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Forecasting Long Memory Processes Subject to Structural Breaks: Evidence from CAC40 and SP500 Stock Markets

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Abstract: *We investigate the potential of structural changes and long memory properties in returns and volatility of the two major precious stock markets (SP500 and CAC40). Broadly speaking, a random variable is said to exhibit long memory behavior if its autocorrelation function is not integrable, while structural changes can induce sudden and significant shifts in the time-series behavior of that variable. The results from implementing several parametric and semi-parametric methods indicate strong evidence of long range dependence in the daily conditional return and volatility processes for the stock markets. Despite the divergence of the economic situation and the geographical positions of the countries making up our sample, the FIGARCH and FIEGARCH models mainly turn out to be the most accurate models for predicting the volatility of the stock market.*

KEYWORDS: stock markets, forecasts, fiegarch

INTRODUCTION

It is now widely agreed that oil price and stock markets return volatilities are time varying, with persistent dynamics. This is true across assets, asset classes, time periods, and regions. Furthermore, asset return volatilities are central to finance, whether in asset pricing, portfolio allocation, or market risk measurement. Therefore, the field of financial econometrics dedicates considerable attention to time-varying volatility and associated tools designed for its measurement modeling and forecasting.

In the last few decades a growing number of studies have focused attention on the analysis and forecasting of volatility, due to its important role in financial markets. Portfolio managers, option traders and market makers all are interested in the opportunity of forecasting, with a

reasonable level of accuracy, this significant magnitude, in order to get either higher proceeds or less risky positions.
This p
ersistence in volatility is a common empirical result in financial economics and was studied extensively in Baillie et al (1996) and Andersen et al (2003), Koopman et al. (2005) and Corsi (2009). Whereas stock markets have largely been found to include very little autocorrelation, it has been noted in a large number of works across different asset classes that autocorrelation in diverse measures of volatility does exist at significant levels and remains over a large number of works across diverse asset classes that autocorrelation in various measures of volatility does exist at significant levels and remainder over a large number of lags.

The recent more general long-memory literature, in contrast, pays comparatively little attention to confusing long memory and structural break. It is important, for example, that the otherwise masterful investigations by Bilke (2005), Lee (2005), Gourieroux and Jasiak (2001), don't so much as mention the subject. The opportunity of confusing long memory and structural breaks has of course arisen occasionally, in a numerous of literatures including applied exchange rates Sakoulis et al. (2010)'s, econometrics[1], and mathematical statistics[2], but those warnings have had little shock.

A new contribution of Sakoulis et al. (2010) argues that the persistence of forward premium is affected by the presence structural breaks. This study casts some doubt on 3 their finding, and argues that the presence of structural breaks in Sakoulis et al. (2010) is a result of model mis-specification, which ignores a proper lag structure that is usually related with AR regression or unit root test.

In addition to Sakoulis et al. (2010)'s study, we also process the forward discount as an AR(1) model with structural breaks along with a lag structure. We provide evidence that when a proper lag length is included in Sakoulis et al. (2010)'s process, the structural breaks do not have some effects on the autoregressive parameter of the lagged forward discount. Results obtained from imposing a lag structure on Sakoulis et al. (2010)'s process are robust to those from that without lag structure.

With the consideration of problems described more than, the results of Choi and Zivot (2007) and Kellar and Sarantis (2008) show that the forward exchange rate exhibits both fractionally integrated behavior and structural change.

The purpose of this paper is to propose an easy-to implement approach for forecasting a long memory process subject to structural change. The conventional forecasting technique based on post-break data could be sub-optimal for the break detection approach can lead to false results concerning the number of structural changes even the data generating process truly follows an

[1]For example see Hidalgo and Robinson, (1996), Lobato and Savin, (1997).
[2]For example see, Bhattacharya, et al, (1983), Künsch, (1986) and Teverovsky and Taqqu, (1997).

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fractionally integrated process without breaks (for example Granger and Hyung, 2004 and Hsu, 2001) or there could be more than one structural change.

This paper is organized as follows. Section 2 presents a review of empirical literature about modeling and forecasting financial returns. Section 3 presents the GARCH-class of models used in this study. Section 4 discusses the empirical findings of both estimation and forecasting. Finally, Section 5 concludes the paper.

LITERATURE REVIEW

This paper fits two long memory volatility models, Fractionally Integrated GARCH (FIGARCH) and (FIEGARCH) that allow for asymmetry. Baillie et al (1996) find that these processes have considerable success in modeling in daily stock market and oil price and we will investigate whether these GARCH models can indicate the long memory proprieties of stock markets returns.

Baillie (1996) shows that long memory processes have the attribute of having very strong autocorrelation persistence previous to differencing, and there by being non-stationary, whereas the first differenced series does not demonstrate persistence and is stationary. Though the long memory property of these stock price series is not evident from just first differencing alone, but has resulted from analysis of risk measures, in fact financial returns.

Econometric methodology

Modeling and forecasting conditional variance with GARCH-class of models has been examined in several studies (for example Pagan and Schwert, 1990, Brailsford and Faff, 1996, Franses, Neele, and Van Dijk, 1998 and Loudon, et al, 2000). Moreover, comparing normal density with non-normal ones has been also investigated by many researchers (see, for example Hsieh, 1989, Baillie and Bollerslev, 1989, Peters, 2000 or Lambert and Laurent, 2001). In this work, we couple these two advances by examining a variety of GARCH-class of models, the GARCH, EGARCH, FIGARCH and FIEGARCH processes where innovations follow different errors distributions (Normal, Student-t and Skewed Student-t). In-sample and out-of-sample forecasts evaluation are made using different loss functions

GARCH class of models

Let us consider an univariate time series y_t. If ψ_{t-1} is the information set (i.e. all the information available) at time $t-1$, we can describe its functional form as:

$$y_t = E\left[y_t \middle| \psi_{t-1}\right] + \varepsilon_t \qquad (1)$$

where $E\left[.|.\right]$ denotes the conditional expectation operator and ε_t is the disturbance term (or unpredictable part), with $E\left[\varepsilon_t\right] = 0$ and $E\left[\varepsilon_t \varepsilon_s\right] = 0 \forall t \neq s$. The ε_t term in equation (1) is the innovation of the process. The conditional expectation is the expectation conditional to every past information available at time $t-1$. The Autoregressive Conditional Heteroscedastic (ARCH) process of Engle (1982) is any $\{\varepsilon_t\}$ of the form:

$$\varepsilon_t = z_t \sigma_t \tag{2}$$

where z_t is an independently and identically distributed (i.i.d.) process, $E(z_t)=0$, $\text{var}(z_t)=1$ and where σ_t is a time-varying, positive and measurable function of the information set at time $t-1$. By definition, ε_t is serially uncorrelated with mean zero, but its conditional variance equals $\sigma^2{}_t$ and, consequently, may change over time, contrary to what is assumed in OLS estimations. Particularly, the ARCH (q) process is specified by:

$$\sigma^2{}_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 \tag{3}$$

The processes considered in this paper are all ARCH-type. They differ on the functional form of $\sigma^2{}_t$ but the basic logic is the same

The GARCH(p,q) process

Bollerslev (1986) have extended the ARCH models to the generalized ARCH (GARCH) models. The latter is characterized by an autoregressive moving average form in the conditional variance σ_t^2.

For the GARCH (*p*, *q*) process the conditional variance is expressed as,

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2$$

$$= w + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \tag{4}$$

Where $w > 0, \alpha_1, \ldots, \alpha_q, \beta_1, \ldots, \beta_p \geq 0$, $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \ldots + \beta_p L^p$ are the lag polynomials with orders of *q* and *p* respectively for stability and covariance stationary of the $\{\varepsilon_t\}$ process. To ensure nonnegative of the conditional variance, it is assumed that all the roots of the polynomial $[1-\beta(L)]$ are laying outside the unit circles.

In this paper we use the GARCH (1,1) process indicated as following:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{5}$$

EGARCH(p,q) process

Our first asymmetric GARCH model is the Exponential GARCH (EGARCH) process of Nelson (1991):

$$\ln \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i g\left(z_{t-i}\right) + \sum_{j=1}^{p} \beta_j \ln \sigma_{t-j}^2 \qquad (6)$$

Where $z_t = \dfrac{\varepsilon_t}{\sigma_t}$ is the normalized residuals series.

The value of $g\left(z_t\right)$ depends on several elements. Nelson (1991) notes that, *"to accommodate* the asymmetric relation among stock returns and volatility changes, the value of $g\left(z_t\right)$ must be a function of both the magnitude and the sign of z_t:" That is why he propose to express the function $g\left(.\right)$ as:

$$g\left(z_t\right) = \theta_1 z_t + \theta_2 \left[\left|z_t\right| - E\left|z_t\right|\right] \qquad (7)$$

Another advantage of this condition is that i does not require any stationary restriction. Notice also that $E\left|z_t\right|$ depends on the supposition made on the unconditional density.

In this paper we use the GARCH (1,1) process indicated as following: The EGARCH (1,1) model may then be expressed as:

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \alpha_1 \left(\left|z_{t-1}\right| - E\left(\left|z_{t-1}\right|\right)\right) + \gamma z_{t-1} + \beta \ln\left(\sigma_{t-1}^2\right) \qquad (8)$$

The FIGARCH(p,d,q) process

Baillie (1996) introduced FIGARCH process as a popular parametric approach to test the long memory property in the volatility of financial return series. In contrast to an stationary time series in which shocks die out at an exponential rate, or a non-stationary time series in which there is no mean reversion, shocks to an $I\left(d\right)$ time series with $d \in \left(0,1\right)$ decay at a very slow hyperbolic rate. The FIGARCH (p, d, q) process is given by:

$$\varphi\left(L\right)\left(1-L\right)^d \varepsilon_t^2 = \alpha + \left[1 - \beta\left(L\right)\right] w_t \qquad (9)$$

Conditional variance of ε_t is:

$$\sigma_t^2 = \frac{\alpha}{\left[1-\beta(1)\right]} + \gamma(L)\varepsilon_t^2 \tag{10}$$

where, $\gamma(L) = \gamma_1 L_1 + \gamma_2 L_2 + \ldots + \gamma_K L_K$

when $0 < d < 1$ the coefficients capture the short term dynamics of volatility while fractional difference coefficient d process the long term characteristics of volatility.

FIGARCH process is the extension of IGARCH process when d=1. In the IGARCH models shocks to the conditional variance are completely persistent and therefore the unconditional variance does not exist. In addition, when $d = 0$, FIGARCH process gives the same output with GARCH process and when $d = 1$, FIGARCH process gives the same output with IGARCH process.
In its simplest form the FIGARCH(1,d,1) is given by,

$$\varphi(L)(1-L)^d \varepsilon_t^2 = w + \alpha\left(1-\beta(L)\right)vt \tag{11}$$

The FIEGARCH(p,d,q) process

In order to accommodate asymmetries between positive and negative shocks, called the leverage effect, Bollerslev and Mikkelsen (1996) extend the FIGARCH process to FIEGARCH, to correspond with Nelson's (1991) Exponential GARCH model to allow for asymmetry. The FIEGARCH(p,d,q) model is given as:

$$\ln\left(\sigma_t^2\right) = w + \phi(L)^{-1}(1-L)^{-d}\left[1+\alpha(L)\right]g\left(z_{t-1}\right) \tag{12}$$

where $g(z_t) = \theta z_t + \gamma\left[|z_t| - E|z_t|\right]$, the first term θz_t is the sign effect, and the second term $\gamma\left[|z_t| - E|z_t|\right]$ is the magnitude effect. All the roots of $\phi(L)$ and $\lambda(L)$ are an autoregressive polynomial and a moving average polynomial in the lag operator L and lie outside the unit circle, and both polynomials do not have a common root. When $d = 1$, the FIEGARCH(p,d,q) process reduces to EGARCH of Nelson (1991), and when $d = 1$, the process becomes integrated EGARCH (IEGARCH).
In particular, the FIEGARCH (1,d,1) process may be conveniently expressed as:

$$(1-\beta L)(1-L)^d \log\left(\sigma_t^2\right) = w + \alpha\left(|z_{t-1}| - E\left(|z_{t-1}|\right)\right) + \gamma z_{t-1} \tag{13}$$

2.2.5 The APARCH process

Generally, the inclusion of a power term acts so as to underline the periods of relative tranquility and volatility by amplify the outliers in that series. Squared terms are consequently so often used in processes. If a data series is normally distributed than we are capable to completely

illustrate its distribution by its first two moments[3]. If we recognize that the data may have a non-normal error distribution, other power transformations can be more appropriate.
Distinguishing the possibility that a squared power term may not perforce be optimal, Ding, et al (1993) indicated a novel class of ARCH process called the Power-ARCH (PARCH) process. Rather than imposing a structure on the data, the Power-ARCH class of processes estimates the optimal power term. Ding, et al, (1993) also specified a generalized asymmetric version of the Power-ARCH process (APARCH). The APARCH (p,q) process can be indicated as:

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i} \right)^{\delta} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta} \tag{14}$$

where $\alpha_0 > 0$, $\delta \geq 0$, $\beta_j \geq 0$, $\alpha_i \geq 0$ and $-1 < \gamma_i \leq 1$.

This process couples the flexibility of a varying exponent δ with the asymmetry parameter γ_i to take the "leverage effect" into account. Furthermore, the APARCH comprises ARCH, GARCH and GJR as following:

ARCH when $\delta = 2$, $\gamma_i = 0 \ (i = 1,....q)$ and $\beta_j = 0 \ (j = 1,....p)$

The HYGARCH process

Davidson (2001) extended the class of FIGARCH processes to HYGARCH (p,α,d,q) processes which stands for hyperbolic GARCH. HYGARCH processes replace the operator $(1-L)^d$ in FIGARCH process by $\left[(1-\alpha)+\alpha(1-L)^d\right]$. The parametrization of HYGARCH-processes is given as following:

$$\sigma_t^2 = \frac{w}{\left[1-\beta(L)\right]} + \left\{1 - \left[1-\beta(L)\right]^{-1} \phi(L) \left\{1+\alpha\left[(1-L)^d\right]\right\}\right\} \varepsilon_t^2 \tag{15}$$

The parameters α and d are assumed to be non-negative. HYGARCH processes nest GARCH processes (for $\alpha = 0$), FIGARCH-processes (for $\alpha = 1$) and IGARCH- processes (for $\alpha = d = 1$).

[3]For more detail see McKenzie and Mitchell, (2001).

The FIEGARCH and FIAPARCH process

The phenomenon of fractional integration has been prolonged to other GARCH types of processes, including the Fractionally Integrated EGARCH (FIEGARCH) of Bollerslev and Mikkelsen (1996) and the Fractionally Integrated APARCH (FIAPARCH) of Tse (1998).

equally to the GARCH (p,q) process, the EGARCH (p,q) can be prolonged to account for long memory by factorizing the autoregressive polynomial $\left[1-\beta(L)\right]=\phi(L)(1-L)^{d}$ where all the roots of $\phi(z)=0$ lie outside the unit circle. The FIEGARCH (p, d, q) is indicated by:

$$\ln\left(\sigma_t^2\right)=w+\phi(L)^{-1}(1-L)^{-d}\left[1+\alpha(L)\right]s\left(z_{t-1}\right) \tag{16}$$

And the FIAPARCH (*p*, *d*, *q*) process can be indicated as:

$$\sigma_t^{\delta}=w+\left\{1-\left[1-\beta(L)\right]^{-1}\phi(L)(1-L)^{-d}\right\}\left(\left|\varepsilon_t\right|-\gamma\varepsilon_t\right)^{\delta} \tag{17}$$

Different types of conditional distribution functions are discussed in literature. These are the Normal distribution which we used in the previous section, the standardized Student-t distribution and the generalized error distribution and their skewed versions.

Different errors distributions can be employed when using the four GARCH process as described before. In this paper, in addition to the Normal distribution largely employed in empirical literature, we use the t-students, skewed t-Students and the Generalized Error Distributions (GED).

The Normal distribution is by far the most widely used distribution when estimating and forecasting GARCH processes. If we express the mean equation as in equation (1) and $\varepsilon_t = z_t \sigma t$, the log-likelihood function of the standard normal distribution is indicated as:

$$L_T=-\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi)+\ln\left(\sigma_t^2\right)+z_t^2\right] \tag{18}$$

where T is the number of observations.

A second distribution also largely employed when using the GARCH-class of models is the t-Student distribution. This distribution has been employed in many studies to account for fat tailed, see for instance. Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989), Bollerslev, et al (1992), Palm (1996), Pagan (1996), and Palm and Vlaar (1997) among others showed that the Student-t distribution better captures the observed kurtosis in the return time series. The density $f^*(z|v)$ of the Standardized Student-t Distribution can be expressed as:

$$f^*(z|v) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)} \frac{1}{\left(1+\frac{z^2}{v-2}\right)^{\frac{v+1}{2}}} = \frac{1}{\sqrt{v-2}B\left(\frac{1}{2},\frac{v}{2}\right)} \frac{1}{\left(1+\frac{z^2}{v-2}\right)^{\frac{v+1}{2}}} \tag{19}$$

Where $v > 2$ is the shape parameter and $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ the Beta function. Note, when setting $\mu = 0$ and $\sigma^2 = \frac{v}{(v-2)}$ equation (11) results in the usual one-parameter expression for the Student-t distribution.

Generalized Error Distribution

Nelson (1991) suggested considering the family of Generalized Error Distributions, GED, already employed by Box and Tiao (1973), and Harvey (1981). $f^*(z|v)$ can be indicated as:

$$f^*(z|v) = \frac{v}{\lambda_v 2^{1+1/v}\Gamma(1/v)} e^{-\frac{1}{2}\left|\frac{z}{\lambda_v}\right|^v} \tag{20}$$

Where

$$\lambda_v = \left(\frac{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right)^{1/2} \tag{21}$$

with $0 < v \le \infty$. Note, that the density is standardized and thus has zero mean and unit variance. Arbitrary location and scale coefficients μ and σ can be commenced via the transformation

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$z \rightarrow \frac{z-\mu}{\sigma}$. Since the density is symmetric, odd central moments of the GED are zero and those of even order can be calculated as:

$$\mu_{2r} = \sigma^{2r} \mu^{*}_{2r} = \sigma^{2r} \frac{\left(2^{1/v} \lambda_v\right)^{2r}}{\Gamma\left(\frac{1}{v}\right)} \Gamma\left(\frac{2r+1}{v}\right) \qquad (22)$$

Skewness γ_1 and kurtosis γ_2 are given by:

$$\gamma_1 = \frac{\mu_3}{\mu^{3/2}_{2}} = 0, \gamma_2 = \frac{\mu_4}{\mu^{2}_{2}} - 3 = \frac{\Gamma\left(\frac{1}{v}\right)\Gamma\left(\frac{5}{v}\right)}{\Gamma\left(\frac{3}{v}\right)^2} - 3 \qquad (23)$$

For $v=1$ the GED reduces to the Laplace distribution, for $v=2$ to the Normal distribution, and for $v \rightarrow \infty$ to the uniform distribution as a special case.

EMPIRICAL RESULTS

Data

In this section, we describe the data and our empirical findings. Data consist in 6019 daily observations of the SP500 and the CAC40 returns stock markets. It covers a 23 years period, from 02/08/1997 to 30/02/2021. The estimation process is run using 23 years of data while the remaining 6 years are used for forecasting. The indices prices are transformed into their returns so that we obtain stationary series. The transformation is

$$r_t = 100 * \left[\ln\left(y_t\right) - \ln\left(y_{t-1}\right)\right] \qquad (24)$$

Table 1. Descriptive statistics of return series

	T	Mean	Std. dev.	Skewness	kurtosis	J.B	ARCH	Q(10)
SP500	6019	0.00012	0.831	-0.482	7.921	8316	0.542	16.721
CAC40	6019	-0.0005	0.751	-0.512	6.892	7892	0.678	15.823

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Table 1 report descriptive statistics of the SP500 and CAC40 return series. Mean returns were very small compared to the standard deviations. All the values of the Skewness statistics were negative as well as more pronounced in the CAC40 index return, suggesting a greater probability of large decreases in index returns compared to the series of SP500. The Skewness values show that the marginal distributions are asymmetrical to the left. The high values of the Kurtosis statistics are consistent with fat tails in the return distributions and are more pronounced in the index returns than in S&P500.

In fact, the hypothesis of normality is rejected for all series since the high value of Jarque-Bera test. Moreover, the ARCH effects were likely to be found in all the return series since the significant value of the Lagrange multiplier (ARCH-LM) is statistic. The existence of this effect supports our decision to employ a GARCH modeling approach to examine the volatility dynamic and transmission between S&P500 and CAC40 stock markets. The CAC40 returns were more volatile than the other indices as measured by standard deviation. The stock markets indices have the same volatility.

Table 2. The diagnostic tests of the standardized residuals

	model		Log-Likelihood	Akaike	Q(10)	ARCH(10)
S&P500	FIGARCH	coefficient	-3812.281	1.421	4.567	0.378
		p-prob	-	-	0.723	0.421
	FIEGARCH	coefficient	-3829.218	1.662	4.111	0.387
		p-prob	-	-	0.686	0.429
	FIAPARCH	coefficient	-3891.382	1.712	3.985	0.538
		p-prob	-	-	0.524	0.413
	HYGARCH	coefficient	-3821.385	1.428	4.812	0.589
		p-prob	-	-	0.528	0.628
CAC40	FIGARCH	coefficient	-4217.384	1.432	4.391	0.482
		p-prob	-	-	0.678	0382
	FIEGARCH	coefficient	-4512.812	1.623	4.287	0.581
		p-prob	-	-	0.862	0.387
	FIAPARCH	coefficient	-4548.237	1.723	4.813	0.428
		p-prob	-	-	0.218	0.674
	HYGARCH	coefficient	-4382.382	1.862	4.388	0.589
		p-prob	-	-	0.428	0.381

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Table 2 indicate that the FIEGARCH process product that the FIAPARCH. Note that the largest values of Log-Likelihood are greater in absolute values for the FIEGARCH and FIAPARCH models. This indicates the relevance of these processes compared to the other models considered. Regarding the Akaike statistics, we notice that the largest values are accepted for models FIEGARCH and FIAPARCH and HYGARCH. FIEGARCH is the best fitting model for SP500. FIGARCH is the best fitting model for CAC40. The Log-Likelihood value is always negative. The bigger Log-Likelihood measurement indicates that FIAPARCH is the best model to reveal forecasting process. Using the Akaike information the FIEGARCH process was adjudged to be the best model in the case of S&P500 returns although the ARCH-effect.

Table 3. Process evaluation forecasts

process	S&P500		CAC40	
	MSE	MAD	MSE	MAD
FIGARCH	0.0023	0.0054	0.00201	0.0083
FIEGARCH	0.00098	0.0018	0.0028	0.00366
FIAPARCH	0.0029	0.00017	0.0098	0.0046
HYGARCH	0.0082	0.0061	0.00123	0.0028

For stock markets, based on the MSE the FIAPARCH performs best. Further, the FIAPARCH process under give less errors to be the best model in the case of CAC40 returns compared to the S&P500. Lastly, based on the S&P500, the FIEGARCH gives less prediction error. Hence this confirms the selection of FIAPARCH process as good models since it evident from the MAE evaluation measure. The believe that our study contributes significantly to the literature by evaluating the difference process of GARCH family (FIGARCH- FIEGARCH- HYPERGARCH- FIAPARCH).

CONCLUSION

We have made an attempt a brief review and comparison on FIGARCH, FIEGARCH, FIAPARCH and HYGARCH models.

We believe that our study contributes significantly to the literature by evaluation different process of GARCH family (FIGARCH, FIEGARCH, APARCH and HYPERGARCH). The FIEGARCH is the best process

Results from this paper points to the need for more empirical analysis on the S&P500 and CAC40 stock markets. For this research, we will compare time varying FIGARCH, FIEGARCH and HYPERGARCH type process that will factor in structural breaks to structural breaks adjusted FIEGARCH type process.

Moreover, for most of the precious metals considered, this dual long memory is found to be adequately captured by an FIFGARCH and FIAPARRCH process, which also provides better out-of-sample forecast accuracy than several popular volatility process. Finally, evidence shows that conditional volatility of stock markets is better explained by long memory than by structural breaks.

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