International Journal of Mathematics and Statistics Studies, 13 (1), 50-71, 2025
Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)
Website: https://www.eajournals.org/
Publication of the European Centre for Research Training and Development -UK

Establishing a New Matrix and Matrix Order Transformation by that New Matrix

S. M. Naser
B.Sc. (hons.): Mathematics, Pabna Science and Technology University
M.Sc.: Mathematics, Bangladesh University of Engineering and Technology
E-mail: s.m.naser.090313@gmail.com

doi: https://doi.org/10.37745/ijmss.13/vol13n15071 Published April 27, 202

Citation: Naser S.M. (2025) Establishing a New Matrix and Matrix Order Transformation by that New Matrix, *International Journal of Mathematics and Statistics Studies*, 13 (1), 50-71

Abstract: *This is an original research work with new inventions which will show the Matrix Order Transformation based on Perimeter Matrix, which I invented previously. This method will be generalised for n-th term to convert a matrix of order* $n \times n$ *into a matrix of order* 3×3 *and regain the original matrix of order* $n \times n$ *from the resultant matrix of order* 3×3. *For the n-th term I named it as the Method of Determinant Block Transformation.*

KEYWORDS: matrix order transformation, block transformation, block simplification, core matrix, core value, matrix equivalent theorem

INTRODUCTION

The role of matrices in linear algebra and its applications are so much. Applications of matrices cover not only the field of mathematics but also the other field of science and real-life sciences such as – cryptography, wireless communication, probability theory and statistics, electronics, engineering, computer science etc. Matrices have been used by different mathematicians all over the world from ancient time to now. There are so many kinds of matrices. Some of them are Square matrix, Identity matrix, Diagonal matrix and Zero matrix [1] etc.. Mathematics has a long history which claims it begins from Africa and spread to China, India and Middle East. Structure-wise mathematical study flourished in Europe, specifically from Greece.

Historically, matrices were introduced at 1800 century. Before that it was named as arrays. Chinese mathematicians claim that matrices were used by them in 10^{th}-2^{nd} BCE to solve simultaneous equations and determinants, which is noted in the China book '*The Nine Chapter On The Mathematical Art*' ([2], [3]). The theory of *Determinant* was vastly studied and started the use of it by Italian mathematician Gerolamo Cardano in 16^{th} century. His rule was for 2×2 determinants where German mathematician Leibnitz discussed for larger determinants. In Japan determinant was introduced by Seki Takakasu in 1683[4].

To find the solutions of unknowns of algebraic equations, Gaussian elimination and Cramer's rule are very important. Cramer's rule was published by Swiss mathematician Gabriel Cramer in 1750 and Gaussian elimination was published by German mathematician Carl Friedrich Gauss in 1849 [5]. French mathematician Cauchy-Binet expanded their study on pseudo determinants [6]. Later, Matrices hugely developed by British mathematician Arthur Cayley in the middle of nineteenth century [2].

There are so many applications of matrices. It is mainly used to solve the system of linear equations. The basic operations of matrices are solving system of linear equations, elementary row operations, Gaussian Elimination, matrix addition, matrix multiplication, matrix inversion, matrix transpose, matrix trace, matrix determinant, eigenvalues and eigenvectors, transformation of the Cartesian plane and linear transformation [7].

PREFACE AND DESCRIBING SOME NEW DEFINITIONS TO CONTINUE THE METHODOLOGY

While I was studying my M-Sc. in BUET (2019 to 2021) and doing my research work on twin primes (number theory) I was simoultaneously thinking on different areas of mathematics like Matrix, Modern Algebra etc. In that time, I found this matrix constructed in my mind and find out a way to matrix order transformation based on this matrix invented by me.

PERIMETER MATRIX

I have already defined this matrix in my previous publication [1]. Once again, this is a new matrix among the other existing matrices. So, before defining it let's see the nature of it.
Look onto the following 3 square matrices of dimensions 3×3, 4×4, 5×5 respectively.

i) $$\boldsymbol{A}_{\mathbf{3}\times\mathbf{3}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

ii) $$\boldsymbol{A}_{\mathbf{4}\times\mathbf{4}} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 0 & 0 & 6 \\ 7 & 0 & 0 & 8 \\ 9 & 1 & 2 & 3 \end{bmatrix}$$

iii) $$\boldsymbol{A}_{\mathbf{5}\times\mathbf{5}} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 0 & 0 & 0 & 7 \\ 8 & 0 & 0 & 0 & 9 \\ 1 & 0 & 0 & 0 & 2 \\ 3 & 4 & 5 & 6 & 7 \end{bmatrix}$$

From the above matrices $A_{3\times3}$, $A_{4\times4}$, $A_{5\times5}$we see a common thing; that is the core entries of each matrices are 0's. The dimension $m \times m$ (where in (i) $m = 3$, in (ii) $m = 4$ and in (iii)

$m = 5$) are increasing, the matrices are getting bigger following a certain manner increasing dimension 1×1 by 1 from 3 to 5. All of the entries of the perimeter has a value 1 to 9 (which means any negative or non-negative number; if we take any entry randomly then it could be 99 or -876. For the convenience, we are taking simple numbers like $1,2,3,\ldots,9$.) from the decimal digits and the value of the entries other than the perimeter entries are 0's.

So for the above cases we can find out two types of entries. The entries which has the values from 1 to 9 and the entries which has the value 0's. If we look at those matrices we see that the values of the entries of the core (those entries other than the perimeter entries) of those matrices are 0's and the value of the perimeter entries are integers other than 0 (none of the row or column be completely 0).

We can define those entries as following.

RULE.1

Any or both value of i or j of the entries a_{ij} of a matrix $\boldsymbol{A}$ must be one of maximum or minimum values of the dimension $m \times m$ of the matrix $\boldsymbol{A}$. In any case of a matrix, the minimum value of the dimension is 1 and the maximum value is m.

RULE.2

The value of the perimeter entries could be negative number or any number (generally not 0).

RULE.3

The value of core entries must be 0.

EXAMPLE 2.1

Suppose the second matrix $\boldsymbol{A}_{\boldsymbol{3}\times\boldsymbol{3}}$, whose dimension is 3×3, and the values of i, j are from1 to 3.

$$\boldsymbol{A}_{\boldsymbol{3}\times\boldsymbol{3}} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

Here, the perimeter entries are 1,2,3,4,5,6,7 and 8.
Where,

$$a_{11} = 1$$
$$a_{12} = 2$$
$$a_{13} = 3$$
$$a_{21} = 4$$
$$a_{23} = 5$$
$$a_{31} = 6$$
$$a_{32} = 7$$
$$a_{33} = 8$$

And the core entry, $a_{22} = 0$.
So, we can define a perimeter matrix by the following way.

DEFINITION 2.1

A square matrix $\boldsymbol{A}_{\boldsymbol{m}\times\boldsymbol{m}}$ is said to be a perimeter matrix if the values of the perimeter entries are negative or nonnegative numbers $(\mathbb{Z} - 0)$ and the value of the core entries are 0's.
More specifically, A square matrix $\boldsymbol{A}_{\boldsymbol{m}\times\boldsymbol{m}}$ is said to be a perimeter matrix if the values of the entries of the matrix follow a certain manner that any or both values of i or j of the entry $\boldsymbol{a}_{\boldsymbol{ij}}$ must be one of maximum or minimum value of the dimensions $m \times m$ of the matrix $\boldsymbol{A}_{\boldsymbol{m}\times\boldsymbol{m}}$ and the core value is 0. In any case of a matrix the minimum value of the dimension is 1 and the maximum value is m.

DENOTATION 2.2

We can denote a matrix **A** as perimeter matrix as following,

$$\boldsymbol{\rho}[\boldsymbol{A}]$$

Let's see one more example.

EXAMPLE 2.2

Let the following matrix $\boldsymbol{\rho}[\boldsymbol{A}]$ as,

$$\boldsymbol{\rho}[\boldsymbol{A}] = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 15 \\ 7 & 0 & 0 & 0 & 0 & 82 \\ 9 & 0 & 0 & 0 & 0 & 10 \\ 11 & 0 & 0 & 0 & 0 & 12 \\ 13 & 0 & 0 & 0 & 0 & -14 \\ 15 & 16 & 18 & 19 & 20 & 21 \end{bmatrix}$$

We can say that above matrix $\boldsymbol{\rho}[\boldsymbol{A}]$ is a perimeter matrix as it follows the rules to be a perimeter matrix [1].

METHOD OF DETERMINANT BLOCK TRANSFORMATION

Logically, matrix order transformation is near to impossible. Here, I supposed a key or connection between two order of same matrices. Then I imagine a set of key connected to each other like a chain. Simply, my method is like you throw a rope down from the top of a well. Then riding on the rope you come down to the bottom of the well and by the same procedure you come back to the top of the well and take the rope.

In this chapter, a completely new method- the method of ***Determinant Block Transformation*** has been introduced. This method contains two phases-the method of ***Block Simplification*** to simplify a matrix of order $n \times n$ into a matrix of order 3×3 and again it's reverse method the method of ***Reverse Block Simplification*** to obtain the matrix of order $n \times n$ from the resultant matrix of order 3×3, operating the provided steps hints. For the convenience, this research work will proposed few definitions and theorems. Those definitions, theorems and methods are new to the matrix theory. So, I will name all of those, based on the matters or matrix operations used to describe all of those new findings, for the convenience and better understandings of the readers.

BASIC DEFINITIONS

In this section, I am going to define some new incidents of matrix theory. Readers may have a look on those to understand the methodologies, provided in this chapter in the later sections.

Definition 3.1.

Let a square matrix A of order n. Keep the perimeter entries i.e. entries a_{ij} where the values of i and j or at least one of i and j is 1 or n, and block the rest of all core entries. Then replace the core block matrix by a zero matrix of the same order or the dimensions of the core block matrix. Then the resultant matrix may said to be the ***perimeter matrix*** *of* A and the replaced block matrix may said to be the ***core matrix*** *of* A.

Denotation.

As all of those operations are new to matrix theory, so there are no notations for those definitions. So for the convenience of the readers and to express symbolically, I have introduced some notations. Let the notation $\boldsymbol{\rho(A)}$ or $\boldsymbol{{}_PA}$ for the ***perimeter matrix*** *of* $\boldsymbol{A}$ and $\boldsymbol{C(A)}$ or $\boldsymbol{{}_CA}$ for the ***Core matrix*** *of* $\boldsymbol{A}$.

Example 3.1.

Let, the following square matrix A of order 5 as,

$$\boldsymbol{A} = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}$$

Then if we block all of the entries of $\boldsymbol{A}$, except the perimeter entries of $\boldsymbol{A}$ and then replacing the block matrix or core matrix of $\boldsymbol{A}$ by the zero matrix of the same order or the same dimensions of the core matrix, we get two matrices as like the procedure of the following figure.

$$A = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}$$

Fig-2.1.a: Blocking the entris of A except the perimeter entries

$${}_PA = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 7 \\ 1 & 0 & 0 & 0 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}$$

Fig-2.1.b: Perimeter matrix pA of the matrix A

$${}_CA = \begin{bmatrix} 2 & 9 & 1 \\ 6 & 5 & 4 \\ 8 & 7 & 3 \end{bmatrix}$$

Fig-2.1.c: Core matrix cA of the matrix A .

Definition 3.2.

After a step of ***Block Simplification*** (defined in later section) of a matrix $A_{n\times n}$, let it transforms into the matrix $\mathrm{A}_{(\mathrm{n}-1)\times(\mathrm{n}-1)}$. Then the Core Matrix of the matrix $\mathrm{A}_{(\mathrm{n}-1)\times(\mathrm{n}-1)}$ may said to be the ***Core Value of the Matrix*** $A_{n\times n}$.

Denotation.

For the convenience of the description, let the ***Core Value of the Matrix*** $A_{n\times n}$ may be denoted by $[\![{}_{\boldsymbol{C}}\mathbf{A}]\!]$**.**

Example 3.2.

Let the matrix A as following,

$$A = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}.$$

Then ***the core matrix*** of A is

$${}_{C}A = \begin{bmatrix} 2 & 9 & 1 \\ 6 & 5 & 4 \\ 8 & 7 & 3 \end{bmatrix}.$$

And ***the core value*** of A is

$$[\![{}_{C}A]\!] = \begin{bmatrix} \begin{vmatrix} 2 & 9 \\ 6 & 5 \end{vmatrix} & \begin{vmatrix} 9 & 1 \\ 5 & 4 \end{vmatrix} \\ \begin{vmatrix} 6 & 5 \\ 8 & 7 \end{vmatrix} & \begin{vmatrix} 5 & 4 \\ 7 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} -44 & 31 \\ 2 & -13 \end{bmatrix}.$$

Definition 3.3.

The entries located at the corners of a square matrix A of order $n \times n$ may said to be the ***corner entries*** *of* A. There are 4 corners of a $n \times n$ square matrix. The entries $\boldsymbol{a_{11}}, \boldsymbol{a_{1n}}, \boldsymbol{a_{n1}}, \boldsymbol{a_{nn}}$ are row wise *the* ***1st entry of the first corner, the 1st entry of the 2nd corner, the 1st entry of the 3rd corner, the 1st entry of the 4th*** corner of a $n \times n$ matrix respectively. The *corners* may define based on the location of those entries; putting ***first, second, third*** and ***fourth*** before the ***corners*** based on their positions. As those are new findings here, so to express symbolically following the usual system of symbolize the elements of a matrix, we may express the entry or the entries of the corners of a $n \times n$ square matrix A as A_{c_i} , where $i = 1,2,3, \ldots < \frac{n}{2}$ and $c = 1, 2, 3, 4$. When we define it, we may say *the* $\boldsymbol{1st\ entries},\ \boldsymbol{the\ 2nd\ entries},\ \boldsymbol{the\ 3rd\ \ entries}$, ...

respectively for the values of $i = 1, 2, 3, \ldots$ of $\boldsymbol{the\ first\ corner,\ the\ 2nd\ corner,\ the\ 3rd\ corner\ or\ the\ 4th\ corner}$ respectively for the values of $c = 1, 2, 3\ or\ 4$.

Example 3.3.

Let the following matrix A of order 5 as,

$$\boldsymbol{A} = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}.$$

Here the ***2nd entries*** of the ***1st corner***, the ***2nd corner***, the ***3rd corner*** and the ***4th corner*** are respectively are $\boldsymbol{A_{1_2}} = (\boldsymbol{1}, \boldsymbol{6})$, $\boldsymbol{A_{2_2}} = (\boldsymbol{0}, \boldsymbol{3})$, $\boldsymbol{A_{3_2}} = (\boldsymbol{1}, \boldsymbol{4})$, and $\boldsymbol{A_{4_2}} = (\boldsymbol{6}, \boldsymbol{3})$.

THE METHOD OF BLOCK TRANSFORMATION.

As, this is a new method of the matrix theory, so, based on the implemented matrix operations I named it as the ***Method of Block Transformation***. By applying this method, we transform a square matrix of order n into a square matrix of order 3 and again can transform that square matrix of order 3 into the original square matrix of order n, which was at the beginning of this operation. So, this whole operation can be operated by dividing into ***two phases***. Based on the operation, I named the ***1st-phase*** by the ***Method of Block Simplification,*** as in first phase we transform a square matrix of order n into a square matrix of order 3. I named the ***2nd-phase*** **by the *Method of Reverse Block Simplification***, as in this method we transform the resultant matrix of order 3 into the original matrix of order n. Methodologies of these two phases are given in the following two sections. As, by applying this method over a square matrix of order n, we transform it into a square matrix of order 3 and again transform that resultant square matrix of order 3 into the original square matrix of order n, so the needed steps should be $(n-3) + (n-3) = 2.(n-3)$; where in each of the phases, $(n-3)$ steps needed, where $n > 3$.

GENERAL FORM OF THE METHOD OF BLOCK SIMPLIFICATION.

As it is a new method of the matrix theory, so I named it by the ***Method of Block Simplification*** to describe the whole method easily. Suppose a matrix of order n. We may follow few steps to make it into a matrix of order 3 (Actually the number of the steps to transform an $n \times n$ matrix into a 3×3 matrix is $(n-3)$). After each of the steps, it reduces by 1 dimension from each of the sides. i.e., an $n \times n$ order matrix will be an $(n-1) \times (n-1)$ order matrix after the first step and an $(n-2) \times (n-2)$ order matrix after the second step. Proceeding in this way finally we obtain a matrix of order 3. i.e., we have to generalize $(n-3)th$ steps for a matrix of order n, where $n > 3$. The following operations represent the general form of the Method of Block Simplification.

METHODOLOGY 3.2.1.

Let the matrix $A_{n\times n}$ of order n as following,

$$A=\begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ \vdots & \vdots & & & & \vdots \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{bmatrix}.$$

Where,$n>3$.

Step generation.

In each of the steps, the ***Method of Block Simplification*** is applied as follows.

Step-1:

Applying the ***Method of Block Simplification*** over $\boldsymbol{A}$, transforms $\boldsymbol{A}$ into $\boldsymbol{A^1}$, as following,

$$\boldsymbol{A^1}=\begin{bmatrix} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & . & . & . & \begin{vmatrix} a_{1(n-1)} & a_{1n} \\ a_{2(n-1)} & a_{2n} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & . & . & . & \begin{vmatrix} a_{2(n-1)} & a_{2n} \\ a_{3(n-1)} & a_{3n} \end{vmatrix} \\ \vdots & \vdots & & & & \vdots \\ \begin{vmatrix} a_{(n-1)1} & a_{(n-1)2} \\ a_{n1} & a_{n2} \end{vmatrix} & \begin{vmatrix} a_{(n-1)2} & a_{(n-1)3} \\ a_{n2} & a_{n3} \end{vmatrix} & . & . & . & \begin{vmatrix} a_{(n-1)(n-1)} & a_{(n-1)n} \\ a_{n(n-1)} & a_{nn} \end{vmatrix} \end{bmatrix}.$$

Where $\boldsymbol{A^1}$ is of order $(n-1)\times(n-1)$.

$$=\begin{bmatrix} b_{11} & b_{12} & . & . & . & b_{1(n-1)} \\ b_{21} & b_{22} & . & . & . & b_{2(n-1)} \\ \vdots & & & & & \vdots \\ b_{(n-1)1} & b_{n2} & . & . & . & b_{(n-1)(n-1)} \end{bmatrix}.$$

Where $b_{ij}=\begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}$, $i=1,2,\dots,n-1$ and $j=1,2,\dots,n-1$.

STEP HINTS:

The following step hints would be given to find out the original matrix $\boldsymbol{A}$ from the resultant matrix $\boldsymbol{A^1}$.

Steps. $(n-(n-1))=1$

Step-1. The perimeter matrix of the core matrix of $\boldsymbol{A}$,

$$_p(_C(\boldsymbol{A})) = \begin{bmatrix} a_{22} & a_{23} & \cdots & a_{2(n-1)} \\ a_{32} & 0 & \cdots\ 0 & a_{2(n-1)} \\ \vdots & \vdots & & \vdots \\ \vdots & 0 & \cdots\ 0 & \vdots \\ a_{(n-1)2} & a_{(n-1)3} & \cdots & a_{(n-1)(n-1)} \end{bmatrix}.$$

The 2nd entries of the 1st corner of $\boldsymbol{A}$, $\boldsymbol{A_{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}})$.

The 2nd entries of the 4th corner of $,\boldsymbol{A_{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right)$.

Step Generating Function.

The step generating function is

$$\boldsymbol{b_{ij}} = \begin{vmatrix} \boldsymbol{a_{ij}} & \boldsymbol{a_{i(j+1)}} \\ \boldsymbol{a_{(i+1)j}} & \boldsymbol{a_{(i+1)(j+1)}} \end{vmatrix}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n-1} \text{ and } \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{n-1}.$$

Example 3.4.

I am going to illustrate the Method of Block Simplification as following,

<u>Problem</u>

Let the matrix $\boldsymbol{A}$,

$$\boldsymbol{A} = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}.$$

Here the ***2nd entries*** of the ***1st corner***, the ***2nd corner***, the ***3rd corner*** and the ***4th corner*** are respectively are $\boldsymbol{A_{1_2}} = (\boldsymbol{1}, \boldsymbol{6})$, $\boldsymbol{A_{2_2}} = (\boldsymbol{0}, \boldsymbol{3})$, $\boldsymbol{A_{3_2}} = (\boldsymbol{1}, \boldsymbol{4})$, and $\boldsymbol{A_{4_2}} = (\boldsymbol{6}, \boldsymbol{3})$. Make the first step ***Block Simplification*** of $\boldsymbol{A}$**.**

<u>Solution</u>

Given that,

$$\boldsymbol{A} = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}.$$

After applying one step ***Block Simplification*** method over $\boldsymbol{A}$, $\boldsymbol{A}$ becomes into $\boldsymbol{A^1}$ as following,

$$\boldsymbol{A^1} = \begin{bmatrix} \begin{vmatrix} 5 & 1 \\ 6 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 9 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 9 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 7 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} 6 & 2 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 9 \\ 6 & 5 \end{vmatrix} & \begin{vmatrix} 9 & 1 \\ 5 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 4 & 7 \end{vmatrix} \\ \begin{vmatrix} 3 & 6 \\ 1 & 8 \end{vmatrix} & \begin{vmatrix} 6 & 5 \\ 8 & 7 \end{vmatrix} & \begin{vmatrix} 5 & 4 \\ 7 & 3 \end{vmatrix} & \begin{vmatrix} 4 & 7 \\ 3 & 6 \end{vmatrix} \\ \begin{vmatrix} 1 & 8 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 8 & 7 \\ 4 & 5 \end{vmatrix} & \begin{vmatrix} 7 & 3 \\ 5 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 6 \\ 3 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 4 & 3 & 3 & -7 \\ 30 & -44 & 31 & -5 \\ 18 & 2 & -13 & 3 \\ -12 & 12 & 6 & -15 \end{bmatrix}$$

Where $b_{ij} = \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}$, $a_{ij} \in \boldsymbol{A}$, $b_{ij} \epsilon \boldsymbol{A^1}$.

Step Hints:

Steps: 1

The perimeter matrix of the core matrix of $\boldsymbol{A}$,

$$_{\boldsymbol{p}}(_{\boldsymbol{C}}(\boldsymbol{A})) = \begin{bmatrix} 2 & 9 & 1 \\ 6 & 0 & 4 \\ 8 & 7 & 3 \end{bmatrix}.$$

The 2nd entries of the 1st corner of $\boldsymbol{A}$, $\boldsymbol{A_{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}}) = (3,30)$.

The 2nd entries of the 4th corner of $,\boldsymbol{A_{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right) = (3,6)$.

Step Generating Function.

The step generating function is

$$b_{ij} = \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}, a_{ij} \in \boldsymbol{A}, b_{ij} \epsilon \boldsymbol{A^1}.$$

GENERAL FORM OF THE METHOD OF REVERSE BLOCK SIMPLIFICATION.

In this section, after applying this method over the resultant matrix of order $(n-1)$ from the ***section 3.2.1***, we obtain the original matrix of order n originated in the ***section 3.2.1***, operating the given steps hints. As this is reverse to the applied ***Block Simplification Method*** in the ***section 3.2.1***, so we may call it the ***Method of Reverse Block Simplification***.

METHODOLOGY 3.2.2.

To operate this phase, step hints would be given. Based on the resultant matrix of order $(n-1)$ and steps hints, the methodology of the transformation of that matrix of order $(n-1)$ into the matrix of order n is operated.

Let the given resultant matrix $\boldsymbol{A^1}$ of order $(n-1)$ is as following,

$$\boldsymbol{A^1} = \begin{bmatrix} b_{11} & b_{12} & . & . & . & b_{1(n-1)} \\ b_{21} & b_{22} & . & . & . & b_{2(n-1)} \\ \cdot & & & & & \cdot \\ \vdots & & & & & \vdots \\ b_{(n-1)1} & b_{n2} & . & . & . & b_{(n-1)(n-1)} \end{bmatrix}.$$

To obtain the matrix $\boldsymbol{A}$ from the matrix $\boldsymbol{A^1}$, the following hints would be given.

Step hints:

The following step hints would be given.

Steps. $(n-(n-1)) = 1$

Step-1. The perimeter matrix of the core matrix of $\boldsymbol{A}$,

$$_{\boldsymbol{p}}(_{\boldsymbol{C}}(\boldsymbol{A})) = \begin{bmatrix} a_{22} & a_{23} & . & . & . & & a_{2(n-1)} \\ a_{32} & 0 & . & . & . & 0 & a_{2(n-1)} \\ \cdot & \vdots & & & & & \cdot \\ \vdots & \cdot & & & & & \vdots \\ & 0 & . & . & . & 0 & \\ a_{(n-1)2} & a_{(n-1)3} & . & . & . & & a_{(n-1)(n-1)} \end{bmatrix}.$$

The 2nd entries of the 1st corner of $\boldsymbol{A}$, $\boldsymbol{A_{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}})$.

The 2nd entries of the 4th corner of $,\boldsymbol{A_{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right)$.

Step Generating Function.

The step generating function is

$$\boldsymbol{b_{ij}} = \begin{vmatrix} \boldsymbol{a_{ij}} & \boldsymbol{a_{i(j+1)}} \\ \boldsymbol{a_{(i+1)j}} & \boldsymbol{a_{(i+1)(j+1)}} \end{vmatrix}, \boldsymbol{i = 1, 2, \dots, n-1} \text{ and } \boldsymbol{j = 1, 2, \dots, n-1}.$$

Step generation.

The ***Method of Reverse Block Simplification*** is applied as follows.

Step-1.

Applying the ***Method of Reverse Block Simplification*** over the matrix $\boldsymbol{A^1}$ of order $(n-1)$, we transform it into the matrix $\boldsymbol{A}$ of order n.

Given that,

$$\boldsymbol{A^1} = \begin{bmatrix} b_{11} & b_{12} & . & . & . & b_{1(n-1)} \\ b_{21} & b_{22} & . & . & . & b_{2(n-1)} \\ \cdot & & & & & \cdot \\ \vdots & & & & & \vdots \\ b_{(n-1)1} & b_{n2} & . & . & . & b_{(n-1)(n-1)} \end{bmatrix}.$$

And the 2nd entries of the 1st corner of $\boldsymbol{A}$, $\boldsymbol{A_{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}})$.

Step:1.

Step generating function,

$$b_{ij} = \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}.$$

The perimeter matrix of the core matrix of $\boldsymbol{A}$,

$$_{p}(_{c}(\boldsymbol{A})) = \begin{bmatrix} a_{22} & a_{23} & . & . & \cdot & a_{2(n-1)} \\ a_{32} & 0 & . & . & . \; 0 & a_{2(n-1)} \\ . & \vdots & & & & . \\ \vdots & \dot{0} & . & . & . \; 0 & \vdots \\ a_{(n-1)2} & a_{(n-1)3} & . & . & . & a_{(n-1)(n-1)} \end{bmatrix}.$$

We have to find out all of the values of the known entries bolded in the following matrix,

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \boldsymbol{a_{12}} & . & . & \cdot & a_{1n} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & . & . & \cdot & a_{2n} \\ \cdot & \cdot & & & & \cdot \\ \vdots & \vdots & & & & \vdots \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{bmatrix} \dots\dots\dots..(1)$$

Step Generation:

Using the step generating function we can find,

$$\boldsymbol{b_{11}} = \begin{vmatrix} a_{11} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{vmatrix} \qquad \dots\dots\dots(2)$$

Putting the values of known entries $\boldsymbol{b_{11}}$, $\boldsymbol{a_{12}}$, $\boldsymbol{a_{21}}$, $\boldsymbol{a_{22}}$ in equation (2), we will find out the value of the unknown entry a_{11}.

In this manner, applying the step generating function we will find out the values of all unknown entries of $\boldsymbol{A}$ and putting those values in (1) we will construct $\boldsymbol{A}$ from $\boldsymbol{A^1}$**.**

Example 3.5

From the last part of example 3.4 , we have:

Problem:

Steps: 1

The perimeter matrix of the core matrix of $\boldsymbol{A}$,

$$_{p}(_{c}(\boldsymbol{A})) = \begin{bmatrix} 2 & 9 & 1 \\ 6 & 0 & 4 \\ 8 & 7 & 3 \end{bmatrix}.$$

The 2nd entries of the 1st corner of $\boldsymbol{A}$, $\boldsymbol{A_{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}}) = (1{,}6)$.

The 2nd entries of the 4th corner of ,$\boldsymbol{A_{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right) = (6{,}3)$.

And

$$\boldsymbol{A^1} = \begin{bmatrix} 4 & 3 & 3 & -7 \\ 30 & -44 & 31 & -5 \\ 18 & 2 & -13 & 3 \\ -12 & 12 & 6 & -15 \end{bmatrix}$$

Find out $\boldsymbol{A}$ from $\boldsymbol{A^1}$.

Solution:

Given that,

$$\boldsymbol{A^1} = \begin{bmatrix} 4 & 3 & 3 & -7 \\ 30 & -44 & 31 & -5 \\ 18 & 2 & -13 & 3 \\ -12 & 12 & 6 & -15 \end{bmatrix},$$

where $b_{11} = 4, b_{12} = 3, \dots, b_{44} = -15$ and the perimeter matrix of the core matrix $\boldsymbol{A}$,

$$_{\boldsymbol{p}}(_{\boldsymbol{C}}(\boldsymbol{A})) = \begin{bmatrix} 2 & 9 & 1 \\ 6 & 0 & 4 \\ 8 & 7 & 3 \end{bmatrix},$$

$$\boldsymbol{A_{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}}) = (1,6).$$
$$\boldsymbol{A_{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right) = (6,3)$$

Let, the resultant matrix of order $\mathbf{5} \times \mathbf{5}$ is,

$$A = \begin{bmatrix} a_{11} & \boldsymbol{a_{12}} & a_{13} & a_{14} & a_{15} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & \boldsymbol{a_{23}} & \boldsymbol{a_{24}} & a_{25} \\ a_{31} & \boldsymbol{a_{32}} & a_{33} & \boldsymbol{a_{34}} & a_{35} \\ a_{41} & \boldsymbol{a_{42}} & \boldsymbol{a_{43}} & \boldsymbol{a_{44}} & \boldsymbol{a_{45}} \\ a_{51} & a_{52} & a_{53} & \boldsymbol{a_{54}} & a_{55} \end{bmatrix}$$

Where the values of bolded entries are known from $\boldsymbol{A_{1_2}}$ and $_{\boldsymbol{p}}(_{\boldsymbol{C}}(\boldsymbol{A}))$ and putting them in $\boldsymbol{A}$ we can find $\boldsymbol{A}$ as following,

$$A = \begin{bmatrix} a_{11} & 1 & a_{13} & a_{14} & a_{15} \\ 6 & 2 & 9 & 1 & a_{25} \\ a_{31} & 6 & a_{33} & 4 & a_{35} \\ a_{41} & 8 & 7 & 3 & 6 \\ a_{51} & a_{52} & a_{53} & 3 & a_{55} \end{bmatrix} \qquad \dots\dots\dots (1)$$

By step generating function we have,

$$\boldsymbol{b_{11}} = \begin{vmatrix} a_{11} & \boldsymbol{a_{12}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} \end{vmatrix}$$

Putting all the known values in the above matrix we have,

$$\mathbf{4} = \begin{vmatrix} a_{11} & \mathbf{1} \\ \mathbf{6} & \mathbf{2} \end{vmatrix}$$

$$=> 4 = 2a_{11} - 6 \times 1$$

$$=> a_{11} = 5$$

Again,

$$\boldsymbol{b_{12}} = \begin{vmatrix} \boldsymbol{a_{12}} & a_{13} \\ \boldsymbol{a_{22}} & \boldsymbol{a_{23}} \end{vmatrix}$$

Putting all the known values,

$$3 = \begin{vmatrix} 1 & \boldsymbol{a_{13}} \\ 2 & 9 \end{vmatrix}$$

$$=> 3 = 9 - 2\boldsymbol{a_{13}}$$

$$=> \boldsymbol{a_{13}} = 3$$

By this manner continuously applying ***Block Simplification*** over $\boldsymbol{A^1}$ for all of the (2×2) determinant entries of $\boldsymbol{A^1}$**,** we got,

$$a_{14} = 0$$
$$a_{31} = 3$$
$$a_{33} = 5$$
$$a_{35} = 7$$
$$a_{25} = 3$$
$$a_{15} = 7$$
$$a_{41} = 1$$
$$a_{55} = 1$$
$$a_{53} = 5$$
$$a_{52} = 4$$
$$a_{51} = 2$$

Putting the values of all the unknown entries of (1) into (1), we get

$$\boldsymbol{A} = \begin{bmatrix} 5 & 1 & 3 & 0 & 7 \\ 6 & 2 & 9 & 1 & 3 \\ 3 & 6 & 5 & 4 & 7 \\ 1 & 8 & 7 & 3 & 6 \\ 2 & 4 & 5 & 3 & 1 \end{bmatrix}$$

n-th TERM GENERALISATION

We can run the method n times for n, where $n > 3$.

Let the following example of square matrix, where $n = 6$.
As an example, let a matrix A of order 6.

$$A = \begin{bmatrix} 3 & 2 & 1 & 5 & 6 & 4 \\ 4 & 7 & 5 & 6 & 2 & 1 \\ 1 & 2 & 6 & 7 & 4 & 5 \\ 3 & 6 & 5 & 4 & 1 & 2 \\ 5 & 7 & 6 & 4 & 2 & 1 \\ 2 & 4 & 3 & 1 & 6 & 5 \end{bmatrix}.$$

To simplify this matrix into the matrix of order 3, we will proceed step by step. To do this, we take all the possible second order block matrix of the entries from A, where all the entries, except the first entry of each of the corner of A are repeated. The determinants of those second order block matrices are the entries of the succeeding matrix of order 5 maintaining the sequences of the second order block matrices. We follow the same operation for all the other steps.

Step generation. The total steps for this procedure are $(n-3) = (6-3) = 3$.

Step 1. Applying this method, after the first step, the matrix A transforms into the matrix A_1 as following

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 & 5 & 6 & 4 \\ 4 & 7 & 5 & 6 & 2 & 1 \\ 1 & 2 & 6 & 7 & 4 & 5 \\ 3 & 6 & 5 & 4 & 1 & 2 \\ 5 & 7 & 6 & 4 & 2 & 1 \\ 2 & 4 & 3 & 1 & 6 & 5 \end{bmatrix} \xrightarrow{\textit{Applying Method of Block Simplification}} \mathrm{A}_1 = \begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 4 & 7 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 7 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 5 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 5 & 6 \\ 6 & 2 \end{vmatrix} & \begin{vmatrix} 6 & 4 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 4 & 7 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 7 & 5 \\ 2 & 6 \end{vmatrix} & \begin{vmatrix} 5 & 6 \\ 6 & 7 \end{vmatrix} & \begin{vmatrix} 6 & 2 \\ 7 & 4 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 6 \\ 6 & 5 \end{vmatrix} & \begin{vmatrix} 6 & 7 \\ 5 & 4 \end{vmatrix} & \begin{vmatrix} 7 & 4 \\ 4 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 3 & 6 \\ 5 & 7 \end{vmatrix} & \begin{vmatrix} 6 & 5 \\ 7 & 6 \end{vmatrix} & \begin{vmatrix} 5 & 4 \\ 6 & 4 \end{vmatrix} & \begin{vmatrix} 4 & 1 \\ 4 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} 5 & 7 \\ 2 & 4 \end{vmatrix} & \begin{vmatrix} 7 & 6 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 6 & 4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 2 \\ 1 & 6 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 6 & 5 \end{vmatrix} \end{bmatrix}$$

Fig- 2.4.a: The original matrix A

Fig-2.4.b: Matrix A1 after ***Block Simplification***

Fig-2.4: Method of ***Block Simplification***

Then the matrix A^1 is

$$A^1 = \begin{bmatrix} 13 & 3 & -19 & -26 & -2 \\ 1 & 32 & -1 & 10 & 6 \\ 0 & -26 & -11 & -9 & 3 \\ -9 & 1 & -4 & 4 & -3 \\ 6 & -3 & -6 & -6 & 4 \end{bmatrix}.$$

The 2nd entries of the 1st corner of $\boldsymbol{A}$, $\boldsymbol{A}_{\boldsymbol{1}_{\boldsymbol{2}}} = (\boldsymbol{a}_{\boldsymbol{12}}, \boldsymbol{a}_{\boldsymbol{21}}) = (2,4)$.
The 2nd entries of the 4th corner of $\boldsymbol{A}$,$\boldsymbol{A}_{\boldsymbol{4}_{\boldsymbol{2}}} = \left(\boldsymbol{a}_{(\boldsymbol{n-1})\boldsymbol{n}}, \boldsymbol{a}_{\boldsymbol{n}(\boldsymbol{n-1})}\right) = (1,6)$.

$$ {}_{\boldsymbol{p}}({}_{\boldsymbol{C}}(\boldsymbol{A})) = \begin{bmatrix} 7 & 5 & 6 & 2 \\ 2 & 0 & 0 & 4 \\ 6 & 0 & 0 & 1 \\ 7 & 6 & 4 & 2 \end{bmatrix}, $$

Step 2.

Applying the same procedure over A^1, we get

$$ A_2 = \begin{bmatrix} 413 & 605 & -216 & -136 \\ -26 & -378 & 119 & 84 \\ 234 & 115 & -80 & 15 \\ 21 & -18 & -64 & 82 \end{bmatrix}. $$

The 2nd entries of the 1st corner of $\boldsymbol{A^1}$, $\boldsymbol{A^1}_{\boldsymbol{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}}) = (3,1)$.

The 2nd entries of the 4th corner of $\boldsymbol{A^1}$,$\boldsymbol{A^1}_{\boldsymbol{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right) = (= -3, -6)$.

$$ {}_{\boldsymbol{p}}({}_{\boldsymbol{C}}(\boldsymbol{A^1})) = \begin{bmatrix} 32 & -1 & 10 \\ -26 & 0 & -9 \\ 1 & -4 & 4 \end{bmatrix} $$

Step 3.

Applying the same procedure processed in the *step 1*, A_2 transforms into A_3 as following

$$ A^3 = \begin{bmatrix} -140384 & -\ 9653 & -1960 \\ 85462 & 16555 & 8505 \\ -\ \ 6627 & 8800 & -5600 \end{bmatrix}. $$

Here, A^3 is the resultant matrix of order 3.

The 2nd entries of the 1st corner of $\boldsymbol{A^2}$, $\boldsymbol{A^2}_{\boldsymbol{1_2}} = (\boldsymbol{a_{12}}, \boldsymbol{a_{21}}) = (3,1)$.

The 2nd entries of the 4th corner of $\boldsymbol{A^2}$,$\boldsymbol{A^2}_{\boldsymbol{4_2}} = \left(\boldsymbol{a_{(n-1)n}}, \boldsymbol{a_{n(n-1)}}\right) = (= 15, -64)$

The perimeter matrix of the core matrix of $\boldsymbol{A^2}$ ${}_{\boldsymbol{p}}({}_{\boldsymbol{C}}(\boldsymbol{A^2})) = \begin{bmatrix} -378 & 119 \\ 115 & -80 \end{bmatrix}$

So the question maybe like this,

- **Let the following example of square matrix A,of order 6×6**

$$ \boldsymbol{A} = \begin{bmatrix} \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{5} & \mathbf{6} & \mathbf{4} \\ \mathbf{4} & \mathbf{7} & \mathbf{5} & \mathbf{6} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{6} & \mathbf{7} & \mathbf{4} & \mathbf{5} \\ \mathbf{3} & \mathbf{6} & \mathbf{5} & \mathbf{4} & \mathbf{1} & \mathbf{2} \\ \mathbf{5} & \mathbf{7} & \mathbf{6} & \mathbf{4} & \mathbf{2} & \mathbf{1} \\ \mathbf{2} & \mathbf{4} & \mathbf{3} & \mathbf{1} & \mathbf{6} & \mathbf{5} \end{bmatrix}. $$

Then keep applying the method of block simplification over A, as long as we get a matrix of order 3×3 equivalent to the original matrix with given key values

THE METHOD OF REVERSE BLOCK SIMPLIFICATION TO TRANSFORM THE RESULTANT MATRIX OF ORDER 3 INTO THE ORIGINAL MATRIX OF ORDER 6.

In this section, after applying this method over the resultant matrix of order 3 from the section 2.8, after the $(n-3)=(6-3)=3$ steps later, we obtain the original matrix of order 6, originated in the section 2.4.1. The resultant matrix of order 3 from the section 3.4.1 is

$$A_3 = \begin{bmatrix} -140384 & -9653 & -1960 \\ 85462 & 16555 & 8505 \\ -6627 & 8800 & -5600 \end{bmatrix}.$$

Steps hints. To obtain the original matrix from the matrix of order 3×3, the following step hints would be given.

Steps: $(n-3)=(6-3)=3.$

Step 1: The perimeter entries of the core matrix of A_2,

$$_p(_k(A_2)) = \begin{bmatrix} -378 & 119 \\ 115 & -80 \end{bmatrix}.$$

The 2nd entries of the 1st corner of A_2, $A_{2_{\mathbf{1_2}}} = (605, -26)$.
The 2nd entries of the 4th corner of A_2, $A_{2_{\mathbf{4_2}}} = (15, -64)$.

Step 2: The perimeter entries of the core matrix of A_1,

$$_p(_k(A_1)) = \begin{bmatrix} 32 & -1 & 10 \\ -26 & & -9 \\ 1 & -4 & 4 \end{bmatrix}.$$

The 2nd entries the of 1st corner of A_1, $(A_1)_{1_2} = (3, 1)$.
The 2nd entries of the 4th corner of A_1, $(A_1)_{4_2} = (-3, 6)$.

Step 3: The perimeter entries of the core matrix of A,

$$_p(_k(A)) = \begin{bmatrix} 7 & 5 & 6 & 2 \\ 2 & & & 4 \\ 6 & & & 1 \\ 7 & 6 & 4 & 2 \end{bmatrix}.$$

The 2nd entries of the 1st corner of A, $(A)_{1_2} = (3, 1)$.

The 2nd entries of 4th corner of A, $(A)_{4_2} = (-3, 6)$.

Step generation. By the following 3 steps we will obtain the required original matrix of order 6.

Step 1. By operating this step, the matrix A_3 would transform into the matrix A_2. Given the resultant square matrix A_3 of order 3×3

$$A_3 = \begin{bmatrix} -140384 & -9653 & -1960 \\ 85462 & 16555 & 8505 \\ -6627 & 8800 & -5600 \end{bmatrix}.$$

The perimeter entries of the core matrix of A_2,

$$_p(_k(A_2)) = \begin{bmatrix} -378 & 119 \\ 115 & -80 \end{bmatrix}.$$

The 2nd entries of 1st corner of A_2, $A_{2_{1_2}} = (605, -26)$.
The 2nd entries of 4th corner of A_2, $A_{2_{4_2}} = (15, -64)$.

Let the intended matrix A_2 of order 4 as,

$$A_2 = \begin{bmatrix} a_{11} & 605 & a_{13} & a_{14} \\ -26 & -378 & 119 & a_{24} \\ a_{31} & 115 & -80 & 15 \\ a_{41} & a_{42} & -64 & a_{44} \end{bmatrix}.$$

Apply the formula $b_{ij} = \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}$ to find out any of the right hand side entries, where the other 3 of the right hand side entries and b_{ij} would always be given if we maintain the sequence of this operation of the Method of Reverse Block Simplification to find out the unknown entries of the intended matrix A_2.

Then,

$$b_{11} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \Rightarrow a_{11} = \frac{b_{11} + a_{12}a_{21}}{a_{22}} = \frac{-140384 + 605.(-26)}{-378} = 413.$$

$$b_{33} = \begin{vmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{vmatrix} \Rightarrow a_{44} = \frac{b_{33} + a_{34}a_{43}}{a_{33}} = \frac{-5600 + (15 \times (-64))}{-80} = 82.$$

$$b_{12} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \Rightarrow a_{13} = \frac{a_{12}a_{23} - b_{12}}{a_{22}} = \frac{605\,.119 - (-9653)}{-378} = -216.$$

$$b_{21} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \Rightarrow a_{31} = \frac{a_{21}a_{32} - b_{21}}{a_{22}} = \frac{(-26)\,.115 - 85462}{-378} = 234.$$

$$b_{23} = \begin{vmatrix} a_{23} & a_{24} \\ a_{33} & a_{34} \end{vmatrix} \Rightarrow a_{24} = \frac{\mathrm{a}_{23}\mathrm{a}_{34} - \mathrm{b}_{23}}{\mathrm{a}_{33}} = \frac{119\,.15 - 8505}{-80} = 84.$$

$$b_{32} = \begin{vmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{vmatrix} \Rightarrow a_{42} = \frac{\mathrm{a}_{32}\mathrm{a}_{43} - \mathrm{b}_{32}}{\mathrm{a}_{33}} = \frac{(-64\,.115) - (-8800)}{-80} = -18.$$

$$b_{13} = \begin{vmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{vmatrix} \Rightarrow a_{14} = \frac{\mathrm{a}_{13}\mathrm{a}_{24} - \mathrm{b}_{13}}{\mathrm{a}_{23}} = \frac{(-216)\,.84 - (-1960)}{119} = -136.$$

$$b_{31} = \begin{vmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{vmatrix} \Rightarrow a_{41} = \frac{\mathrm{a}_{31}\mathrm{a}_{42} - \mathrm{b}_{31}}{\mathrm{a}_{32}} = \frac{(-216)\,.84 - (-6627)}{115} = -136.$$

Putting all of those values, the intended matrix A_2 would be

$$A_2 = \begin{bmatrix} 413 & 605 & -216 & -136 \\ -26 & -378 & 119 & 84 \\ 234 & 115 & -80 & 15 \\ 21 & -18 & -64 & 82 \end{bmatrix}.$$

Step 2. Applying the same procedure of the *Step 1* over A_2, transform A_2 into A_1.

$$A_1 = \begin{bmatrix} 13 & 3 & -19 & -26 & -2 \\ 1 & 32 & -1 & 10 & 6 \\ 0 & -26 & -11 & -9 & 3 \\ -9 & 1 & -4 & 4 & -3 \\ 6 & -3 & -6 & 22 & 4 \end{bmatrix}.$$

Step 3. Applying the same procedure of the *Step 1*, transform A_1 into A as following,

$$A = \begin{bmatrix} 3 & 2 & 1 & 5 & 6 & 4 \\ 4 & 7 & 5 & 6 & 2 & 1 \\ 1 & 2 & 6 & 7 & 4 & 5 \\ 3 & 6 & 5 & 4 & 1 & 2 \\ 5 & 7 & 6 & 4 & 2 & 1 \\ 2 & 4 & 3 & 1 & 6 & 5 \end{bmatrix}.$$

Then this matrix A is the required original matrix A.

Now, based on above calculation, we can construct the following question:

- Given that, $A^3 = \begin{bmatrix} -140384 & -9653 & -1960 \\ 85462 & 16555 & 8505 \\ -6627 & 8800 & -5600 \end{bmatrix}$, ${}_p(_C(\boldsymbol{A^2}))\begin{bmatrix} -378 & 119 \\ 115 & -80 \end{bmatrix}$,

$$\boldsymbol{A^2}_{\boldsymbol{1_2}} = (605,-26), \boldsymbol{A^2}_{\boldsymbol{4_2}} = (15,-64), {}_{\boldsymbol{p}}({}_{\boldsymbol{c}}(\boldsymbol{A^1})) = \begin{bmatrix} 32 & -1 & 10 \\ -26 & 0 & -9 \\ 1 & -4 & 4 \end{bmatrix}, \boldsymbol{A^1}_{\boldsymbol{1_2}} = (3,1), \boldsymbol{A^1}_{\boldsymbol{4_2}} = (-3,-6);\ {}_{\boldsymbol{p}}({}_{\boldsymbol{c}}(\boldsymbol{A})) = \begin{bmatrix} 7 & 5 & 6 & 2 \\ 2 & 0 & 0 & 4 \\ 6 & 0 & 0 & 1 \\ 7 & 6 & 4 & 2 \end{bmatrix}, \boldsymbol{A}_{\boldsymbol{1_2}} = (2,4) \boldsymbol{A}_{\boldsymbol{4_2}} = (1,6).$$

Then find out the original matrix A of order 6×6.,

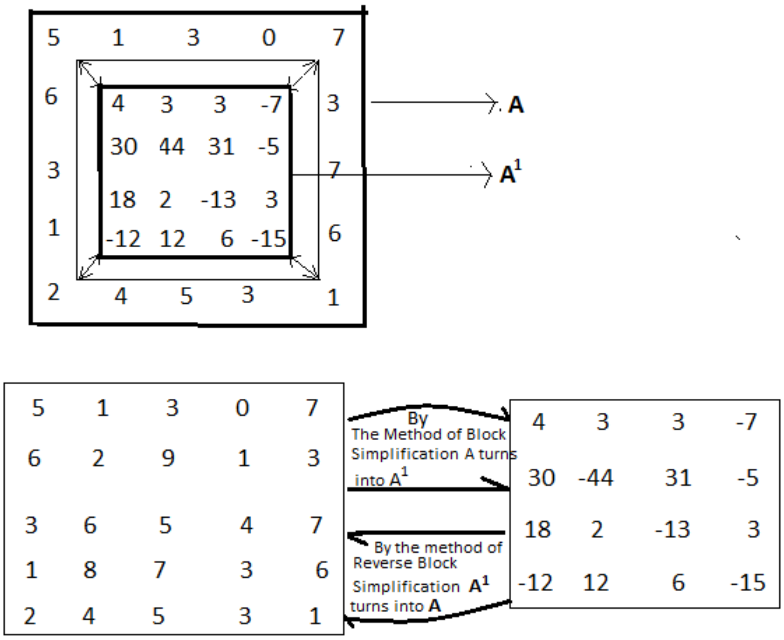
Theorem.1.
A square matrix of order $n \times n$ is equal or same of matrix A^1 of order $(n-1)(n-1)$ if it satisfies the method of determinant block transformation $b_{ij} = \begin{vmatrix} a_{ij} & a_{i(j+1)} \\ a_{(i+1)j} & a_{(i+1)(j+1)} \end{vmatrix}$ for all $a_{ij} \in A$ and $b_{ij} \in A^1$ and $i, j \in n$. i.e.,

$$A = A^1$$

Or,

$$A_{n \times n} = A_{(n-1)\times(n-1)}$$

For the given determinant function with $\rho(c(A))$,the first corner's second entries and fourth corners second entries.



THE METHOD OF DETERMINANT BLOCK TRANSFORMATION

CONCLUSION

Solving the system of linear equations using arrays or matrices is one of the oldest methods of mathematics, which is advancing day by day with new methods and applications of it. In shortly it can be said that the existence of this modern world based on computer and electronics would not be possible without the applications of matrices and cryptography. This research work has proposed the *Method of Block Transformation* to convert a matrix of order n into a matrix of order 3 and regain the original matrix of order n.

FUTURE WORK

While I was working on Perimeter Matrix and Block Simplification method, I discovered a new types of matrix cipher using both of those idea. I will later publish the method of matrix cryptography based on the idea of perimeter matrix and block simplification.

REFERENCES

[1] S. M. Naser, *Introducing a Matrix and its Characteristics*, American Scientific Research Journal for Engineering, Technology, and Science (ASRJETS). **77** (2021), no. 1, 90-94.
[2] S. M. Naser, *Cryptography: From the Ancient History to Now, It's Applications and a New Complete Numerical Model*, International Journal of Mathematics and Statistics Studies. ECRTD-UK. **9** (2021), no. 3, 11-30.
[3] M. Flagg, *solving a System of Linear Equations Using Ancients Chinese Methods,* Linear algebra, Digital common @ Ursinus, 2017.
[4] K. Chemla, *Different concepts of equations in the Nine Chapters on Mathematical Procedures and in the commentary on it by Liu Hui (3rd century),* Historica Scientarum. **4** (1994), no. 2, 113–137.
[5] A. Akhtyamov, M. Amram, M. Dagan and A. Mouftahkov, *Cramer's Rule for Nonsingular $m \times n$ Matrices*, The Teaching of Mathematics. **xx** (2017), no. 1, 13-19.
[6] B. Olayiwola, *Short history on Determinant of a Matrix*, Abacus (Mathematics Science Series). **44** (2019), no. 1, 538-542.
[7] O. Knill, *Cauchy-Binet for Pseudo Determinants*, Linear Algebra and its Applications. ScienceDirect. **459** (2014), 522-547, DOI 10.1016/j.laa.2014.07.013.
[8] Thawte, *History of Cryptography: An Easy-to-Understand History of Cryptography*, Thawte, Inc., 3013.
[9] D. Pandya, K. R. Narayan, S. Thakkar, T. Madhekar and B. S. Thakare, *Brief History of Encryption*, International Journal of Computer Applications. **131** (2015), no. 9, 28-31.
[10] S. Prabhavathi and T. Chandrapushpam, *Role of Matrices in Cryptography*, International Journal of Mathematics and It's Applications. **6** (2018), no. 1, 25-28.
[11] A. P. U. Siahaan, *Rail Fence Cryptography in Securing Information*, International Journal of Scientific & Engineering Research, **7** (2016), issue 7, 535-538.
[12] J. C. T. Arroyo & A. J. P. Dellma, *A modified Polybius Cipher with a New Element-in-grid Sequencer*, International Journal of Advanced Trends in Computer Science and Engineering. **9** (2020), no. 3, 3249-3255.
[13] J. C. T. Arroyo, C. E. Dumdumaya & A. J. P. Dellma, *Polybius square in Cryptography: A Brief Review of Literature*, International Journal of Advanced Trends in Computer Science and Engineering. **9** (2020), no. 3, 3798-3808.
[14] N. Vijayaraghavan, S. Narasimhan & M. Baskar, *A Study on the Analysis of Hill's Cipher in Cryptography*, International Journal of Mathematics Trends & Technology (IJMTT). **54** (2018), no. 7, 519-522.

[15] M. D. L. Siahaan & A. P. U. Siahaan, *Application of Hill Cipher Algorithm in Securing Text Messages,* International Journal for Innovative Research in Multidisciplinary Field, **4 (**2018), Issue 10, 55-59.