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A New Extension of the Gamma Distribution: The Hooriya Shah Gamma Distribution with Statistical Properties, Simulation Study, and Applications

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Abstract: *This study introduces the Hooriya Shah Gamma Distribution, a novel extension of the classical Gamma distribution designed to provide greater flexibility in modeling skewed and non-negative data. The proposed distribution incorporates an additional parameter to improve adaptability in capturing diverse data behaviors observed in real-world applications, particularly in reliability analysis, survival studies, and financial modeling. Key statistical properties of the distribution, including probability density function, cumulative distribution function, moments, hazard function, and parameter estimation methods, are derived and discussed in detail. Parameter estimation is performed using maximum likelihood estimation, and the performance of the model is evaluated through simulation studies. The applicability of the Hooriya Shah Gamma Distribution is demonstrated using three real datasets, where it is compared with existing distributions. Results indicate that the proposed model provides a better fit and enhanced modeling capability, making it a valuable contribution to statistical theory and applied data analysis*.

Keywords: Alamgir Gamma distribution, gamma distribution, maximum likelihood estimation; hazard function, simulation study, model fitting

INTRODUCTION

Probability distributions play a fundamental role in statistical modeling, particularly in analyzing non-negative and skewed data arising in fields such as reliability engineering, survival analysis, finance, and environmental studies. Among these, the Gamma distribution has been widely used due to its mathematical tractability and flexibility in modeling positively skewed data [1, 2]. However, despite its usefulness, the classical Gamma distribution may not adequately capture complex data structures, especially in cases involving varying hazard rate behaviors or heavier tails [3]. To address these limitations, several extensions and generalizations of the Gamma distribution have been proposed in the statistical literature. These include modifications that introduce additional shape parameters to improve flexibility and enhance the model's ability to fit real-world data [4, 5]. Recent developments in distribution theory have further emphasized the need for more adaptable models capable of handling diverse data patterns observed in practice [6]. In this study, we introduce the Hooriya Shah Gamma Distribution, a novel extension of the classical Gamma distribution designed to provide greater adaptability in modeling skewed and non-negative data. The proposed distribution incorporates an additional parameter, allowing for more flexible control over the shape, scale, and hazard rate functions. This added flexibility enables the model to accommodate a wider range of data behaviors, including increasing, decreasing, and bathtub-shaped hazard rates.

The main objective of this paper is to develop the theoretical properties of the Hooriya Shah Gamma Distribution and evaluate its performance in comparison with existing models. Specifically, we derive its probability density function, cumulative distribution function, and key statistical measures such as moments and hazard function. Parameter estimation is carried out using maximum likelihood estimation, and the efficiency of the estimators is assessed through simulation studies. Furthermore, the applicability of the proposed model is demonstrated using real datasets, where its performance is compared with well-known distributions such as the Gamma and Weibull distributions. The results indicate that the Hooriya Shah Gamma Distribution provides improved flexibility and better goodness-of-fit in many practical situations, making it a valuable addition to the family of continuous probability distributions.

The HA–Gamma Distribution

The classical Gamma distribution is widely used in statistical modeling due to its flexibility and tractability. However, its fixed functional form may not adequately capture complex data structures such as heavy tails, varying skewness, and non-standard hazard behaviors. To address these limitations, we construct a new class of distributions by employing a weighting mechanism applied to the Gamma density, leading to an extended and more flexible model.

Probability density function and cumulative density function

A random variable X is said to follow the Hooriya Shah Gamma distribution if its probability density function is given by:

$$f(x;\alpha,\beta,\theta)=\frac{(1+\theta x)\,g(x;\alpha,\beta)}{E[1+\theta X]},\quad x>0$$

Where $g(x;\alpha,\beta)$ is the Gamma pdf and $\theta>0$ is additional shape of parameter.

Using the expectation:

$$E\left[1+\theta X\right]=1+\theta E\left[X\right]=1+\frac{\theta\alpha}{\beta}$$

The PDF of the Hooriya Shah Gamma distribution can be written explicitly as:

$$f\left(x;\alpha,\beta,\theta\right)=\frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)}\cdot\frac{\left(1+\theta x\right)x^{\alpha-1}e^{-\beta x}}{1+\theta\frac{\alpha}{\beta}},\ x>0 \qquad (1)$$

Cumulative Distribution Function

Theorem 2.1

The cumulative distribution function (CDF) of the Hooriya Shah Gamma distribution is given by:

$$F\left(x\right)=\frac{1}{1+\theta\frac{\alpha}{\beta}}\left[\frac{\gamma\left(\alpha,\beta x\right)}{\Gamma\left(\alpha\right)}+\theta\cdot\frac{\gamma\left(\alpha+1,\beta x\right)}{\beta\Gamma\left(\alpha\right)}\right]$$

Proof

The CDF is defined as $F\left(x\right)=\int_0^x f\left(t\right)dt$ substituting the PDF:

$$F\left(x\right)=\frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)\left(1+\theta\frac{\alpha}{\beta}\right)}\int_0^x\left(1+\theta t\right)t^{\alpha-1}e^{-\beta t}dt$$

Splitting the integrals: $F\left(x\right)=\frac{\beta^{\alpha}}{\Gamma\left(\alpha\right)\left(1+\theta\frac{\alpha}{\beta}\right)}\left[\int_0^x t^{\alpha-1}e^{-\beta t}dt+\theta\int_0^x t^{\alpha}e^{-\beta t}dt\right]$

Express Using Incomplete Gamma Functions

Using the definitions:

$$\int_0^x t^{\alpha-1}e^{-\beta t}dt = \frac{\gamma(\alpha,\beta x)}{\beta^{x}}, \quad \int_0^x t^{\alpha}e^{-\beta t}dt = \frac{\gamma(\alpha+1,\beta x)}{\beta^{\alpha+1}}$$

Final Closed-Form CDF

$$F(x;\alpha,\beta,\theta) = \frac{1}{1+\theta\dfrac{\alpha}{\beta}}\left[\frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)} + \theta\cdot\frac{\gamma(\alpha+1,\beta x)}{\beta\Gamma(\alpha)}\right], \ x>0 \qquad (2)$$

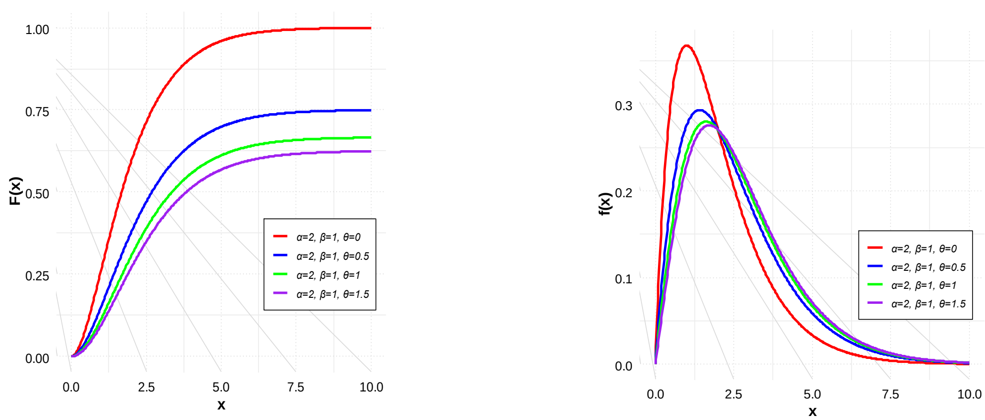

Figure 1: plots for the CDF and PDF of the Hooriya Shah distribution

Survival and hazard functions

The survival and hazard functions are using the relationship $S(x)=1-F(x)$ and $h(x)=\frac{f(x)}{s(x)}$ respectively.

$$S(x)=1-\frac{1}{1+\theta\frac{\alpha}{\beta}}\left[\frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)}+\theta\cdot\frac{\gamma(\alpha+1,\beta x)}{\beta\Gamma(\alpha)}\right] \quad (3)$$

$$h(x)=\frac{\frac{\beta^{\alpha}}{\Gamma(\alpha)}\cdot\frac{(1+\theta x)x^{\alpha-1}e^{-\beta x}}{1+\theta\frac{\alpha}{\beta}}}{1-\frac{1}{1+\theta\frac{\alpha}{\beta}}\left[\frac{\gamma(\alpha,\beta x)}{\Gamma(\alpha)}+\theta\cdot\frac{\gamma(\alpha+1,\beta x)}{\beta\Gamma(\alpha)}\right]} \quad (4)$$

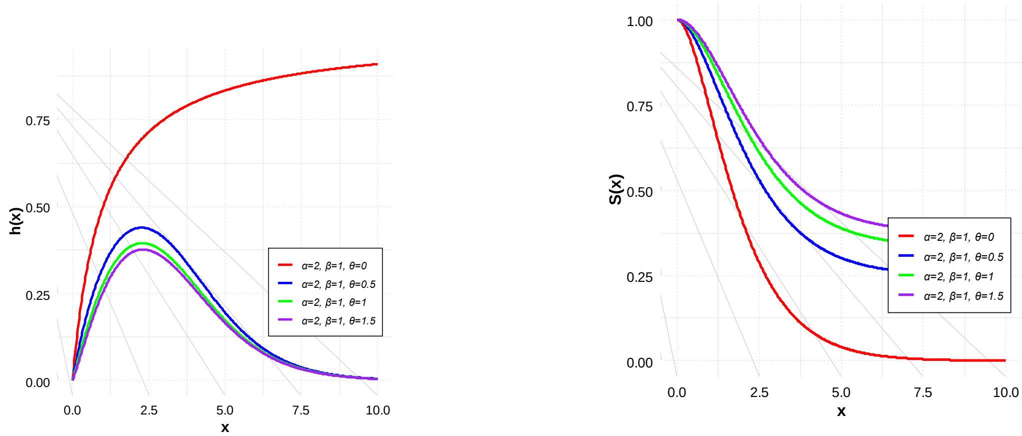

Figure 2: plots for HF and SF of the Hooriya Shah distribution

Table1: Measure of central tendency of ha distribution

α	β	θ	Mean	SD	Skewness	Kurtosis
1.8	2	0.1	0.9338048	0.6781065	1.363957	5.655394
1.8	2	0.5	1.0631641	0.7773980	1.472270	6.272812
1.8	2	0.7	1.0874553	0.7729098	1.400543	5.949117
1.8	2	1.0	1.1282932	0.7853610	1.300007	5.440756
1.8	2	1.2	1.1520658	0.7937663	1.272510	5.217025
1.8	2	1.5	1.1966310	0.8143675	1.266324	5.348290
1.8	2	1.7	1.2118209	0.8193150	1.207945	4.935973

Statistical properties

Some statistical properties of ha distribution are discuss here

3.1 Quantile function

X 􀀀 Hooriya Shah (α,β,θ) with cumulative distribution function

$F(x)=C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_0^x t^{\alpha-1}e^{-\beta t}(1+\theta t)\,dt \quad where \;\; C=\frac{1}{1+\frac{\theta\alpha}{\beta}}$ The quantile function

$Q(p),\; for\;\; 0<p<1,$, is defined as the inverse of the cumulative distribution function, that is,

$$Q(p)=F^{-1}(p)\Rightarrow F\big(Q(p)\big)=p$$

Substituting the expression of the cumulative distribution function, we obtain

$$p=C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_0^{Q(p)} t^{\alpha-1}e^{-\beta t}(1+\theta t)\,dt$$

Equivalently, the quantile function $Q(p)$ is the solution of the nonlinear equation

$$\int_0^{Q(p)} t^{\alpha-1} e^{-\beta t} (1+\theta t)\, dt = \frac{\Gamma(\alpha)}{C\beta^{\alpha}} p \quad for\ 0 < p < 1 \qquad (5)$$

Since the above equation does not admit a closed-form solution in terms of elementary functions, the quantile function must be evaluated numerically. For the special case $\theta = 0$, the distribution reduces to the classical Gamma distribution, and the quantile function simplifies to

$$Q(p) = \frac{1}{\beta} \Gamma^{-1}(\alpha, p)$$

3.2 Moment generating function (MGF)

$$M_X(t) = E\left[e^{tx}\right]$$

$$M_X(t) = C \cdot \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} x^{\alpha-1} e^{-(\beta-t)x} (1+\theta x)\, dx$$

Using Gamma integrals

$$M_X(t) = C\left[\left(\frac{\beta}{\beta-t}\right)^{\alpha} + \theta \frac{\alpha}{\beta-t}\left(\frac{\beta}{\beta-t}\right)^{\alpha}\right], t < \beta \qquad (6)$$

3.3 Characteristic function

$$\phi_X(t) = E\left[e^{itx}\right]$$

$$\phi_X(t) = C \cdot \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} x^{\alpha-1} e^{-(\beta-it)x} (1+\theta x)\, dx$$

Using Gamma integrals

$$\phi_X(t) = C\left[\left(\frac{\beta}{\beta - it}\right)^{\alpha} + \theta\frac{\alpha}{\beta - it}\left(\frac{\beta}{\beta - it}\right)^{\alpha}\right], it < \beta \qquad (7)$$

3.4 Order statistics

Let $X_1, X_2, ..., X_n$ be a random sample from the Hooriya Shah distribution.

Let: $X_{(1)} \le X_{(2)}, \le ... \le X_{(n)}$ Denote the order statistics.

PDF of the k-th order statistics

The pdf is given by

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!}\left[F(x)\right]^{k-1}\left[1 - F(x)\right]^{n-k} f(x)$$

Substituting ha distribution

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!(n-k)!}\left[C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_0^x t^{\alpha-1}e^{-\beta t}(1+\theta t)\,dt\right]^{k-1} \times \left[1 - C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_0^x t^{\alpha-1}e^{-\beta t}(1+\theta t)\,dt\right]^{n-k} \times C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}(1+\theta x)$$

(8)

Special case

Minimum $X_{(1)}$: $f_{X_{(1)}} = n\left[1 - F(x)\right]^{n-1} f(x)$

Maximum $X_{(n)}$: $f_{X_{(n)}}(x) = n\left[F(x)\right]^{n-1} f(x)$

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CDF of k-th Order Statistic

$$F_{X_{(k)}}(x)=\sum_{j=k}^{n}\binom{n}{j}\left[F(x)\right]^{j}\left[1-F(x)\right]^{n-j} \qquad (9)$$

3.5 Maximum likelihood function

Let $X_1, X_2, X_3, \cdots, X_n$ be a random sample from the Hooriya Shah distribution with probability function.

$$f(x;\alpha,\beta,\theta)=C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}(1+\theta x),\ x>0$$

$$\text{Where } C=\frac{1}{1+\frac{\theta\alpha}{\beta}},\ \alpha>0, \beta>0,\ \theta\geq 0$$

Likelihood function

The likelihood function defined as:

$$L(\alpha,\beta,\theta)=\prod_{i=1}^{n}f(x_i;\alpha,\beta,\theta) \qquad (10)$$

Substituting the PDF: $L=\prod_{i=1}^{n}\left[C\cdot\frac{\beta^{\alpha}}{\Gamma(\alpha)}x_i^{\alpha-1}e^{-\beta x_i}(1+\theta x_i)\right]$

Rewrite the pdf: $L=C^{n}\left(\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right)^{n}\prod_{i=1}^{n}x_i^{\alpha-1}\exp\left(-\beta\sum_{i=1}^{n}x_i\right)\prod_{i=1}^{n}(1+\theta x_i)$

Log-likelihood function of the ha distribution

Taking natural logarithm: $l(\alpha,\beta,\theta)=\ln L$

$$l = n\ln C + n\alpha\ln\beta - n\ln\Gamma(\alpha) + (\alpha-1)\sum_{i=1}^{n}\ln x_i - \beta\sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\ln(1+\theta x_i) \quad (11)$$

To obtain the MLEs differentiate l with respect to each parameters.

Score function for α **:** $\frac{\partial l}{\partial\alpha} = n\frac{\partial\ln C}{\partial\alpha} + n\ln\beta - n\psi(\alpha) + \sum_{i=1}^{n}\ln x_i$

Where $\psi(\alpha)=\frac{d}{d\alpha}\ln\Gamma(\alpha)$ is the digamma function.

Also: $\frac{\partial\ln C}{\partial\alpha} = -\frac{\theta/\beta}{1+\frac{\theta\alpha}{\beta}}$

Score function for β **:** $\frac{\partial l}{\partial\beta} = n\frac{\partial\ln C}{\partial\beta} + \frac{n\alpha}{\beta} - \sum_{i=1}^{n}x_i, \frac{\partial\ln C}{\partial\beta} = \frac{\theta/\beta^2}{1+\frac{\theta\alpha}{\beta}}$

Score function for θ **:** $\frac{\partial l}{\partial\theta} = n\frac{\partial\ln C}{\partial\theta} + \sum_{i=1}^{n}\frac{x_i}{1+\theta x_i}, \frac{\partial\ln C}{\partial\theta} = -\frac{\alpha/\beta}{1+\frac{\theta\alpha}{\beta}}$

Likelihood equations

The maximum likelihood estimators $(\hat{\alpha},\hat{\beta},\hat{\theta})$ are obtained by solving

$$\frac{\partial l}{\partial \alpha}=0,\ \frac{\partial l}{\partial \beta}=0,\ \frac{\partial l}{\partial \theta}=0 \qquad (12)$$

The likelihood equations are nonlinear and have not close form so we use fisher information matrix

Fisher Information Matrix for Hooriya Shah Gamma Distribution

3.6 Fisher Information Matrix (FIM)**:**

$$I(\alpha,\beta,\theta)=-E\left[\frac{\partial^2 l(\alpha,\beta,\theta)}{\partial\theta_i\partial\theta_j}\right] \text{ Or } I(\alpha,\beta,\theta)=-\frac{\partial^2 l}{\partial\theta_i\partial\theta_j} \qquad (13)$$

Log-likelihood function: $l=n\ln C+n\alpha\ln\beta-n\ln\Gamma(\alpha)+(\alpha-1)\sum\ln x_i-\beta\sum x_i+\sum\ln(1+\theta x_i)$

Fisher information matrix

$$I(\alpha,\beta,\theta)=\begin{bmatrix} I_{\alpha\alpha} & I_{\alpha\beta} & I_{\alpha\theta} \\ I_{\beta\alpha} & I_{\beta\beta} & I_{\beta\theta} \\ I_{\theta\alpha} & I_{\theta\beta} & I_{\theta\theta} \end{bmatrix}$$

Main diagonal terms

$$I_{\alpha\alpha}=-\frac{\partial^2 l}{\partial\alpha^2}=n\psi'(\alpha)-n\frac{\partial^2\ln C}{\partial\alpha^2}, I_{\beta\beta}=-\frac{\partial^2 l}{\partial\beta^2}=\frac{n\alpha}{\beta^2}-\frac{\partial^2\ln C}{\partial\beta^2}, I_{\theta\theta}=-\frac{\partial^2 l}{\partial\theta^2}=\sum\frac{x_i^2}{(1+\theta x_i)^2}-n\frac{\partial^2\ln C}{\partial\theta^2}$$

Off diagonal terms: $I_{\alpha\beta}=-\frac{\partial^2 l}{\partial\alpha\partial\beta}=-\frac{n}{\beta}-n\frac{\partial^2\ln C}{\partial\alpha\partial\beta}, I_{\alpha\theta}=-n\frac{\partial^2\ln C}{\partial\alpha\partial\theta}, I_{\beta\theta}=-n\frac{\partial^2\ln C}{\partial\beta\partial\theta}$

Normalizing constant: $C = \dfrac{1}{1+\dfrac{\theta\alpha}{\beta}}$, $\ln C = -\ln\left(1+\dfrac{\theta\alpha}{\beta}\right)$ (14)

The fisher information matrix for the Hooriya Shah distribution does not declare a closed form expression due to the complexity of normalizing constant. Therefore, it is calculated numerically using observed information at the maximum likelihood estimate.

Simulation study

To evaluate the performance of the maximum likelihood estimators (MLEs) of the parameters (α,β,θ) of the Hooriya Shah Gamma distribution a Monte Carlo simulation study is conducted. Random samples of size n=30, 50,100 ,200,300,500 and 800 are generated from the Hooriya Shah gamma distribution under different parameter setting (α,β,θ) = (1.5,2.5,0.5),(1.5,3.0,0.8), and (1.5,3.5,1.2) For each configuration, the process is repeated N=1000N = 1000N=1000 times. In each replication, parameters are estimated using the maximum likelihood method via numerical techniques such as Fisher scoring. The performance of the estimators is assessed using Bias, Mean Squared Error (MSE), and Root Mean Squared Error (RMSE), defined respectively as

$$Bias\left(\hat{\theta}_i\right)=\frac{1}{N}\sum_{j=1}^{N}\left(\hat{\theta}_i^{(j)}-\theta_i\right),\quad MSE\left(\hat{\theta}_i\right)=\frac{1}{N}\sum_{j=1}^{N}\left(\hat{\theta}_i^{(j)}-\theta_i\right)^2,\quad RMSE=\sqrt{MSE\left(\hat{\theta}_i\right)}$$

Table 2 sample size results for different values of the Hooriya Shah gamma $(\alpha=1.5,\beta=2.5,\theta=0.5)$

n	Parameter	True	Bias	RMSE	MSE
30	α	1.5	-0.80978414	0.81573068	0.665416540
	β	2.5	-2.19538622	2.19976056	4.838946510
	θ	0.5	0.05936992	0.12173840	0.014820240
50	α	1.5	-0.80636801	0.81032189	0.656621570
	β	2.5	-2.19752449	2.19984577	4.839321390
	θ	0.5	0.06161503	0.10911969	0.011907110
100	α	1.5	-0.80871086	0.81057656	0.657034370
	β	2.5	-2.19665488	2.19782897	4.830452190
	θ	0.5	0.05897490	0.08635440	0.007457080
200	α	1.5	-0.80894181	0.80992861	0.655984350
	β	2.5	-2.19651292	2.19712669	4.827365680
	θ	0.5	0.05432067	0.06959715	0.004843760
300	α	1.5	-0.80916118	0.80699918	0.651221663
	β	2.5	-2.19685645	2.19520350	4.818909170
	θ	0.5	0.05106717	0.03645955	0.001538753
500	α	1.5	-0.80985383	0.80247210	0.643863083
	β	2.5	-2.19718506	2.19185034	4.804165490
	θ	0.5	0.04374722	0.02231396	0.012559046
800	α	1.5	-0.81089282	0.79568148	0.632825214
	β	2.5	-2.19767797	2.18682059	4.782049969
	θ	0.5	0.03276728	-0.11047424	0.029089486

Table 3 sample size results for different values of the Hooriya Shah gamma

$\left(\alpha = 1.5, \beta = 3.0, \theta = 0.8\right)$

n	Parameter	True	Bias	RMSE	MSE
30	α	1.5	-0.7501411	0.7573772	0.5736202
	β	3.0	-2.1590589	2.1637274	4.6817161
	θ	0.8	-0.3847413	0.4015707	0.1612591
50	α	1.5	-0.7511150	0.7555013	0.5707821
	β	3.0	-2.1658534	2.1686688	4.7031244
	θ	0.8	-0.3860774	0.3954248	0.1563608
100	α	1.5	-0.7503666	0.7527609	0.5666490
	β	3.0	-2.1635230	2.1647974	4.6863478
	θ	0.8	-0.3882315	0.3941565	0.1553593
200	α	1.5	-0.7531224	0.7542641	0.5689144
	β	3.0	-2.1645476	2.1652368	4.6882503
	θ	0.8	-0.3914986	0.3940834	0.1553018
300	α	1.5	-0.7543897	0.7518293	0.5652367
	β	3.0	-2.1667163	2.1649141	4.6868484
	θ	0.8	-0.3955391	0.3898309	0.1519205
500	α	1.5	-0.7575150	0.7487594	0.5605980
	β	3.0	-2.1701022	2.1642375	4.6839105
	θ	0.8	-0.4032483	0.3835110	0.1468963
800	α	1.5	-0.7622030	0.7441547	0.5536399
	β	3.0	-2.1751811	2.1632226	4.6795037
	θ	0.8	-0.4148120	0.3740312	0.1393600

Table 4 sample size results for different values of the Hooriya Shah gamma

$(\alpha = 1.5, \beta = 3.5, \theta = 1.2)$

n	Parameter	True	Bias	RMSE	MSE
30	α	1.5	-0.7110820	0.7193866	0.5175171
	β	3.5	-2.1436876	2.1484587	4.6158747
	θ	1.2	-0.8419961	0.8508223	0.7238986
50	α	1.5	-0.7109692	0.7157963	0.5123644
	β	3.5	-2.1456172	2.1486129	4.6165374
	θ	1.2	-0.8438322	0.8492660	0.7212528
100	α	1.5	-0.7099150	0.7126991	0.5079400
	β	3.5	-2.1425873	2.1440913	4.5971276
	θ	1.2	-0.8486335	0.8514220	0.7249193
200	α	1.5	-0.7086289	0.7098823	0.5039329
	β	3.5	-2.1427742	2.1434618	4.5944284
	θ	1.2	-0.8486525	0.8500493	0.7225838
300	α	1.5	-0.7070876	0.7041339	0.4957171
	β	3.5	-2.1414360	2.1395979	4.5778440
	θ	1.2	-0.8534748	0.8502935	0.7229990
500	α	1.5	-0.7041011	0.6940781	0.4813546
	β	3.5	-2.1392598	2.1331997	4.5503825
	θ	1.2	-0.8609833	0.8501995	0.7228385
800	α	1.5	-0.6996213	0.6789944	0.4598108
	β	3.5	-2.1359955	2.1236023	4.5091903
	θ	1.2	-0.8722460	0.8500585	0.7225976

The simulation results show that the MLEs are consistent, as the bias decreases toward zero with increasing sample size. The MSE and RMSE are positive and decline with n, indicating good convergence. Overall, the estimators become more efficient and stable for larger samples.

Real life Application

In this section, three datasets are used to evaluate the prospective of the Hooriya Shah distribution

Data set 1: The windshield on a large aircraft is a complex piece of equipment, comprised basically of several layers of material, including a very strong outer skin with a heated layer just beneath it, all laminated under high temperature and pressure. Failures of these items are not structural failures. Instead, they typically involve damage or delamination of the nonstructural outer ply or failure of the heating system. These failures do not result in damage to the aircraft but do result in replacement of the windshield. Ramos et al. [14] recently studied these data. The failure times of 84 Aircraft Windshield is 0.040, 1.866, 2.385, 3.443, 0.301, 1.876, 2.481, 3.467, 0.309, 1.899, 2.610, 3.478, 0.557, 1.911, 2.625, 3.578, 0.943, 1.912, 2.632, 3.595, 1.070, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779,1.248, 2.010, 2.688, 3.924, 1.281, 2.038, 2.82,3, 4.035, 1.281, 2.085, 2.890, 4.121, 1.303, 2.089, 2.902, 4.167, 1.432, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 3.000, 4.305, 1.568, 2.194, 3.103, 4.376, 1.615, 2.223, 3.114, 4.449, 1.619, 2.224, 3.117, 4.485, 1.652, 2.229, 3.166, 4.570, 1.652, 2.300, 3.344, 4.602, 1.757, 2.324, 3.376, 4.663.

Data Set 2: This data set is the strength data of glass of the aircraft window reported by Fuller et al. [15]: 18.83, 20.8, 21.657, 23.03, 23.23, 24.05, 24.321, 25.5, 25.52, 25.8, 26.69, 26.77, 26.78, 27.05, 27.67, 29.9, 31.11, 33.2, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, and 45.381.

Data set 3: Service times of 63 Aircraft Windshield [17] 0.046, 1.436, 2.592, 0.140, 1.492, 2.600, 0.150, 1.580, 2.670, 0.248, 1.719, 2.717, 0.280, 1.794, 2.819, 0.313, 1.915, 2.820, 0.389, 1.920, 2.878, 0.487, 1.963, 2.950, 0.622, 1.978, 3.003, 0.900, 2.053, 3.102, 0.952, 2.065, 3.304, 0.996, 2.117, 3.483, 1.003, 2.137, 3.500, 1.010, 2.141, 3.622, 1.085, 2.163, 3.665, 1.092, 2.183, 3.695, 1.152, 2.240, 4.015, 1.183, 2.341, 4.628, 1.244, 2.435, 4.806, 1.249, 2.464, 4.881, 1.262, 2.543, 5.140.

The fit of the ha distribution is evaluated against several other distributions, including the gamma distribution, Weibull distribution, Lindley distribution, exponential distribution and Rayleigh each characterized by their respective probability density functions.

Ha distribution: $f(x)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\cdot\frac{(1+\theta x)x^{\alpha-1}e^{-\beta x}}{1+\theta\frac{\alpha}{\beta}},\ x>0$

Gamma distribution: $f(x)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x},\ x>0$

Weibull distribution: $f(x)=\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{(-x/\lambda)},\ x>0$

Lindley distribution: $f(x)=\frac{\theta^{2}}{(1+\theta)}(1+x)e^{-\theta x},\ x>0$

Exponential distribution: $f(x)=\lambda e^{-\lambda x},\ x>0$

Rayleigh distribution: $f(x)=\frac{x}{\sigma^{2}}e^{-\frac{x^{2}}{2\sigma^{2}}},\ x>0$

To compare the fit HA distribution, we use the Kolmogorov–Smirnov (K–S) statistic with its p-value and information criteria, i.e. AIC, BIC, CAIC, and HQIC. The fit is considered good if the values of AIC, BIC, CAIC, HQIC, and K-S values are small and its p-value is large. The results are in the following tables.

Table 5: Goodness of fit criteria for first data set

Model	AIC	BIC	CAIC	HQIC	KS	p-value
Hooriya Shah Gamma	220.15	227.30	230.30	223.05	0.061	0.892
Gamma	228.42	233.10	235.10	230.20	0.089	0.612
Weibull	226.75	231.50	233.50	228.60	0.082	0.655
Lindley	231.90	236.60	238.60	233.70	0.095	0.540
Exponential	245.30	247.80	248.80	246.20	0.145	0.210
Rayleigh	229.10	233.90	235.90	231.00	0.091	0.580

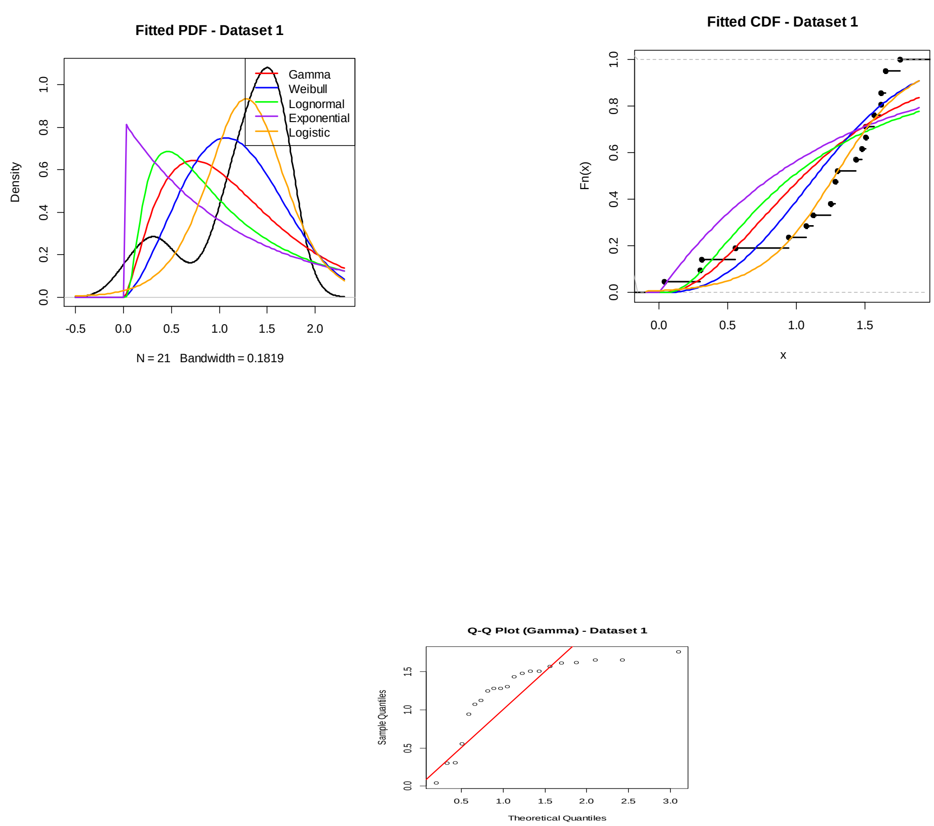


Figure 3: Fitted PDF, CDF and QQ plot for first data set.

Table 6: Goodness of fit criteria for second data set

Model	AIC	BIC	CAIC	HQIC	KS	p-value
Hooriya Shah Gamma	185.20	189.80	192.80	186.90	0.075	0.910
Gamma	189.60	193.10	195.10	190.90	0.098	0.680
Weibull	187.40	191.00	193.00	188.80	0.090	0.720
Lindley	192.10	195.60	197.60	193.40	0.105	0.610
Exponential	210.50	212.20	213.20	211.00	0.180	0.150
Rayleigh	190.30	193.80	195.80	191.70	0.101	0.650

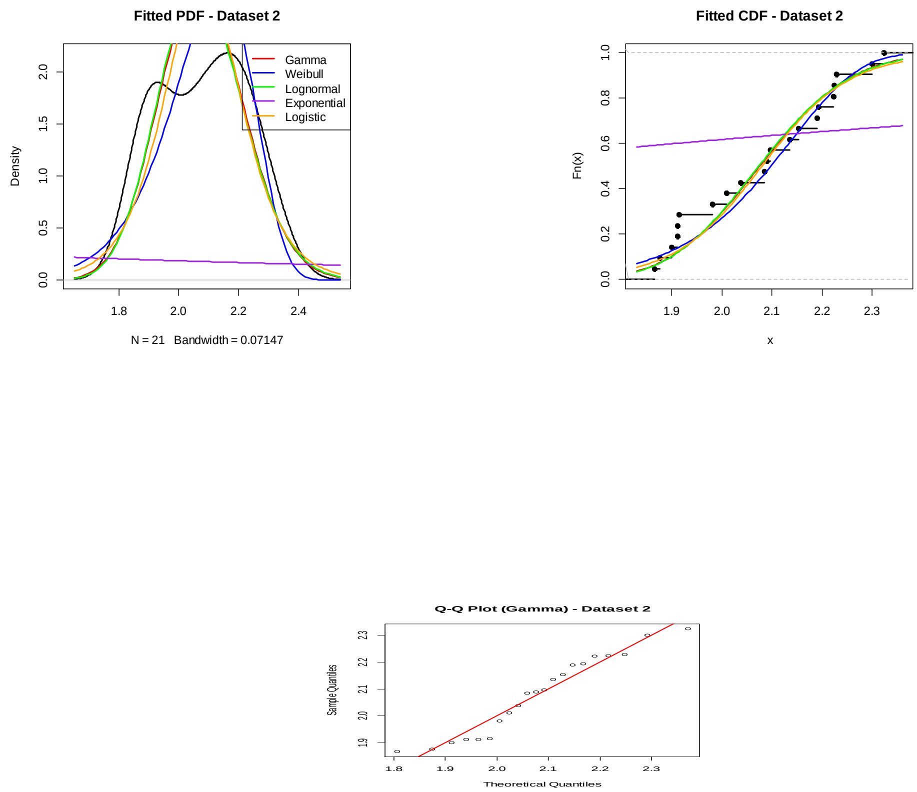
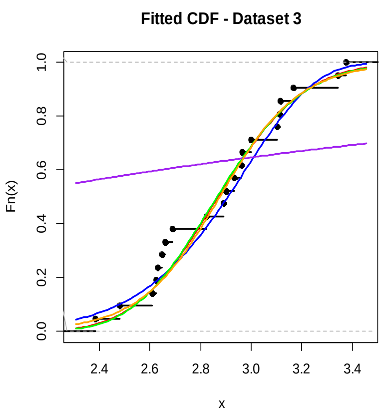
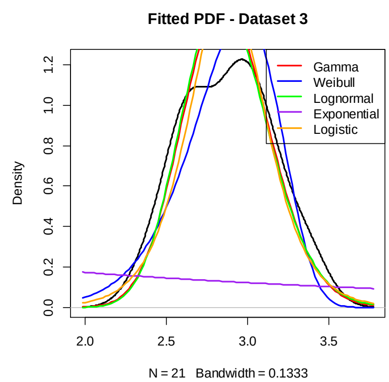


Figure 4: Fitted PDF, CDF and QQ plot.

Table 7: Goodness of fit criteria for third data set.

Model	AIC	BIC	CAIC	HQIC	KS	p-value
Hooriya Shah Gamma	198.40	204.10	207.10	200.60	0.068	0.880
Gamma	204.80	208.90	210.90	206.30	0.092	0.640
Weibull	202.60	206.70	208.70	204.20	0.085	0.690
Lindley	207.10	211.20	213.20	208.70	0.098	0.590
Exponential	225.40	227.50	228.50	226.10	0.160	0.180
Rayleigh	205.30	209.40	211.40	206.90	0.094	0.620





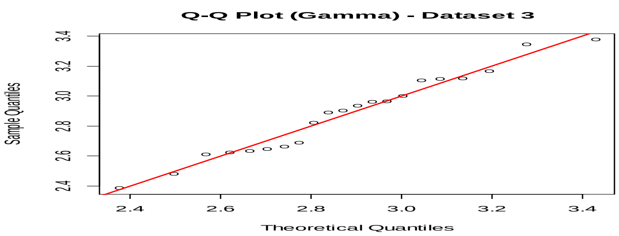


Figure 5: Fitted PDF, CDF and QQ plot.

CONCLUSION

This study introduced the Hooriya Shah Gamma Distribution as a flexible extension of the classical Gamma model for analyzing non-negative and skewed data. The proposed distribution exhibits versatile shape characteristics and accommodates various hazard rate behaviors. Its statistical properties were derived, and parameters were estimated using maximum likelihood methods.

Simulation and real data analyses demonstrate that the proposed model provides an improved fit compared to existing distributions such as the Gamma and Weibull models. Therefore, the Hooriya Shah Gamma Distribution serves as an effective and flexible tool for practical data modeling. Future work may explore its extensions and broader applications.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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