting the viscous friction term, it can be expressed as:

$$\mathbf{F} = \left(\mathbf{F}_c \tanh\left(4\frac{\|\mathbf{v}_t\|}{\mathrm{v}_s}\right) + (\mathbf{F}_s - \mathbf{F}_c\,)\frac{\frac{\|\mathbf{v}_t\|}{\mathrm{v}_s}}{\left(\frac{1}{4}\left(\frac{\|\mathbf{v}_t\|}{\mathrm{v}_s}\right)^2 + \frac{3}{4}\right)^2} \right) \mathrm{sgn}(\mathbf{v}_t) + \mu_v \mathbf{v}_t \tag{7}$$

The Equation 7 clearly shows that the model consists of Coulomb friction and Stribeck friction terms.

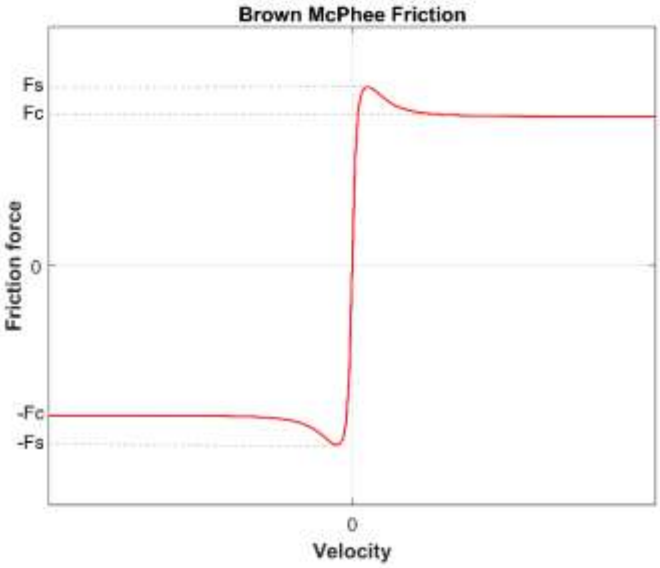

Figure 3. **Representation of Brown and McPhee friction model.**

It is assumed in the above static friction models that there exists no motion while static friction is engaged, which is however not true. There exist compliance in the contact due to external forces. This minute displacement is called pre-sliding displacement [18-19]. It is important to consider this pre-sliding displacement for better simulation accuracy. Another important point about the static friction model is that it yields zero friction force at a standstill which, is however not accurate when compared to a real-life situation. This can be achieved by adding a hysteresis effect.

Karnopp's approach was adopted mostly for the dynamic friction modeling but did not considered the pre-sliding effect [20]. Haessig and Friedland considered pre-sliding displacement in the bristle model only to determine if the struck condition is broken or not [21]. Pre-sliding displacement was not incorporated with the output position of the body.

Dahl was the first one to do so. He described a spring-like behavior between the contacts under static friction. This pre-sliding phenomenon can be represented by a small hysteresis around the zero relative velocity. The friction force value is low for decreasing velocity when compared to increasing velocity. This dynamical behavior can be explained by the frictional memory caused by a lag in the friction force. Frictional lag is nothing but the delay in the change of friction force as a function of velocity.

Dahl's model is based on the stress-strain curve in classical solid mechanics [13,14]. When stress in the contact increases, the friction also increases. Dahl modeled the stress-strain phenomenon by a differential equation given below:

$$\frac{d\mathbf{F}}{dx} = \sigma_0 \left(1 - \frac{\mathbf{F}}{\mathbf{F}_c} \operatorname{sgn}(\mathbf{v}_t)\right)^{\alpha} \tag{8}$$

Where **F** is the friction force, **Fc** is the Coulomb force, x is the displacement, σ_0 is the stiffness coefficient and α is a parameter that determines the shape of the stress-strain curve. Generally, the value of α is taken as one for simplicity. Through Equation 8, we can say that the value of friction will never be greater than **Fc**.

To obtain a time-dependent function, Equation 8 can be written as:

$$\frac{d\mathbf{F}}{dt} = \frac{d\mathbf{F}}{dx}\frac{dx}{dt} = \frac{d\mathbf{F}}{dt}\mathbf{v}_t = \sigma_0\left(1 - \frac{\mathbf{F}}{\mathbf{F}_c}\operatorname{sgn}(\mathbf{v}_t)\right)^{\alpha}\mathbf{v}_t \tag{9}$$

Introducing a new variable **z** which represents average Bristol deflection in dynamic model, and defining it as a function of friction,

$$\mathbf{F} = \sigma_0\mathbf{z} \tag{10}$$

We obtain,

$$\frac{d\mathbf{z}}{dt} = \left(1 - \frac{\sigma_0}{\mathbf{F}_c}\mathbf{z}.\operatorname{sgn}(\mathbf{v}_t)\right)\mathbf{v}_t \tag{11}$$

Figure 4 shows the variation of friction force with respect to velocity for different values of stiffness coefficient. Figure 4b shows that as stiffness increases, displacement needed to achieve maximum friction value decreases, which matches with the basic understanding of the spring-mass system as Dahl defines friction as a spring-mass behavior.

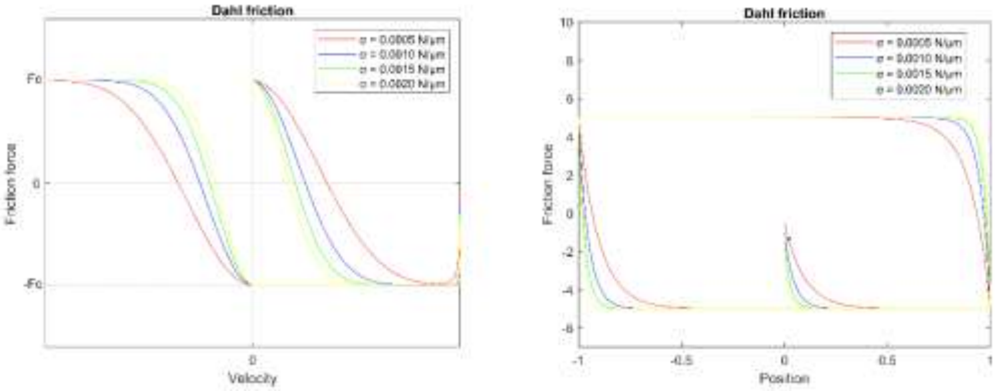

Figure 4. **Variation of dahl friction force (a) With respect to velocity. (b) With respect to position.**

Although Dahl's model takes most of the dynamic phenomena into consideration, it fails to capture the stiction and Stribeck effect.

LuGre model can be described as an update to Dahl's model where stiction and Stribeck effects are also included. The LuGre model is described as follows [17]:

$$\frac{d\mathbf{z}}{dt} = \left(1 - \frac{\sigma_0}{\mathrm{G}(\mathbf{v}_t)}\mathbf{z}.\operatorname{sgn}(\mathbf{v}_t)\right)\mathbf{v}_t \tag{12}$$

$$\mathbf{F} = \sigma_0\mathbf{z} + \sigma_1\frac{d\mathbf{z}}{dt} + \sigma_2\mathbf{v}_t \tag{13}$$

Where σ_1 the damping of the bristles is, σ_2 denotes the viscous friction coefficient, is an arbitrary function that describes the viscous effect and $\mathrm{G}(\mathbf{v}_t)$ is an arbitrary function, which describes the Stribeck curve and can be defined by:

$$\mathrm{G}(\mathbf{v}_t) = \mathbf{F}_c + (\mathbf{F}_s - \mathbf{F}_c\,)\boldsymbol{e}^{-\left(\frac{|\mathbf{v}_t|}{\mathrm{v}_s}\right)^{\text{?'??""""}}} \tag{14}$$

Figure 5 shows variation of friction force with respect to velocity for different values of Stribeck's velocity.

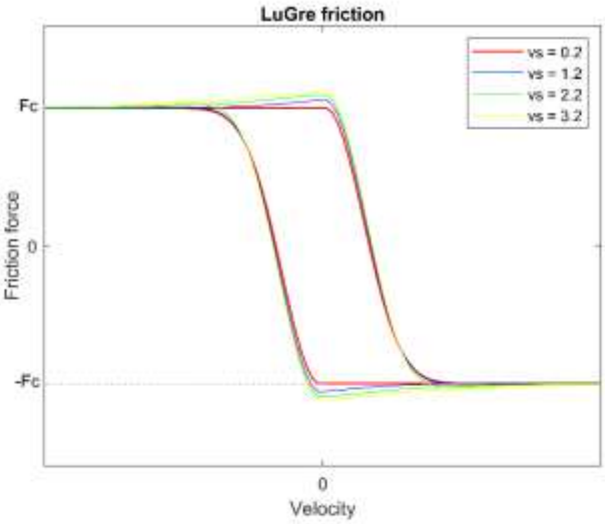

Figure 5. **Variation of LuGre friction force with respect to velocity for different values of Stribeck's velocity**

Although dynamic models are capable of simulating complex friction characteristics, they need high computational power and cost too. Hence, a decision must be taken to choose an appropriate model based on efficiency and accuracy.

MULTIBODY DYNAMIC FORMULATION

A multibody system can be stated as a combination of bodies, rigid or flexible, which are connected by the joints. An analysis, performed over time, of a system that relies on inertial effects to determine motion is called dynamic analysis. Makkonen [22] stated that the dynamic analysis provides the time-history solution for all of the displacements, velocities, accelerations, and internal reaction forces in a mechanical system driven by a set of external forces and excitations.

Joints and contacts not only connect the bodies but also restrict the relative motion between them. From a mathematical perspective [23], such constraints are in form of an algebraic equation as a function of position vector q.

$$\boldsymbol{\Phi} \equiv \boldsymbol{\Phi}(\mathbf{q}) = \mathbf{0} \tag{15}$$

Which is a matrix containing a collection of all algebraic constraints. The number of equations is equal to the number of constraints induced by the joints. The first time derivative of Equation 15 yields the velocity constraint equation expressed as [23]:

$$\boldsymbol{\Phi}_{\mathbf{q}}\dot{\mathbf{q}} = \mathbf{0} \tag{16}$$

Where $\boldsymbol{\phi}_{\mathbf{q}}$ denotes the Jacobian matrix $[\boldsymbol{\partial\phi/\partial q}]$ and $\dot{\mathbf{q}}$ contains the velocity terms. The second time derivative of Equation 15 results in [23]:

$$\boldsymbol{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} + (\boldsymbol{\Phi}_{\mathbf{q}}\dot{\mathbf{q}})_{\mathbf{q}}\dot{\mathbf{q}} = \mathbf{0} \tag{17}$$

$$\mathbf{\Phi_q \ddot{q}} = -(\mathbf{\Phi_q \dot{q}})_\mathbf{q}\mathbf{\dot{q}} \tag{18}$$

Nikravesh referred the term $-(\mathbf{\Phi_q \dot{q}})_\mathbf{q}\mathbf{\dot{q}}$ as right side of kinematic acceleration equation, given by [23]:

$$\boldsymbol{\gamma} = -(\mathbf{\Phi_q \dot{q}})_\mathbf{q}\mathbf{\dot{q}} \tag{19}$$

$$\mathbf{\Phi_q \ddot{q}} = \boldsymbol{\gamma} \tag{20}$$

Newton Euler equations describe the combined translational and rotational dynamics of a rigid body [4]. These equations are the base for complicated ‘multibody’ formulation that describes the system of rigid bodies connected by joints or other constraints.

Newton Euler equations are expressed in the general form as follows:

$$\mathbf{M\ddot{q}} = \mathbf{g} + \mathbf{g}^{(c)} \tag{21}$$

Where, **M** is the diagonal body mass matrix, $\mathbf{\ddot{q}}$ denotes acceleration matrix, **g** denotes body force vector containing all the external forces acting on the system, and $\mathbf{g}^{(c)}$ denotes reaction forces or internal forces acting on a specific body as these equations are written in a specific body’s frame of reference. The term $\mathbf{g}^{(c)}$ can be expressed in terms of Jacobian matrix and Lagrange multiplier as follows [23]:

$$\mathbf{g}^{(c)} = \boldsymbol{\phi}_\mathbf{q}{}^{T}\boldsymbol{\lambda} \tag{22}$$

Hence, the Newton Euler equations from Equation 21 and 22 can be expressed as follows:

$$\mathbf{M\ddot{q}} = \mathbf{g} + \mathbf{\Phi_q}{}^{\mathbf{T}}\boldsymbol{\lambda} \tag{23}$$

Adding Equation 20 to the acceleration constraint Equation 23, the equation of motion of a general constrained multibody system can be written as [24-25]:

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi_q}{}^{\mathbf{T}} \\ \mathbf{\Phi_q} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{\ddot{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \tag{24}$$

The inclusion of friction in multibody dynamic simulation can be difficult due to its nonlinear nature and numerical instabilities associated with it. As it was previously stated that friction force highly depends on the normal contact force, therefore the first step in friction implementation is to analyze normal force. For contact cases where the interaction forces are treated as external forces (i.e., impact simulation using penetration approach), both normal and friction forces are simply included in the vector of forces g and calculated independently in each time step. However, the normal force in the examples simulated is quite simple to calculate as it is constant and is simply defined instead of calculating.

APPLICATION EXAMPLES

SIMPLE SPRING MASS SYSTEM

In this section, a simple spring-mass mechanism is modeled and simulated to examine the problems associated with introducing and simulating friction with the multibody system. Figure 6 shows the representation of a spring-mass system. The mechanism consists of a single mass body or a block and a spring element attached to the body.

The geometrical and inertial properties of the system are listed in Table 1. The friction exists between the ground and the block. During the simulation, the block only moves in a positive x-direction. Hence, it can be that the system only has one degree of freedom. The other end of the spring is given a certain velocity in the positive x-direction. The initial conditions of the system are listed in Table 2. Beyond the reaction and inertial forces, the only force considered is gravity, which is acting in the negative y-direction.

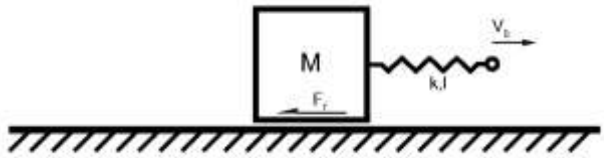

Figure 6. **Representation of initial configuration of spring mass system**

For a comparative study of different friction models, the friction force between the block and the ground was computed as per the different methodologies, namely discontinuous Coulomb, discontinuous Stribeck curve, Brown and McPhee model, Dahl's model, and LuGre model. The parameters used for the implementation of these models are listed in Table 3.

Table 1. **Geometrical and inertial properties of the system**

Component	**Property**	**Symbol**	**Value**
Block	Mass	M	1 Kg
Spring	Length	l	0.1 m
Spring	Stiffness	K	10 N/m

Table 2. **Initial configuration of the system**

Component	**Position**			**Velocity**		
	X (m)	Y (m)	Z (m)	X (m)	Y (m)	Z (m)
Block	0	0	0	0	0	0
Spring end	0.1	0	0	0.4	0	0

Table 3. **Parameters considered for different friction models**

Parameter	**Symbol**	**Value**
Static coefficient of friction	μ_s	0.1 Kg
Kinetic coefficient of friction	μ_k	0.1 m
Stribeck velocity	v_S	0.5
Geometry factor	? ′? ?	2
Stiffness coefficient	σ_0	400
Damping coefficient	σ_1	0.1
Coefficient of viscosity	σ_2	0

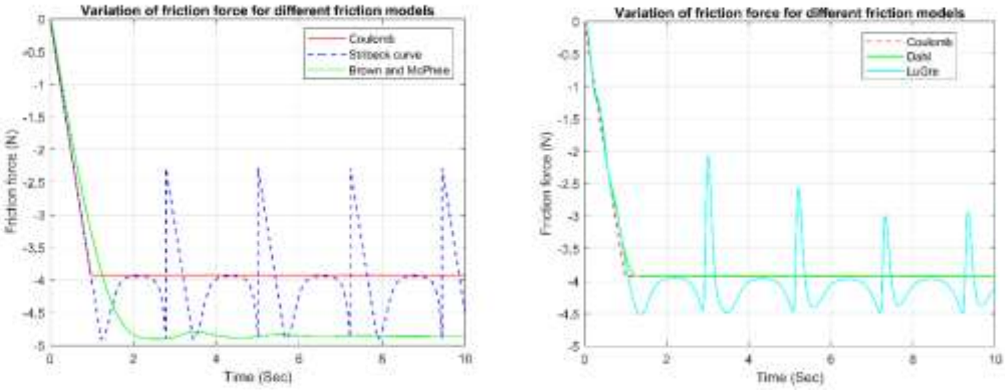

Figure 7. (a) Comparison of friction force acting on the block for different static friction models (b) Comparison of friction force acting on the block for different dynamic friction models

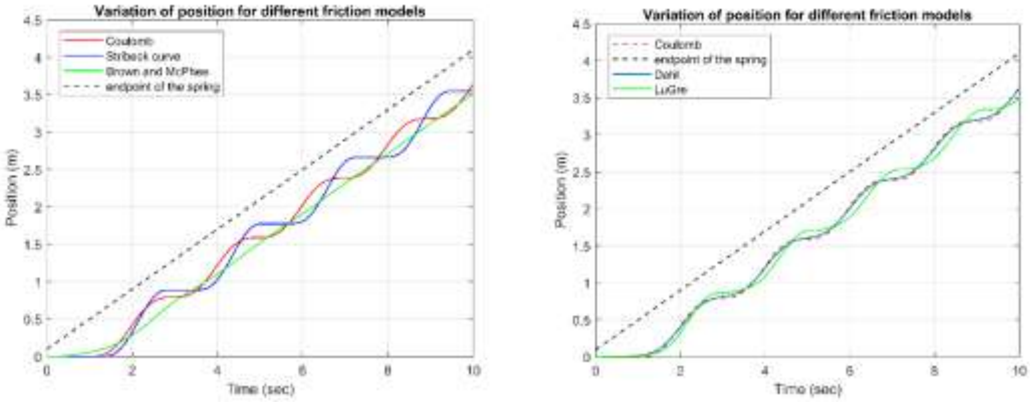

Figure 8. (a) Comparison of position of the block for different static friction models (b) Comparison of position of the block for different dynamic friction models

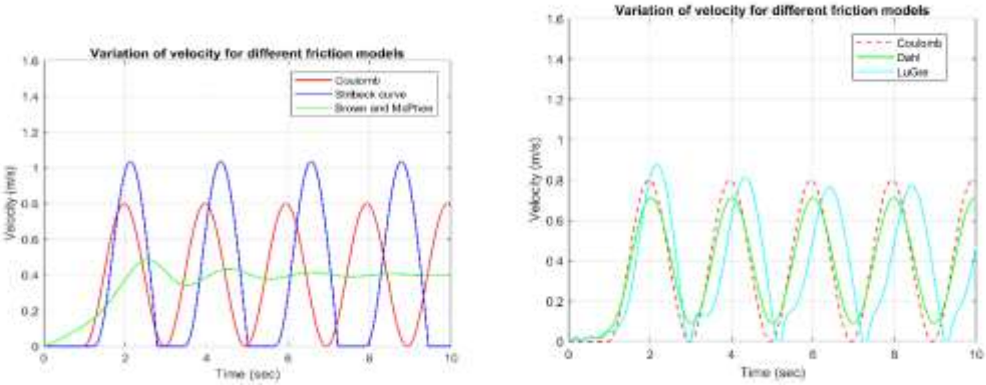

Figure 9. (a) Comparison of velocity of the block for different static friction models (b) Comparison of velocity of the block for different dynamic friction models

The numerical simulation was performed for ten seconds of duration. The variations in position, velocity were compared for different models. Figures 7, 8 and 9 show main differences in the result for different friction models. The Stribeck model shows a sudden change in friction value when velocity becomes zero. This is mainly due to the discontinuity at zero velocity. This discontinuous behavior may cause numerical instability during the simulation. A major change in the result can only be seen in friction value. Models having a static friction term can be seen varying the friction value between static and kinetic friction. Variation of position and velocity does not vary a lot for different models. Hence, it can be said that any friction model whether it is static or dynamic gives acceptable results. However, when the models are compared on the basis of computational efficiency, the difference can be significant. Table 4 shows the computational time of different friction models for the spring-mass mechanism. It can be clearly seen that static models take less time than the dynamic model. A 0.1 second difference may not seem large for this model but when a highly sophisticated model is simulated, the difference can be significant. One of the reasons for the inefficiency of dynamic models can be the stiffness introduced by the extra variables, and the number of calls taken by the function to evaluate the result. The fastest and the most efficient models are Brown and McPhee model and Coulomb model.

Table 4. **Spring mass mechanism computational time for different friction models**

Friction model	**Computational time (s)**
Coulomb Model	0.318
Stribeck Model	0.347
Brown and McPhee Model	0.312
Dahl Model	0.419
LuGre Model	0.429

SIMPLE SPRING MASS SYSTEM

The second application presented in this study is also a spring-mass system but the block is replaced with a disk. Figure 10 shows the representation of the mechanism. The spring is attached to the middle of the disk with a revolute joint. Hence, the disk is free to rotate about its center without causing any change in spring length. The geometrical and inertial properties of the system are listed in the table. The friction exists between the ground and the disk. During the simulation, the disk not only moves in the positive x-direction but also rotates about its center. Hence, it can be said that the system has two degrees of freedom when the disk slips and one degree of freedom when it rolls without slipping. The other end of the spring is given a certain velocity in the positive x-direction. The initial conditions of the system are listed in Table 6. Beyond the reaction and inertial forces, the only force considered is gravity which is acting in the negative y-direction. The parameters used for the implementation of these models are listed in Table 3.

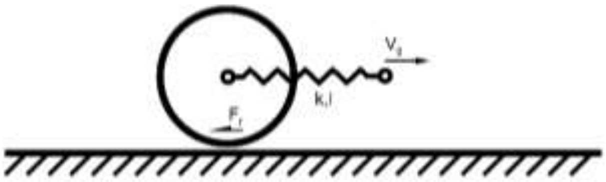

Figure 10. **Representation of initial configuration of spring roller system**

Table 5. Geometrical and inertial properties of the system

Component	Property	Symbol	Value
Roller	Mass	M	1 Kg
Roller	Radius	R	0.1 m
Spring	Length	l	0.1 m
Spring	Stiffness	K	10 N/m

Table 6. Initial configuration of the system

Component	Position			Velocity		
	X (m)	Y (m)	Z (m)	X (m)	Y (m)	Z (m)
Roller	0	0	0	0	0	0
Spring end	0.1	0	0	0.2	0	0

The model is simulated in the same way as the spring-mass system. Different friction models were implemented and simulated. The results obtained were compared and conclusions were made. Figures 11, 12 and 13 show main differences in the result for different friction models.

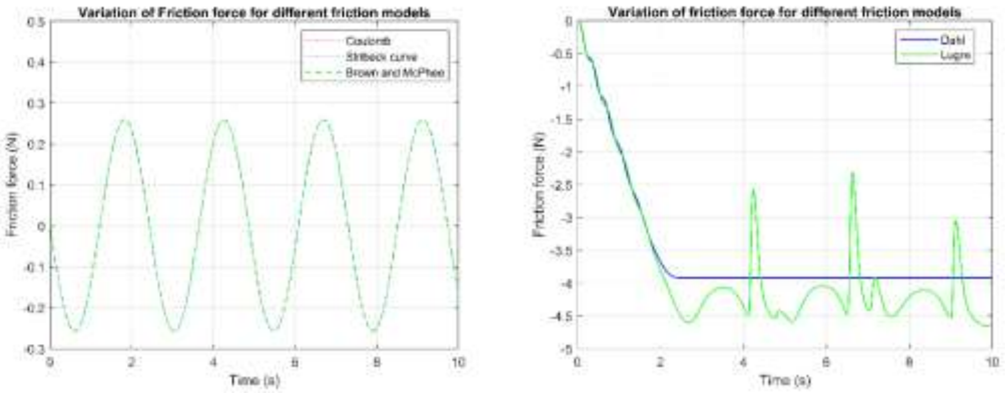

Figure 11. (a) Comparison of friction force acting on the block for different static friction models (b) Comparison of friction force acting on the block for different dynamic friction models

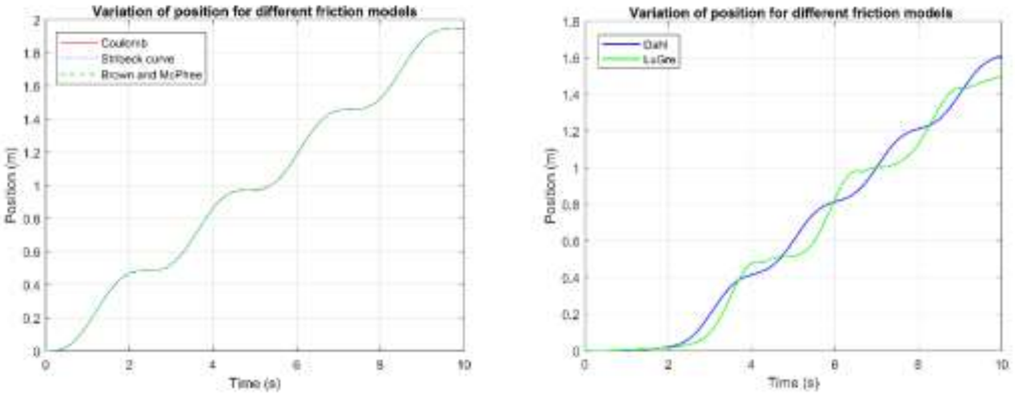

Figure 12. (a) Comparison of position of the block for different static friction models (b) Comparison of position of the block for different dynamic friction models

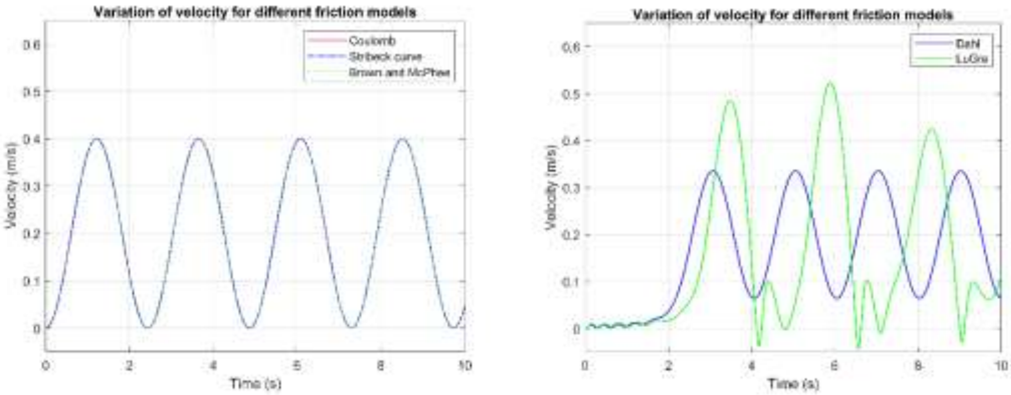

Figure 13. **(a) Comparison of velocity of the block for different static friction models (b) Comparison of velocity of the block for different dynamic friction models**

Table 7 shows the computational time of different friction models for the spring-roller mechanism. The result nearly matches with the previous findings on the spring-mass mechanism. The fastest and the most efficient models are Brown and McPhee model and Coulomb model, which are static models. However, it does not conclude that static models are efficient for every case.

Table 7. **Spring roller mechanism computational time for different friction models**

Friction model	**Computational time (s)**
Coulomb Model	0.314
Stribeck Model	0.351
Brown and McPhee Model	0.374
Dahl Model	0.468
LuGre Model	0.559

DISCUSSION

The conclusion of the research paper highlights the key findings and recommendations based on the study on modeling and simulation of different friction models in multibody dynamics. The study identified various numerical problems associated with modeling friction models and examined the implementation process of important friction models. The authors found that implementing static friction models in the multibody system was relatively straightforward compared to dynamic models. Static models required fewer parameters and were efficient in terms of computational time. However, dynamic models offered the advantage of capturing complex dynamic phenomena such as friction lag and pre-sliding displacement.

To evaluate the different friction models, two examples were modeled and simulated. In the first example, the dynamic behavior of a spring-mass mechanism with friction between a block and the ground was analyzed. In the second example, the dynamic behavior of a spring roller mechanism with friction between a roller and the ground was studied. The authors compared the variation of friction force, position, and velocity over time for these examples.

Implications to Research and Practice

Improved Understanding of Friction Models: The research provides a comprehensive overview and comparison of various friction models in the context of multibody system dynamics simulation. By highlighting the historical development of these models, from Coulomb's initial model to recent advancements by Brown and McPhee, the study enhances our understanding of the evolution of friction modeling in mechanical systems.

Application in Benchmark Problems: The research applies the friction models to benchmark problems involving continuous and rolling contacts. This application allows for a practical assessment of the models' accuracy and efficiency, enabling researchers and practitioners to identify suitable friction models for specific applications.

Similarities and Variances in Friction Force: The study reveals that while some friction models exhibit localized effects, most models demonstrate similar tendencies in terms of resulting friction force. This finding provides insights into the general behavior of different friction models and can guide the selection of an appropriate model based on the desired outcome.

Consideration of Parameter Effects: The research examines the impact of various parameters on the resulting friction force value. By quantifying the effects of these parameters, the study offers valuable insights into the sensitivity of friction models and helps researchers and engineers make informed decisions when selecting and utilizing these models in their simulations.

Facilitating Model Selection: The ultimate goal of this research is to facilitate the selection of an appropriate friction model for specific applications. By providing a comprehensive comparison of static and dynamic friction models, along with their performance metrics such as accuracy and efficiency, the study equips practitioners with valuable information to choose the most suitable friction model for their particular simulation needs.

CONCLUSION

Based on their analysis, the authors drew the following conclusions:

1. For most situations, static friction models that are continuous at zero relative velocity are suitable choices as they effectively capture static friction phenomena. Among the static models, the Brown and McPhee model was identified as the first choice due to its continuity and computational efficiency in simulation.
2. When more detailed and accurate results are required, dynamic friction models should be preferred. The LuGre model was specifically mentioned as capable of capturing more detailed phenomena like friction lag and pre-sliding displacement.

However, it was noted that the increased complexity of dynamic models may lead to less reliable results.

3. The authors emphasized the need for future development of a friction model that can capture complex dynamical phenomena with accurate results while maintaining a low computational cost. Such a model would be highly beneficial for various applications that require both efficiency and accuracy in simulating frictional interactions in multibody systems.

Future Research

Overall, the conclusion highlights the trade-off between computational efficiency and accuracy in modeling friction in multibody dynamics. It emphasizes the significance of selecting an appropriate friction model based on the specific requirements of the simulation and suggests areas for future research and development.

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