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A Quantization for the Toy Model of Black Hole and a Suggestion to the Unification Theory

(Carson) Kai Shun, Lam
British National Oversea
Fellow of Scholar Academic Scientific Society, India
Dignitary Fellow, International Organization for Academic & Scientific Development, India
Email: carson1114@hkbn.net

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Abstract: *In my previous paper about the correlations about the quantum mechanics between four different natural forces, I have suggested that there may be a fifth force or even the existence of a new particle. However, it may be still a mysterious that physicists for nearly hundreds of years that they cannot unify these four natural forces. This may be because there are indeed too many variables for them considerate. Also there is a twin such that the quantum machanics may coexist with the quantum field theory. In the present paper, this author proposes that there may be a misconcept in the computational equation for our relative time or the gaps in the system of measure time between quantum mechanics and general relativity. Hence, one may still cannot unify these natural forces. This author suggests that we need to rewrite parts of our QM – the Schrödinger Equation or even the GR equation. Besides, this author also proposes there may be a bridge converting equation for the quantum mechanics and the general relativity. Moreover, this author has employed the non-trivial zeta zeros to stimulate the black hole or the so-called black hole toy model. In practice, there may be an electromagnetic field laying around the boundary of the black hole as well as the existing of a continumm in the such boundary contour. This author hopes that in such case, we may decode back those high frequency electromagnetic waves into the useful information and may go a further step to have a verification for the famous Stephen Hawking's theory between the black hole radition and the information entropy. In fact, this author has used the HKLam statistical model theory to express the electromagnetic field energy-stress tensor (with the possibility of quantization) to analogically establish a quantized model for the Einstein Gravitational Field Equation. Hence, the problem of the quantum gravity may then be solved. Last but not least, this author may also expect that we – human may finally find a way to unify the quantum mechanics and the general relativity through the modification to the present quantum gravity theory from my suggested bridge converting equation etc.*
KEYWORDS: quantization, Toy model, black hole, unification theory

INTRODUCTION

There are lots of discussion about the Hawking radiation and the information paradox. However, none of the researchers may completely resolve the mystery. This author have tried to handle the problem in another way – the black hole toy model, the Riemann Zeta non trivial zeros layers and the electromagnetic field around the boundary or the continuum etc for the information restoration. This author hopes that the present paper may act as a pioneer research for those who are interested in the question about the information paradox as well as the related affairs.

LITERATURE REVIEW

Quantum Mechanics – The Schrödinger Equation

When one is talking about the Schrödinger Equation or the quantum mechanics, one may usually refer to the wave properties of a particle or the so-called wave equation [39],p.21-25 & [40], p.22. This author will try to derive the equation from the wave natural of a particle. Let's first consider a quantum-mechanical particle particle of energy E and momentum p and its corresponding wave frequency is:

$$v = \frac{E}{h}$$

and the wavelength is: $\lambda = \frac{h}{p}$. By introducing the angular frequency ω and so as the wave number *k,* then we may have: $\omega \equiv 2\pi v = \frac{E}{\hbar}$ where $\hbar = \frac{h}{2\pi}$

As E = $\frac{p^2}{2m}$, we may have $\omega = \frac{p^2}{2m\hbar}$.

Let us assume a particle of momentum p traveling in the positive x direction, then we will have to use a wave traveling in that positive x direction. Hence, the corresponding harmonic waves are:

$$e^{Ikx-I\omega t}, \quad e^{-Ikx+I\omega t}, \quad \sin(kx - \omega t) \text{ and } \cos(kx - \omega t).$$

For the particle which moves in the positive x direction, we may have:

$$\sin(kx - \omega t)$$

For the particle which moves in the negative -x direction, we may have:

$$\sin(kx + \omega t)$$

Thus, by the principle of superposition, the resulting wave is just the sum of the two waves or: sin(kx – ωt) + sin(kx + ωt).

Practically, the particle has equal probabilities for motion in both of the +x direction and also the negative -x direction, the resulting sine function is:

$2 \sin kx \cos \omega t$ which has a zero at t = $\frac{\pi}{2\omega}$ and hence is NOT acceptable.

However, if we consider the wave function $e^{Ikx-I\omega t}$, then the superposition will be:

$$e^{Ikx-I\omega t} + e^{-Ikx-I\omega t} = 2\, e^{-I\omega t} \cos kx$$

which is non-zero everywhere and thus will be accepted.

Assuming $e^{Ikx-I\omega t}$ to be the standard harmonic wave function describing a free particle with a given momentum, we may be interested in the wave equation that satisfied by the wavefunction. Then we differentiate the above wavefunction with respect to position x and time t. Next, we may find that the first derivate of $e^{Ikx-I\omega t}$ w.r.t. time is proportional to the second derivative of it w.r.t position are indeed proportional, hence we may get:

$$k^2 e^{Ikx-I\omega t} = -\omega e^{Ikx-I\omega t}$$

By considering the total energy (potentia energy + kinetic energy) of the particle with mass m and momentum p, we may have:

$$\frac{p^2}{2m} + v(x,t) = E$$

Therefore by using the Plank-Einstein relation, $p = \hbar k$ and multiplying the above equation with the wave function, we may get:

$$\frac{-\hbar^2}{2m} k^2 e^{Ikx-I\omega t} + v(x,t)\, e^{Ikx-I\omega t} = \hbar\omega e^{Ikx-I\omega t}$$

(N.B. E = $\hbar\omega$ and $p = \hbar K$)

But as $k^2 \rightarrow \frac{\partial^2}{\partial x^2}$ and $\omega \rightarrow I\frac{\partial}{\partial t}$, we may thus get:

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}e^{Ikx-I\omega t} + v(x,t)\, e^{Ikx-I\omega t} = \hbar I\frac{\partial}{\partial t}e^{Ikx-I\omega t}.$$

In reality, we may further generalize the above equation [38] to:

$$\frac{-\hbar^2}{2m}\nabla^2 e^{Ikx-I\omega t} + v(x,t)\, e^{Ikx-I\omega t} = \hbar I\frac{\partial}{\partial t}e^{Ikx-I\omega t}$$

This is what the famous **Schrödinger Equation**.

Einstein General Relativity Theory Equation

Next we shall proceed to the derivation of the Einstein's General Relativity Theory Equation or the field equation [41]. First let us have a brief review about the basics of the tensor operator [42]. In fact, the tensor may be viewed as an extension of the matrix or can use more indices, upper or lower. To cite a case, consider the 4x4x4x4x4 tensor:

$$T \equiv \left(T^{\alpha\beta}_{\gamma\delta\varepsilon}\right)_{0\leq\alpha,\beta,\gamma,\delta,\varepsilon\leq 3}$$

Practically, we may consider the spacetime as a four-dimensional manifold. That say, t, x, y, and z. Or for the sake of convenient, we may denote these dimensional coordinates by: $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$. If we pick a four-dimensional row vector that contains four members in a row: $v = (v^0, v^1, v^2, v^3)$. Then the upper index will be denoted by a small Greek letter: $v \equiv (v^\alpha)_{0\leq\alpha\leq 3}$. In fact, we may consider the upper index of the tensor as the number of dimension and the lower index as the number of rank to locate the wanted information or data [43].

If we differentiate the position 4-vector v, then we may get:

$$\frac{\partial}{\partial v} \equiv \frac{\partial}{\partial (v^\alpha)_{0\leq\alpha\leq 3}} = \begin{matrix}\frac{\partial}{\partial x^0}\\ \frac{\partial}{\partial x^1}\\ \frac{\partial}{\partial x^2}\\ \frac{\partial}{\partial x^3}\end{matrix} = \begin{matrix}\frac{\partial}{\partial t}\\ \frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{matrix}$$

. Or we may propose a symbol operator such as "$[\nabla]$" which is similar to the Laplace operator $\nabla = \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ for the present 4-dimension positional vector as prescribed above. i.e. $\frac{\partial}{\partial v} \equiv \frac{\partial}{\partial (v^\alpha)_{0\leq\alpha\leq 3}} = [\nabla](v)$ or $[\nabla](v^\alpha)_{0\leq\alpha\leq 3}$ etc.

To start for the derivation of the General Relativity Field Equation, we may first introduce the Guass's law for gravity or the Guass's flux theorem for gravity [47]. It mentions that the (gravitational) flux (similar to the magnetic or electric one) caused by the gravitational field over a closed surface is proportional to the mass enclosed. Practically, by considering a mass such as the rocket or spacecraft that follows into the gravitational field of a heavier mass such as a star in the curved spacetime of the universe, then in terms of the differential form, we may have:

$$\nabla \cdot g = -4\pi G\rho$$

(N.B. The Guass's law has another form by integration, this author will NOT repeat as the difference may only lie in the format of expression but the concepts are similar.)

But as the gravitational field has a zero curl which means that our gravity is a conservative force, we may rewrite the gravity as a scalar potential or the so-called gravitational potential Φ:

$$g = -\nabla\Phi.$$

When substituting into the Guassian flux equation, we may get [48]:

$$\nabla^2\Phi = 4\pi G\rho$$

where $\Phi = \frac{U}{m} = -\frac{GM}{r}$ and U(r) is the potential energy due to the gravitation field [47].
In addition, we may observe that 4πGρ is highly related to the energy-momentum tensor ($T^{\mu\nu}$) as it is in general describing all energy and matter. Furthermore, $\nabla^2\Phi$ may be used to determine the curvature of the space-time and so as its relationship with the gravity or the energy.
This author wants to note an interesting fact that as the matter is always described by the energy-momentum tensor T_{ab} which is an energy conservative one and hence must satisfy the continuity equation as the case in fluid dynamics etc. In fact, the conservation may be expressed by equation $\partial_a j^a = 0$ where j^a is the current. In the case of energy monentum tensor, the continuity equation is:

$$\nabla^a T_{ab} = 0.$$

Initially, Einstein guesses the relationship between the geometry and the matter is through the Ricci Tensor R_{ab}. Thus, the equation is:

$R_{ab} = kT_{ab}$ where k is some constant but there is no gravitating matter in the case of a vaccum and therefore T_{ab} vanishes. The equation reduces to:

$$R_{ab} = 0.$$

On the other hand, in the presence of matter, the Ricci tensor must satify the contracted Bianchi identities or:

$$\nabla^a R_{ab} = \frac{1}{2}\nabla_b R,$$

Then the problem may arise.
By applying ∇^a to both sides of the $R_{ab} = kT_{ab}$, we may get:

$$\nabla_a R = 0.$$

In order to restore the conversation of energy, the scalar curvature must be a constant everywhere. But imagine a very massive star, its neighbour curvature will be very large but as the distance goes far away from the star, the curvature may decline to a zero. This may then create a contradiction to the fact that the scalar curvature must be a constant. In practice, by rewriting the equation in its equaivalent form, we have:

$\nabla^a\left(R_{ab} - \frac{1}{2}g_{ab}R\right) = 0$ which satifies the continuity equation $\nabla^a T_{ab}$.

In other words, $\left(R_{ab} - \frac{1}{2}g_{ab}R\right) = kT_{ab}$ which will give a consistent result, hence we may define the Einstein Tensor by:

$$G_{ab} = \left(R_{ab} - \frac{1}{2}g_{ab}R\right).$$

For the different kinds of the metric tensor, it is used to capturee all of the geometric and causal structure of the space and time as the case time, distance, volume, curvature, angle etc. In general, we may have three types of metric tensor named as Minkowski metric for the flat space-time, the spherical coordinates for the flat spacetime as well as the black hole Schwarzschild metric. This author will focus in the black hole metric as follow:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right)c^2dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1}dr^2 + r^2 d\Omega^2,$$

The Black hole metric can thus be written as:

$$\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix}.$$

In reality, the Einstein constant is given by:

$$k = \frac{8\pi G}{c^4}.$$

Finally, we conclude that the Einstein Field Equation is therefore:

$$R_{(\mu\nu)} - \frac{1}{2} g_{(\mu\nu)} R + \Lambda g_{(\mu\nu)} = \mathrm{k} T_{(\mu\nu)}$$

Field Theory – Classical Vs Quantum

In the classical field theory like the general relativity, Einstein treats the curved space (-time) as an infinite number of coupled harmonic oscillators or just a mass spring. To go a step, it is just a case of two or three masses connected by some springs or the “infinite box spring”. That say, the curved space is filled with a dense grid with every node in the grid is a mass and every mass is connected by some springs to its nearest neighbors. Such kinds of the “infinite box spring” is then being used by the Physicists to describe those wavelike formats which will propagate through the space when one of the point masses was “pinged” by an external object. This situation may lead to the question of deterministic for some known initial condition of the field, the evolution of the field will be actually based on the configuration of the field at a given time.

On the other hand, for the quantum field theory, one may also consider the aforementioned “infinite box spring” analogy. However, the only different is the random motion of the infinite box spring will be driven by the quantum mechanics. That says, a quantum harmonic oscillator is really exhibiting a random zero-point motion which results a discrete spectrum of excitations above this random ground state. To go a step, one may formulate a “quantum (random-ness) boxspring mattress” to depict the picture of a such kind quantum field. In practice, each of the discrete spectrum of excitations for one quantum harmonic oscillator may thus be interpreted as some particles in the quantum field theory. In such a context, the implication is the quantum field will then achieve certain configurations based under the probability amplitudes or Prob P = Amp $|A|^2$.

Obviously, for the classical field theory such as the case in general relativity, it does NOT use quantum mechanics. Or else when the formulation of the general relativity equation, it does NOT include any concepts of quantization or even the quantum mechanics. Basically, the classical field theory and the quantum field theory are two types of box spring system – infinitely (classical continuum for a large scale matter) Vs quantum (random-ness), (quantum continuum for a small scale matter). Hence, it is very difficult to get the formula for the quantum gravity or “quantize the gravity” as one may need to merge the small scale things into the large scale one. My suggestion is to reformulate/re-establish either the quantum mechanics or the general relativity or both kinds of the equations. Thus, this author proposes an hybrid idea that one may begin to sub-divide the large curved space-time into a very small scale quantized one or just like the so-called “Finite-element method” in the engineering field or this author’s paper named “A Rationalized Visit to Holy Land — Israel ” [60]. That say, a mirror image of mirror image or an infinite number of large harmonic springs such that each of the individual spring contains some quantized small random-ness springs. These small springs will oscillate discretely, random-nessly, quantizely, spectrally and harmonically in each individual section of the large spring etc. In the mirror image way, the collection of these small quantized springs in the quantum field can make up the large individual spring in the classical field.

In brief, this author suggests that the relationship between the quantum field theory and the quantum mechanics are just like the coin’s two sides. To be precise, they are only the business primal-dual way of the simplex method for describing the microscope sturctures. Hence, anologically, this author proposes that the string theory and the loop quantum gravity may also be the business primal-dual way of depicting the macro-universe phemonema.

Major Mathematical and Computational Results

From [1], we may have:

$$\frac{1}{\xi\prime(x)} = \sum_{r\in\mathbb{Z}[1/2],\geq 2} b_r\, r^{-s}$$ which is a generalized Dirichlet series.

Then one may get:

$$\int \frac{1}{\xi\prime(x)}dx = \int \sum_{r\in\mathbb{Z}[1/2],\geq 2} b_r\, r^{-s}$$

Or $$\int \frac{1}{\xi\prime(x)}dx = \int \frac{1}{\xi(x)} \frac{1}{\frac{\xi\prime(x)}{\xi(x)}}dx,$$

i.e. $$\int \frac{1}{\xi\prime(x)}dx = \int \frac{1}{\xi(x)} \frac{1}{\partial(ln\xi(x))}$$

$$= \frac{1}{\xi(x)}\frac{1}{(ln\xi(x))} - \int \frac{1}{-ln(\xi(x))[\xi(x)]^2}\xi'(x)dx$$

$$= \frac{1}{\xi(x)}\frac{1}{(ln\xi(x))} + \int \frac{1}{ln(\xi(x))[\xi(x)]^2}\xi'(x)dx$$

$$= \frac{1}{\xi(x)}\frac{1}{(ln\xi(x))} + \int \frac{d(\xi(x))}{ln(\xi(x))(\xi^2(x))}$$

$$= 2[\frac{1}{\xi(x)}\frac{1}{(ln\xi(x))}] + \int \frac{(\xi(x))}{\partial[ln(\xi(x))(\xi^2(x))]}$$

$$= \ldots = n\,[\frac{1}{\xi(x)}\frac{1}{(ln\xi(x))}]$$

I.e. $\int \frac{1}{\xi\prime(x)}dx = n[\frac{1}{\xi(x)}\frac{1}{(ln\xi(x))}]$ --------- (1)

Case I: Use a Linear Equation to approximate $\int \frac{1}{\xi\prime(x)}dx$

$\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]\,[(ln(\xi(x)) + 1)]$

or $\int \frac{1}{\xi\prime(x)}dx = \int \sum_{r\in\mathbb{Z}[1/2],\geq 2} b_r\, r^{-s}$

$$= \int\int b_r\, r^{-s}$$

$$\int \frac{1}{\xi\prime(x)}dx = \frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]\,[(ln(\xi(x)) + 1)] = \int\int b_r\, r^{-s}J(x)\partial r\partial s$$

where J(x) is the Jacobian matrix of the transformation from $\partial x\partial y$ normal planed coordinate to $\partial r\partial s$, or the curved (spherical) plane coordinate [4].

$\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]\,[(ln(\xi(x)) + 1)] = \int\int b_r\, r^{-s}J(x)\partial r\partial s$ -------------(*)

i.e. $\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]\,[(ln(\xi(x)) + 1)] - \int\int b_r\, r^{-s}J(x)\partial r\partial s = 0$

which is a linear equation in terms of $ln(\xi(x))$.

Indeed, the root of the left hand side in the equation(*) is: $ln(\xi(x)) = -1$ or $\frac{1}{(ln\xi(x))} = 0$ or $\frac{1}{\xi(x)} = 0$,

i.e. Sub-case I: when $\xi(x) = e^{-1}$ or $1/e$,

Sub-case II: when $(ln\xi(x)) = -\infty$ or $\xi(x) = 0$, x = trivial zeros (negative even integers) or non trivial zeros

Sub-case III: when $\xi(x) = \infty$ or $x = 1$.

Moreover, ξ(x) = e^{-1} is also one of the optimum value of the equation [$\int\int\int b_r r^{-s} J(x)\partial r\partial s$]'s outcome polynomial. ξ(x) = 0 when $\frac{1}{(ln\xi(x))}$ = 0, then x equals to the negative even integers or refer to those trivial zeros of the zeta function. Certainly, there may be those non-trivial zeros for the zeta function [5]. Or
ln ξ(x) = $-\infty$ when ξ(x) = 0. That says, we may approximate the singularity of the black hole toy model as the case x = 1 when $\frac{1}{\xi(x)}$ = 0 or ξ(x) = ∞ by the linear equation $\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]$ [$(ln(\xi(x))$ + 1)]. To be precise, the above result may imply the asymptotic safety in quantum gravity or also the non-trivial fixed point [22] etc.

Case II: Use a Quadratic Equation to Approximate $\int \frac{1}{\xi'(x)}dx$

$$\int \frac{1}{\xi'(x)}dx = \frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)]$$

$$\text{or } \int \frac{1}{\xi'(x)}dx = \int \sum_{r\in\mathbb{Z}[1/2],\geq 2} b_r r^{-s}$$

$$= \int\int b_r r^{-s} J(x)\partial r\partial s$$

$$\int \frac{1}{\xi'(x)}dx = \frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)] = \int\int b_r r^{-s} J(x)\partial r\partial s$$

$$\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)] = \int\int b_r r^{-s} J(x)\partial r\partial s$$

i.e. $-\frac{1}{\xi(x)}[\frac{1}{ln(\xi(x))}]^2 + \frac{1}{\xi(x)}\frac{1}{ln(\xi(x))} - \int\int b_r r^{-s} J(x)\partial r\partial s = 0$ -------------(**)
which is a quadratic equation in terms of $\frac{1}{ln(\xi(x))}$.

Thus, we have:

$$\frac{1}{ln(\xi(x))} = \{- \frac{1}{\xi(x)} +/- [(\frac{1}{\xi(x)})^2 - 4(-\frac{1}{\xi(x)})(- \int\int b_r r^{-s} J(x)\partial r\partial s)]^\wedge\frac{1}{2}\}/ [(2)(-\frac{1}{\xi(x)})]$$

$$= \frac{1}{2} +/- [\frac{1}{4} - \xi(x)(- \int\int b_r r^{-s} J(x)\partial r\partial s)]^\wedge\frac{1}{2}$$

$$(\frac{1}{ln(\xi(x))} - \frac{1}{2})^2 - \frac{1}{4} = \xi(x)\int\int b_r r^{-s} J(x)\partial r\partial s \text{ ----------- (***)}$$

Indeed, the optimum (maximum/minimum) point of the left hand side in the equation(***) is: ($\frac{1}{2}$, -$\frac{1}{4}$), or $\frac{1}{ln(\xi(x))} = \frac{1}{2}$, i.e. when ξ(x) = e^2, $\int\int b_r r^{-s} J(x)\partial r\partial s$ will attain its optimum (maximum/minimum) value. Moreover, the roots of (***) is also one of the optimum value(s) of the primitive function [$\int\int\int b_r r^{-s} J(x)\partial r\partial s$]'s outcome polynomial.
But if we integrate the left hand side of the equation (***) at ξ(x) = e^2, one may get:

$$\int \frac{1}{(\xi(x))}[(\frac{1}{ln(\xi(x))} - \frac{1}{2})^2 - \frac{1}{4}]\,dx = \int -\frac{1}{4}\frac{1}{e^2}dx = -\frac{1}{4}\frac{1}{e^2}x + c = \int\int\int b_r r^{-s} J(x)\partial r\partial s \text{ --(***)}$$

Hence, we may evaluate the above triple integral directly from the simple definite integral. Or we may know the root of the equation $-\frac{1}{4}\frac{1}{e^2}$x + c which is also the root of $\int\int\int b_r r^{-s} J(x)\partial r\partial s$. If we may transform $\int\int\int b_r r^{-s} J(x)\partial r\partial s$ into the form of Gauss's Divergence Theorem, i.e. $\int\int\int \nabla\cdot F\,dV = \int\int F\cdot n\,ds$, then this may imply the flux passing through the surface S on the right-side of the Divergence Equation is the same for the volume V over the object. Or even in a higher dimension, we may get the similar argument for the famous generalized Stoke's Theorem: $\int_{\cdot\partial\cdot\Omega\cdot}\omega = \int_{\cdot\Omega\cdot}d\omega$ which is usually applied in the computation of the flows for the surface of the air-plane object and internal structural stress, tension & deformation etc for the air-plane's fluid dynamic. In addition, the flying object can also work with the finite element analysis, the object's stiffness matrix together with my HKLam statistical model theory for prediction or some other kinds of modeling etc. In particular, with reference to [14], for a simple extension, the complex square contour (line integral) may then be reduced into the projection (or the dot product) of a coordinate point or function to the tangental vector space which may actually be another type of the 1-form to the complex functions or complex numbered coordinates in the 2-dimensional differential geometry or a complex plane. That say a complex projective structure or a complex projective space (may be considered as the complex manifold [18]. Indeed, the multi-dimensional complex manifolds may be used to determine the deformation or the curvature form of the complex structures etc.) To go forward a step, we may interpret such space as the quantum pure states of size n.

Actually, for my paper [14], there may be a path homotopy such that the Matlab programming segment code can lead the way locating at those non-trivial zeros of the Riemann Zeta function through the associated fixed-point theory (with the category theory) etc [15]. At the same time, for the complex contour integral to be zero, there must be a non-trivial roots located in the Matlab segmentation square [14] but the converse that if non-zero value is calculated for the complex contour integral, the implication about no zeta roots may NOT be true. That says, one may need the path homotopic theories to search such type of hidden non-trivial zeros for the convergence of the fixed-point spectral sequence(s) [17] & [18]. Or any divergence evidence of such spectral sequence implies there is NO non-trivial zeros for the complex contour integral over the homotopy path H. In fact, this author have once more turned the pure mathematics of algebric topology into the computational applied mathematics after my undergraduate mathematics project that focused in the foundation of mathematics with applications in both language linguistics and symbolic computations etc. This author wants to remark that for the two paths to be path homotopic if only if they have the same starting point and the same ending point [16], thus intutitively and obviously, the square contour used in my Matlab segment [21] is practically a path homotopic.

In brief, we may approximate the integral $\int \frac{1}{\xi'(x)}dx$ by both linear and quadratic equation(s) just like the case I and case II. Hence, one may eventually get a more generalized situation through the taylor series of order 2 approximation by using the commercial mathematical software Maple that will be shown in the coming section.
(N.B. The social category theory was once used in the Sovient Union or even now in some Eastern communism countries for the incorperation of different categorie of people as the case in man, woman & elderly etc. However, as this author's position, technology or knowledge itself is neutral but its good or bad usage may be completely depending on the one's will. That said, if the nuclear bomb technology was **first invented by the** axis of evil's **dictatorship** leaders in World War II (WWII) but **NOT the liberal & democratic** U.S.A., then our free World history may be completely inverted from the middle to late last Century or even nowadays.)

The Generalization – Use a Second Order Taylor Series to Approximate $\int \frac{1}{\xi'(x)} dx$ with Canada Maple (Soft: student licsenced version, 2022)

In practice, what we want to compute is to find the optimum value(s) of $\int \frac{1}{\xi'(x)} dx$. By the Fundamental Theorem of Calculus, $\partial \frac{\int \frac{1}{\xi'(x)} dx}{\partial x}$ is just $\frac{1}{\xi'(x)}$. Hence, for the $\xi'(x)$ to be maximum, then $\frac{1}{\xi'(x)}$ will attain its minimum or the vice versa.

Thus, by this author's previous paper [5], $\xi(x)$ is only:

$\sum \left[taylor\left(\frac{1}{x^{u+vi}}, x = a\right) a = 1..\infty \right]$ or

$$\sum_{k=1}^{\infty}\left(\frac{1}{e^{(u+vi)ln(k)}} - \frac{(u+vi)(x-k)}{ke^{(u+vi)ln(k)}} + \frac{\left(-\frac{u^2+2uvi+vi^2-u-vi}{2k^2} + \frac{(u+vi)^2}{k^2}\right)(x-k)^2}{e^{(u+vi)ln(k)}} + \frac{\left(-\frac{u^3+3u^2vi+3uvi^2+vi^3-3u^2-6uvi-3vi^2+2u+2vi}{6k^3} + \frac{(u^2+2uvi+vi^2-u-vi)(u+vi)}{2k^3} - \frac{(u^2+2uvi+vi^2+u+vi)(u+vi)}{2k^3}\right)(x-k)^3}{e^{(u+vi)ln(k)}}\right)$$

Then $\xi'(x)$ is just:

$$\sum_{k=1}^{\infty}\left(-\frac{u+vi}{ke^{(u+vi)ln(k)}} + \frac{2\left(-\frac{u^2+2uvi+vi^2-u-vi}{2k^2} + \frac{(u+vi)^2}{k^2}\right)(x-k)}{e^{(u+vi)ln(k)}} + \frac{3\left(-\frac{u^3+3u^2vi+3uvi^2+vi^3-3u^2-6uvi-3vi^2+2u+2vi}{6k^3} + \frac{(u^2+2uvi+vi^2-u-vi)(u+vi)}{2k^3} - \frac{(u^2+2uvi+vi^2+u+vi)(u+vi)}{2k^3}\right)(x-k)^2}{e^{(u+vi)ln(k)}}\right)$$

Set $\xi'(x) = 0$ and hence solve for x, we have:

$$\frac{1}{u^2+2uvi+vi^2+3u+3vi+2}\left(\left(u^2+2uvi+vi^2+\sqrt{-u^2-2uvi-vi^2-4u-4vi-3}+4u+4vi+3\right)k\right)$$

or

$$\frac{1}{u^2+2uvi+vi^2+3u+3vi+2}\left(\left(-u^2-2uvi-vi^2+\sqrt{-u^2-2uvi-vi^2-4u-4vi-3}-4u-4vi-3\right)k\right)$$

as the optimum (minimum/maximum) values for $\xi(x)$ or they are just the roots of $\xi'(x)$.

For $\xi'(x)$ to attain its optimum(maximum/minimum) values, we need to differentiate it once more and set it equals to zero, i.e.

$$\xi''(x) = \sum_{k=1}^{\infty}\left(\frac{2\left(-\frac{u^2+2uvi+vi^2-u-vi}{2k^2}+\frac{(u+vi)^2}{k^2}\right)}{e^{(u+vi)ln(k)}} + \frac{6\left(-\frac{u^3+3u^2vi+3uvi^2+vi^3-3u^2-6uvi-3vi^2+2u+2vi}{6k^3}+\frac{(u^2+2uvi+vi^2-u-vi)(u+vi)}{2k^3}-\frac{(u^2+2uvi+vi^2+u+vi)(u+vi)}{2k^3}\right)(x-k)}{e^{(u+vi)ln(k)}}\right)$$

Solving the above equation w.r.t. x, we have:

x = $\frac{(u+vi+3)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^2}\right)}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}$ as the optimum (maximum/minimum) value for the function $\xi'(x)$. Or $\frac{1}{\xi\prime(x)}$ will attain its optimum value at x = $\frac{(u+vi+3)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^2}\right)}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}$.

Hence, the root of $\frac{1}{\xi\prime(x)}$ is: (x - $\frac{(u+vi+3)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^2}\right)}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}$) = $\pm.\frac{\sqrt{-u^2-2uvi-vi^2-4u-4vi-3}}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}$

$$x = \frac{(u+vi+3)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^2}\right)}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)} \pm.\frac{\sqrt{-u^2-2uvi-vi^2-4u-4vi-3}}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)} \text{ -------(*****)}$$

In practice, instead of difficult mathematical concepts with computations, this author have developed a general algorithm for finding the roots of $\frac{1}{\xi\prime(x)}$ and hence the optimum value(s) of $\int \frac{1}{\xi\prime(x)}dx$ by using the Canada's Maple (2022 student license):

Step 1: (Calling) MTM;
Step 2: Set t = ξ'(x) with order 2 Taylor Series;
Step 3: Solve t;
Step 4: Let g5:= Taylor(1/t, x = a);
Step 5: Set g5 with order 2 Taylor Series;
Step 6: (Calling) MTM;
Step 7: Solve (g5);
Step 8: Simplify the two roots.

According to [8], (*****) may become to the capacitance of the coaxial cable when:

$$c = \frac{2\pi\varepsilon}{log\left[\left(\left(\frac{(u+vi+3)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^2}\right)}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}\right)\pm\frac{\sqrt{-u^2-2uvi-vi^2-4u-4vi-3}}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}\right)^2\right]},$$

or

$$c = \frac{\pi\varepsilon}{log\left(\left(\frac{(u+vi+3)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^2}\right)}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}\right)\pm\frac{\sqrt{-u^2-2uvi-vi^2-4u-4vi-3}}{(u+vi+2)\left(\sum_{k=1}^{\infty}\frac{1}{k^u k^{vi} k^3}\right)}\right)}$$

With reference to [2],

$$ln(N+1).\leq.ln(N^{s_2}+1).\leq.\xi(s_2)\leq.\xi(1).\leqslant\xi(s_1).\leq.1+ln(N).\leq.1+ln(N^{s_1}),$$

where $s_2 .\leqslant. 1 .\leqslant. s_1$. Thus, for any x of ξ(x),

$$ln(N+1).\leq.\xi(x).\leq.1+ln(N).$$

Hence, substitute back into (*),

$$\int e\left[1+\frac{1}{ln[1+ln(N)]}\right]dx .\leq. \int\int\int b_r r^{-s} dr ds .\leq. \int e\left[1+\frac{1}{ln(lnN+1)}\right]dx$$

If we go ahead for a step and approximate $\int \frac{1}{\xi\prime(x)}dx = \xi''(x)\ ln(\xi'(x))$ by the second order Taylor series, then one may get:

$$\xi''(x)\ ln(\xi'(x)) = \xi''(x)\ (\xi'(x) - \frac{\xi'(x)^2}{2})$$

By applying the approximation $(1+x)^n = 1 + nx$, we may have:

$$\int e[\frac{1}{ln(1+lnN)}]^2 dx \ .\leq.\ \int\int\int b_r\, r^{-s} drds\ .\leq.\ \int e[\frac{1}{ln(lnN+1)}]^2 dx.$$

I.e. For a 4-dimensional space-time, $f(x,y,z,t)$ of volume, it can be expressed as the volume of rotation of the reciprocal of the square of a log function for a toy model of a black hole. In particular, we may employ $\int.[\frac{1}{ln(x)}]^2 dx$ for the purpose of approximating the black hole toy model.

Therefore, by the following Matlab segment program code, we get the simulated quantization [3] to such black hole toy model:

MatLab Scripting of plotting 3D Revolution of function "$[1/\log(x)]^2$"

```
x = linspace(1, 5, 20); %Creates 20 points between interval [1,5]
y = [1./log(x)].^2; %The revolution function
plot(x,y), axis equal % draw profile
xlabel("x"); ylabel("y");
[X,Y,Z] = cylinder(y);%use cylinder function to rotate
figure
surf(X,Y,Z), axis square xlabel("Z");ylabel("y");zlabel("X")
```

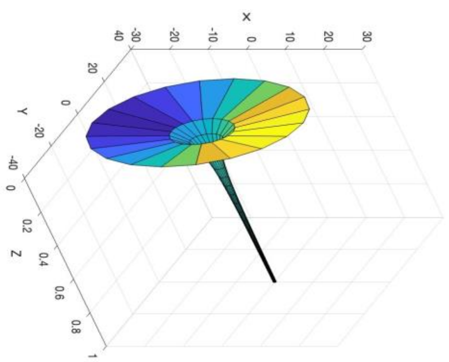

Figure 1: A Matlab Simulated Quantization to the Toy Model of a Black Hole.

Furthermore, as we have already known the root of $\int\int\int b_r\, r^{-s} drds$ is $\frac{1}{e}$, or , one may also approximate one of the root of the two squeezing inequalities or both of:

$$\int e[\frac{1}{ln(1+lnN)}]^2 dx \text{ and } \int e[\frac{1}{ln(lnN+1)}]^2 dx \text{ ---------- } (**)$$

Thus, we may even guess back/approximate the optimum point (maximum/minimum value) of the both squeezing inequalities (**) of the black hole toy model by substituting $ln(N)$ = 1- and 1+ to approach the true value of root of the both squeezing inequalities in (**).

Mathematical Implications – A Conformal Mapping & a Jacobian Matrix

Consider the spherical mapping [8] & [23],

$w = u + vi = e^z = e^{x+yi}$ where $0 < \text{Im } z < a$,

with $u = e^x \cos a$ ---------(eqt 1)

and

$v = e^x \sin a$ ---------(eqt 2)

where $-\infty < x < \infty$ and $y = a$

which transforms the ordinary or normal rectangular square strip in the z-plane into the spherical strip to the w-plane.

In addition, (eqt 2)/(eqt 1) gives us a straight line v = u tan a, which passes through the origin in the w-plane. Then obviously, the aforementioned spherical conformal mapping can be applied for the transformation between $\int \frac{1}{\xi\prime(x)} dx$ and $\int\int b_r\, r^{-s}$. That says, the wanted Jacobian matrix is:

$$\begin{matrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{matrix}$$

Or by expanding the above matrix in its determinant form, we have the requirement:

$$[\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}]|x_0, y_0 \,.\neq.0$$

for a one-to-one mapping.

Using the Cauch-Riemann equations $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$ and $\frac{-\partial u}{\partial y} = \frac{\partial v}{\partial x}$, we may then get:

$$[\frac{\partial u^2}{\partial x} + \frac{\partial v^2}{\partial y}]|x_0, y_0 \,.\neq.0$$

or if f(z) is analytic and $f'(z_0)$ $.\neq.0$, then w = f(z) provides a 1-1 mapping of a neighbourhood of z_0. Obviously both eqt (1) & eqt (2) are analytic and w = u +vi = f(z) with its first derivative ($d\,w/d\,z = d\,e^z/d\,z = e^z$) not equal to zero for all z in the z-plane, hence w = f(z) provides a one-to-one conformal mapping from the w-plane to the z-plane [8]. Thus, the wanted Jacobian matrix for the above transformation should be:

$$\begin{matrix} e^x cosy & -e^x siny \\ e^x siny & e^x cosy \end{matrix}$$

Or we have the following approximation:

$$\int \frac{1}{\xi\prime(x)} dx = \frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)]$$

$$= \int\int b_r\, r^{-s} \begin{matrix} e^x cosy & -e^x siny \\ e^x siny & e^x cosy \end{matrix} \partial r \partial s$$

But both x and y are dummy variables, the result follows immediately:

$$\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)] = \int\int b_r\, r^{-s} \begin{matrix} e^r coss & -e^r sins \\ e^r sins & e^r coss \end{matrix} \partial r \partial s \,.$$

Taylor Approximation of $\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)$

By using the concept of mirror image of the mirror image with the approximated substitution of ln(x) = (x - $\frac{x^2}{2}$), we may get:

$$ln(\xi(x)) = (\xi(x) - \frac{\xi(x)^2}{2}) = (½)(2\,\xi(x) - \xi(x)^2)$$

Thus, $\frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2(ln(\xi(x)) - 1)] = \frac{1}{\xi(x)}\frac{1}{(ln\xi(x))} - \frac{1}{\xi(x)}[\frac{1}{(ln\xi(x))}]^2$

$= \frac{1}{\xi(x)}(\frac{2}{2\xi(x)-\xi(x)^2}) - \frac{1}{\xi(x)}(\frac{2}{2\xi(x)-\xi(x)^2})^2$ ------ (****)

But as $ln(N+1) \leqslant. \sum_{n=1}^{N}\frac{1}{n^x}. \leqslant 1 + lnN$ and $ln(N) = (N - \frac{N^2}{2})$

$$[(N+1) - \frac{(N+1)^2}{2}]\ .\leqslant. \sum_{n=1}^{N}\frac{1}{n^x}.\leqslant.\ 1 + (N - \frac{N^2}{2})$$

By substituting $\sum_{n=1}^{N}\frac{1}{n^x} = [(N+1) - \frac{(N+1)^2}{2}]$ and $\sum_{n=1}^{N}\frac{1}{n^x} = 1 + (N - \frac{N^2}{2}))$ into (****) respectively, we may get:

$$\frac{1}{[1+(N-\frac{N^2}{2})]}\frac{2}{2[1+(N-\frac{N^2}{2})]-[1+(N-\frac{N^2}{2})]^2} = \frac{1}{[1+(N-\frac{N^2}{2})]}\frac{2}{2[1+(N-\frac{N^2}{2})]-[1+2(N-\frac{N^2}{2})]}$$

$$= 2\frac{1}{[1+(N-\frac{N^2}{2})]} = 2\frac{1}{[\frac{[-(N-1)^2]}{2}+\frac{3}{2}]}\ .\leqslant \frac{4}{3}$$

Discussion – Quantization of a Black Hole by the Light Bending Rings and the Information Paradox

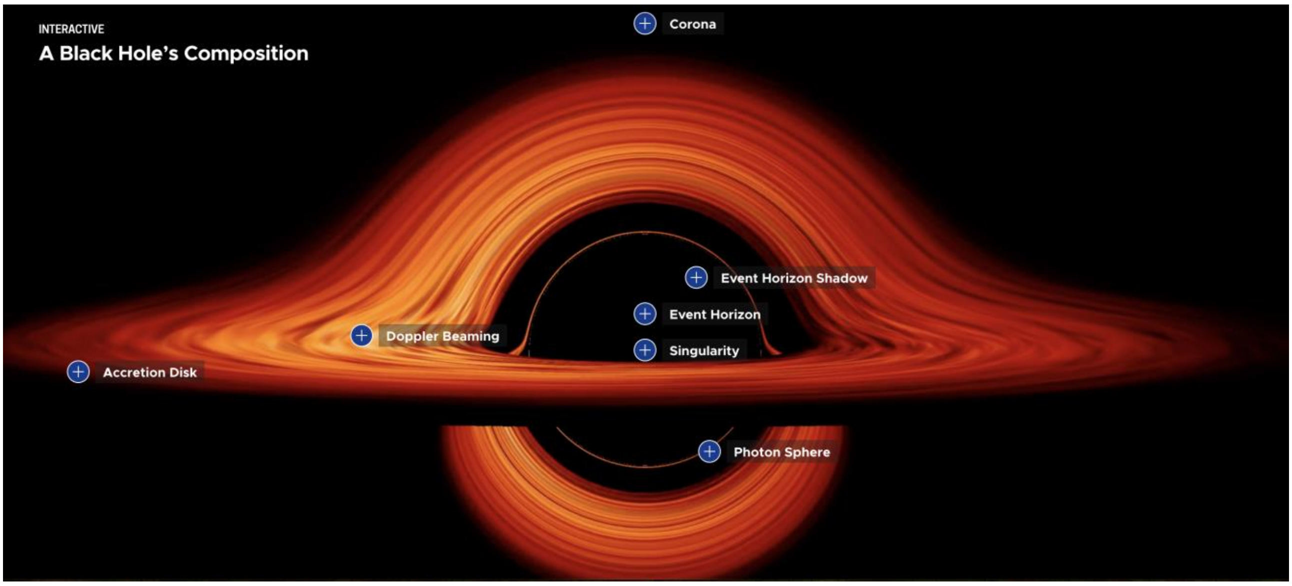

Figure 2: A Black hole photo sample that is obtained from the U.S.A. NASA [25].

In reality, there may be some visible image of the different light path(s) or rings of beam that are bended by the strong gravity of the black hole. Therefore, this author may think forward and suggest an algorithm for the quantization of any visible black hole with bending light paths in a partical way as like the following:

1. Record down those image data for the different bending paths (rings) of the light of a targeted investigating black hole;
2. Compute the corresponding different contour integral(s) of the bending path(s) or the rings of beam, thus according to the Stoke's theorem [27], [28] & [29], the integral(s) of these line contour(s) may imply different surface area(s) of the interested black hole [32];

3. The calculated various surface area(s) may then imply different energy entropy quantization of the black hole [26];
4. Each of the computed area(s) may be corresponding to each of the (area) energy (flux) quantization of the black hole from the boundary of the different bending path(s) or rings of beam;
5. Complete the fully quantization of the investigated black hole.

In practice, for the wanted function $\left(\frac{1}{lnx}\right)^2$ to approximate the toy model of the black hole, we may quantize its surface area piece-wisely by the following method:
Accordingly, the surface area piece-wisely (SAPW) for any function is:
(Surface Area) SA $=\int_a^b$. $[2\pi f(x)\,(1+f'(x)^2)^{1/2}]dx$
In the present toy black hole (PWTBH) model case:

$$SA_{PWTBH} = \int_{\zeta_{n-1}}^{\zeta_n}.[2\pi\left(\frac{1}{lnx}\right)^2(1+[d\left(\frac{1}{lnx}\right)^2/dx]^2)^{1/2}]dx$$

$$= \int_{\zeta_{n-1}}^{\zeta_n}.[2\pi\left(\frac{1}{lnx}\right)^2(1+[\frac{-2}{x}\left(\frac{1}{lnx}\right)^3]^2)^{1/2}]dx$$

By using the Mathematica (Home Liscensed Version) and mkes the Taylor Expansion about a point "a" and integrate for the first two non-trivial Zeta Zeros, i.e. ξ_1 and ξ_2 we may get:

$$4.56854*10^9\left(\frac{864.}{\left(a^{12}\left(1+\frac{4}{(a^2 log[a]^6)}\right)^{2.5} log[a]^{26}\right)}+\frac{1512.}{\left(a^{12}\left(1+\frac{4}{(a^2 log[a]^6)}\right)^{2.5} log[a]^{25}\right)}+\frac{1080.}{\left(a^{12}\left(1+\frac{4}{(a^2 log[a]^6)}\right)^{2.5} log[a]^{24}\right)}+\ldots\right)$$

In order to find the curvature of the wanted logaritmic function, we may need to first find its corresponding arc length parametrization as below:
(N.B. The arc length parametrization is just the $\sqrt{1+[f'(x)]^2}$ which is closely related to the arc length equation.)

Arc-Length Parametrization $=\int_0^t.(1+[\frac{-2}{x}\left(\frac{1}{lnx}\right)^3]^2)^{1/2}\,dx$

In fact, the curvature k for the axis of rotation of a black hole toy model with function $f(t)=\left(\frac{1}{ln(t)}\right)^2$ is:

$$k(t) = \frac{|f''(t)|}{\left[1+(f'(t))^2\right]^{3/2}} = \frac{\left|\frac{6}{t^2(ln(t))^4}+\frac{2}{t^2(ln(t))^3}\right|}{\left[1+\left[\frac{-2}{t}\left(\frac{1}{ln(t)}\right)^3\right]^2\right]^{(3/2)}}$$

After employing Taylor Expansion by Mathematica (liscensed Home Edition), we may get:

$$\frac{|f''(t)|}{\left[1+(f'(t))^2\right]^{3/2}} = \frac{\left(\frac{0.75}{\left((ln(t))^4 t^6\right)}+\frac{0.25}{\left((ln(t))^3 t^5\right)}-0.28125(ln(t))^2 t^2-0.09375(ln(t))^3 t^3\right)}{\left(\frac{1}{\left((ln(t))^6 t^8\right)}\right)^{1.5}}$$

$$= 0.75 + 0.25t(\ln(t))-0.28125t^8(\ln(t))^2-0.09375t^9(\ln(t))^6$$

Let $f(t) = 0.75 + 0.25t(\ln(t)) - 0.28125t^8(\ln(t))^2 - 0.09375t^9(\ln(t))^6$ and
let y = ln(t), then

$f(t,y) = 0.75 + 0.25t*y - 0.28125t^8y^2 - 0.09375t^9y^6$
Solving by Mathematica (Home Liscensed version), we may get:
-1.25713 < t < -1.13027 or t < -1.25713 or t > -0.0902479.
As t > -1.25713 or t < -1.25713, this may imply t = -1.25713 and y = 0.2288+3.1415i. In fact, the above y's complex value will give $f(t)$ an imaginary number valued space curvature. This may be used to model the blackhole's electromagnetic field. To go a step, it may be theoretically possible that one can even model the information containing in such kind of the associated or encoded in the high frequency electromagnetic waves or radiation. Or, we may (with the possibility) finally decode the information from the emitted electromagnetic radiation. If furthermore, the function contains complex variables, then this may implies that $f(t)$ is satisfying the Cauchy-Riemann equations or the mirrored image inverse. Indeed, the $f(t)$ may be a holomorphic function which is analytic and (infinitely) differentiable.

In addition, $\frac{\left|\frac{6}{t^2(ln(t))^4}+\frac{2}{t^2(ln(t))^3}\right|}{\left[1+\left[\frac{-2}{t}\left(\frac{1}{ln(t)}\right)\right]^2\right]^{(3/2)}}$ will attain its minimal curvature or $1+\left[\frac{-2}{t}\left(\frac{1}{ln(t)}\right)\right]^2 = 1$ when $\left[\frac{-2}{t}\left(\frac{1}{ln(t)}\right)\right]^2 = 0$. I.e. When t = ∞, the minimal curvature of the black hole toy model is: $k = 0$.
As the parametric equation $\sqrt{1+[f'(x)]^2}$ is a planar curve, its torsion should be zero.
To get in depth, for each level of non-trivial zeta zeros, there may be a boundary existing between the non-trivial zeta zeros or a continuum [] (that needs to have an in-depth investigation which is beyond the present focus) around the bounded rectangle of each zeta zeros for the proposed electromagnetic fields as shown in the following diagram:

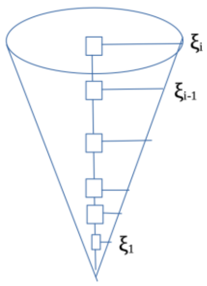

Diagram 3: The mirror inverted cone (or the black hole toy model) with the non-trivial zeta zeros as the different quantized levels.

In practice, the electromagnetic boundary over the above ξ_i & ξ_j will be given by the fact that it is only the contour path integral around the individual ξ_i & ξ_j. We may intuitively get:

$$\frac{2i\pi I}{2j\pi I} = \frac{contourpathintegralof\frac{1}{Zeta(\xi_i)}}{contourpathintegralof\frac{1}{Zeta(\xi_j)}}$$ where I is the imaginary number $\sqrt{-1}$

$$\frac{contourpathintegral\frac{1}{Zeta(\xi_i)}}{contourpathintegral\frac{1}{Zeta(\xi_j)}} = \frac{i}{j} = \text{ratio of the corresponding surfaces integral}$$

where ε_i & ε_j are the permittivity at the layer with zeta zeros ξ_i and ξ_j. Also, we may consider these "i", "j" integral numbers as some kind of quantization as the electromagnetic field is in fact quantized. Or to go forward a step, we may even use these "i", "j" integral numbers as the starting point to formulate the theory of quantum gravity etc. In fact, we may cross the bridge of quantized electromagnetic field [49] into the area of quantized (blackhole) gravity or just like the case of
$\mathbf{k} = \frac{2\pi j_{x,y,z}}{L}$ where $j_{x,y,z} = \pm1, \pm2, \pm3, \pm4, \pm5, \ldots \pm\infty$ and L is the length, k is the wave vector. In practice, as I have shown the existence of the magnetic monopoles around my black-hole toy model, according to the Maxwell's equation, there is also a Dirac strings. At the same time, the Dirac string

may then acts as the solenoid in the Aharonov-Bohm effect which actually implies Dirac quantization rule:

"The product of a magnetic charge and an electric charge must always be an integeral multiple of $\frac{n\hbar}{2}$."

Thus, with reference to the above contour path integral of $\frac{1}{zeta(\xi_i)}$ = constant k where the constant k can be divided by $\frac{n\hbar}{2}$. Or

$$k = \frac{n\hbar}{2} k_1$$

where k_1 is an integer just like the above wave vector.

Indeed, the quantized vector potential, the electric field and the magnetic field are:

$$A(r) = \sum_{k,\mu} \sqrt{\frac{\hbar}{2\omega V\epsilon_0}} \{e^{(\mu)} a^{(\mu)}(\underline{\mathbf{k}}) e^{i\underline{\mathbf{k}}.r} + \bar{e}^{(\mu)} a^{\dagger(\mu)}(\underline{\mathbf{k}}) e^{-i\underline{\mathbf{k}}.r}\}$$

$$E(r) = \sum_{k,\mu} \sqrt{\frac{\hbar}{2\omega V\epsilon_0}} \{e^{(\mu)} a^{(\mu)}(\underline{\mathbf{k}}) e^{i\underline{\mathbf{k}}.r} - \bar{e}^{(\mu)} a^{\dagger(\mu)}(\underline{\mathbf{k}}) e^{-i\underline{\mathbf{k}}.r}\}$$

$$B(r) = I \sum_{k,\mu} \sqrt{\frac{\hbar}{2\omega V\epsilon_0}} \{ (\underline{\mathbf{k}} \times e^{(\mu)})\, a^{(\mu)}(\underline{\mathbf{k}}) e^{ik.r} - (\underline{\mathbf{k}} \times \bar{e}^{(\mu)})\, a^{\dagger(\mu)}(\underline{\mathbf{k}}) e^{-i\underline{\mathbf{k}}.r}\}$$

where $\omega = c|\underline{\mathbf{k}}| = ck = c\left|\frac{2\pi j_{x,y,z}}{L}\right|$.

In practice, if we may establish an anological equivalent model between the quantization of gravitational field and the quantization of the electromagnetic field, then we may naturally quantized the (blackhole) gravity by the quantized electromagnetic field equations. In practice, let us first quantize the Einstein Field equation by the electromagnetic stress-energy tensor as below:

$$R_{(\mu\nu)} - \frac{1}{2} g_{(\mu\nu)} R + \Lambda g_{(\mu\nu)} = kT_{(\mu\nu)}$$

In an usual sitation, the electromagnetic stress-energy tensor is linear or

$$T_{(\mu\nu)} = \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) & \frac{1}{c} S_x & \frac{1}{c} S_y & \frac{1}{c} S_z \\ \frac{1}{c} S_x & -\sigma_{(xx)} & -\sigma_{(xy)} & -\sigma_{(xz)} \\ \frac{1}{c} S_y & -\sigma_{(yx)} & -\sigma_{(yy)} - \sigma_{(yz)} & -\sigma_{(yz)} \\ \frac{1}{c} S_z & -\sigma_{(zx)} & -\sigma_{(zy)} & -\sigma_{(zz)} \end{pmatrix}$$

where the energy-stress tensor can be quantized [50]. But the electromagnetic stress-energy tensor (decomposition) can also be expressed as the linear combination/regression of the following:

$$\begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) & \frac{1}{c} S_x & \frac{1}{c} S_y & \frac{1}{c} S_z \\ \frac{1}{c} S_x & -\sigma_{(xx)} & -\sigma_{(xy)} & -\sigma_{(xz)} \\ \frac{1}{c} S_y & -\sigma_{(yx)} & -\sigma_{(yy)} - \sigma_{(yz)} & -\sigma_{(yz)} \\ \frac{1}{c} S_z & -\sigma_{(zx)} & -\sigma_{(zy)} & -\sigma_{(zz)} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) + \frac{1}{c}\left(S_x + S_y + S_z\right) \\ \frac{1}{c} S_x - \sigma_{(xx)} - \sigma_{(xy)} - \sigma_{(xz)} \\ \frac{1}{c} S_y - \sigma_{(yx)} - \sigma_{(yy)} - \sigma_{(yz)} \\ \frac{1}{c} S_z - \sigma_{(zx)} - \sigma_{(zy)} - \sigma_{(zz)} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) + \frac{1}{c}\left(s_x + s_y + s_z\right) - \delta_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{c}s_x - \sigma_{(xx)} - \sigma_{(xy)} - \sigma_{(xz)} - \delta_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{c}s_y - \sigma_{(yx)} - \sigma_{(yy)} - \sigma_{(yz)} - \delta_3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{c}s_z - \sigma_{(zx)} - \sigma_{(zy)} - \sigma_{(zz)} - \delta_4 \end{pmatrix} + \begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix}$$

i.e. $R_{(\mu\nu)} - \frac{1}{2} g_{(\mu\nu)} R + \mathbf{\Lambda}\, g_{(\mu\nu)} = \mathrm{k}\, T_{(\mu\nu)}$ $.\approx.$ k $\Bigg(\begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) + \frac{1}{c}\left(s_x + s_y + s_z\right) - \delta_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{c}s_x - \sigma_{(xx)} - \sigma_{(xy)} - \sigma_{(xz)} - \delta_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{c}s_y - \sigma_{(yx)} - \sigma_{(yy)} - \sigma_{(yz)} - \delta_3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{c}s_z - \sigma_{(zx)} - \sigma_{(zy)} - \sigma_{(zz)} - \delta_4 \end{pmatrix} \Bigg)$

For the non-linear case of the electromagnetic stress-energy tensor, according to [51] & [52], we may have the following revised tensor:

$$\mathrm{P}^{\mu\nu} := \frac{\partial \mathcal{L}}{\partial F} F^{(\mu\nu)} + \frac{\partial \mathcal{L}}{\partial G} * F^{(\mu\nu)}$$

$$= \begin{pmatrix} 0 & D_1 & D_2 & D_3 \\ -D_1 & 0 & H_3 & -H_2 \\ -D_2 & -H_3 & 0 & H_1 \\ -D_3 & H_2 & -H_1 & 0 \end{pmatrix}$$

In fact, the non-linear electromagnetic energy-stress tensor is:

$$\mathrm{T}^{\mu\nu} = \frac{1}{4\pi}\left(2\frac{\partial \mathcal{L}}{\partial F} F_{(\mu\lambda)} F_{\nu}^{\lambda} + \frac{\partial \mathcal{L}}{\partial G} G\, g_{(\mu\lambda)} - \mathcal{L}_{EM}\, g_{(\mu\lambda)}\right)$$

When we equate the Einstein Relativity Field Equation, one may get:

$$R_{(\mu\nu)} - \frac{1}{2} g_{(\mu\nu)} R + \mathbf{\Lambda} g_{(\mu\nu)} = \mathrm{k}\, \frac{1}{4\pi}\left(2\frac{\partial \mathcal{L}}{\partial F} F_{(\mu\lambda)} F_{\nu}^{\lambda} + \frac{\partial \mathcal{L}}{\partial G} G\, g_{(\mu\nu)} - \mathcal{L}_{EM}\, g_{(\mu\nu)}\right)$$

Or:

$$R_{\mu\nu} = \frac{k}{2\pi}\frac{\partial \mathcal{L}}{\partial F}\left(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda}\right)\frac{\partial A^\lambda}{\partial x_\mu} = \frac{k}{2\pi}\frac{1}{E_r}\frac{q}{4\pi r^2}\left(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda}\right)\frac{\partial A^\lambda}{\partial x_\mu}$$

$$= \frac{k}{2\pi}\frac{1}{E_r}\frac{q}{4\pi r^2}\begin{pmatrix} \frac{\partial A_\lambda}{\partial x^\mu} & 1 \\ \frac{\partial A_\mu}{\partial x^\lambda} & 1 \end{pmatrix}\frac{\partial A^\lambda}{\partial x_\mu}$$

$\mathbf{\Lambda} g_{(\mu\nu)} = \frac{\partial \mathcal{L}}{\partial G} \mathrm{G}\; g_{(\mu\nu)}$ i.e. $\mathbf{\Lambda} = \frac{\partial \mathcal{L}}{\partial G}\mathrm{G} = \frac{k}{2\pi}\frac{1}{G_r}\frac{m}{4\pi r^2}\beta^2\sqrt{\left|f_{\mu\nu}\right|}$

$\frac{1}{2} g_{\mu\nu}\mathrm{R} = \frac{k}{4\pi} g_{(\mu\nu)}\mathcal{L}_{EM}$ i.e. $\mathrm{R} = \frac{k}{2\pi}\mathcal{L}_{EM}$

Similarly, we may find the wanted electromagnetic anological energy-stress tensor model that is equivalent to (i.e. expressed in terms of) (gravitational) metric tensor as:

$$\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) \\ \left(1-\frac{2GM}{rc^2}\right)^{-1} \\ r^2 \\ r^2 sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \left(1-\frac{2GM}{rc^2}\right)^{-1} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r^2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ r^2 sin\theta \end{pmatrix}$$

$$= \begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) - \varepsilon_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \left(1-\frac{2GM}{rc^2}\right)^{-1} - \varepsilon_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ r^2 - \varepsilon_3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ r^2 sin\theta - \varepsilon_4 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{pmatrix}$$

In the mirror image reverse way, we may find the gravitational metric tensor by:

$$\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) \\ \left(1-\frac{2GM}{rc^2}\right) \\ r^2 \\ r^2 sin\theta \end{pmatrix} = \begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix}$$

This author wants to note something interesting that the inverse of the metric tensor multiplying with the electromagnetic tensor or the mirror image converse will still give the same result – a zero vector as these two tensors are just the complementary to each other in the position of zero entries (linear dependent rows/columns for the electromagnetic field tensor). Thus, we cannot have the business primal-dual simplex method that by expressing the potential for both of the electromagnetic field and gravitational field in terms of matrix or tensor, then we may multipy the matrix of the gravitational potential (V_{grav}) or metric tensor by the inverse of the matrix of the electromagnetic potential (V_{elec}). The resulted matrix may then be approximated by this author's HKLam statistical model theory to obtain the wanted linear regression model. Finally, we may get the anological equivalent quantized model for the expected gravitational potential energy.

$$T^{\mu\nu} = \frac{1}{4\pi}\left(2\frac{1}{E_r}\frac{q}{4\pi r^2}\left(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda}\right)\frac{\partial A^\lambda}{\partial x_\mu} + \frac{1}{G_r}\frac{m}{4\pi r^2}\beta^2\sqrt{|f_{\mu\nu}|}\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix} - \right.$$

$$\left.\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix}\right)$$

$$T^{\mu\nu} = \frac{1}{4\pi}\left(2\frac{1}{E_r}\frac{q}{4\pi r^2} + \begin{pmatrix} \frac{\partial A_\lambda}{\partial x^\mu} & 1 \\ \frac{\partial A_\mu}{\partial x^\lambda} & 1 \end{pmatrix}\frac{\partial A^\lambda}{\partial x_\mu} + \right.$$

$$\frac{1}{G_r}\frac{m}{4\pi r^2}\,\mathbf{E}\cdot\mathbf{B}\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix} -$$

$$\left.\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2)\begin{pmatrix} -\left(1-\frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & \left(1-\frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin\theta \end{pmatrix}\right) \text{ ------- (*)}$$

where $F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$ can also be approximated by HKLam Theory.

By the Completing Square Method, we may have:

$\frac{1}{2}(\mathbf{E}^2 - \mathbf{B}^2) = \frac{1}{2}[(\mathbf{E} - \mathbf{B})^2 + 2\mathbf{E}\cdot\mathbf{B} - 2\mathbf{B}^2]g_{(\mu\nu)}$

The equation will attain its minimum when $\mathbf{E} = \mathbf{B}$. Hence, the energy-stress equation is reduced to:

$\frac{1}{4\pi}(2\frac{1}{E_r}\frac{q}{4\pi r^2}(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda})\frac{\partial A^\lambda}{\partial x_\mu}) + [(1+\frac{1}{G_r}\frac{m}{4\pi r^2})\mathbf{E}\cdot\mathbf{B} - \mathbf{B}^2]g_{(\mu\nu)}$

Similarily, when $\frac{1}{G_r}\frac{m}{4\pi r^2} = -1$ (there may be another possible solution equal to 1 but the calculation is similar and this author will not repeat), the equation will be reduced to:

$T^{\mu\nu} = \frac{1}{4\pi}(2\frac{1}{E_r}\frac{q}{4\pi r^2}(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda})\frac{\partial A^\lambda}{\partial x_\mu}) - \mathbf{B}^2 g_{(\mu\nu)}$

Or $g_{(\mu\nu)} = [0 - \frac{1}{4\pi}(2\frac{1}{E_r}\frac{q}{4\pi r^2}(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda})\frac{\partial A^\lambda}{\partial x_\mu})] / \mathbf{B}^2$

$= [0 - \frac{1}{4\pi}(2\frac{1}{E_r}\frac{q}{4\pi r^2}(\frac{\partial A_\lambda}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\lambda})\frac{\partial A^\lambda}{\partial x_\mu})] / \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)^2 + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)^2 + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)^2$

When $\frac{\partial A_\lambda}{\partial x^\mu} = \frac{\partial A_\mu}{\partial x^\lambda}$, then the above equation is a zero and is a minimum for the (gravitational) metric tensor $g_{\mu\nu}$ equation expression. This fact implies the quantization of the electromagnetic field may also help us quantize the metric tensor $g_{\mu\nu}$ according to the quantized values of $\frac{\partial A_\mu}{\partial x^\lambda}$. Or according to the quantized vector potential equation -- $A(r) = \sum_{k,\mu}\sqrt{\frac{\hbar}{2\omega V\epsilon_0}}\,\{e^{(\mu)}a^{(\mu)}(\underline{\mathbf{k}})e^{i\underline{\mathbf{k}}\cdot r} + \bar{e}^{(\mu)}a^{\dagger(\mu)}(\underline{\mathbf{k}})e^{-i\underline{\mathbf{k}}\cdot r}\}$. In practice, the linear regression model (initial starting point or the pioneer) for the (quantized) $g_{\mu\nu}$ is:

$$\begin{pmatrix} \frac{\partial A_\lambda}{\partial x^\mu} + 1 \\ \frac{\partial A_\mu}{\partial x^\lambda} + 1 \end{pmatrix} = \begin{pmatrix} \frac{\partial A_\lambda}{\partial x^\mu} + 1 - \varepsilon_1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{\partial A_\mu}{\partial x^\lambda} + 1 - \varepsilon_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix}$$

e

i.e. We may need to find the optimal value for the above regression model [53] and then solve the corresponding partial differential equations such as

$A_\lambda = (\varepsilon_{opt\_1}-1)x^\mu + c_i$ or $A_\mu = (\varepsilon_{opt\_2}-1)x^\lambda + c_j$. Or a pair of the business primal-dual in the simplex method. If we further suppose there was an radiation field that looks as a plane wave and propagates in the z-direction and is linearly polarized in the x-direction, then the real part of the 4-potential plane wave is:

$A_\mu = f(z-t)\,(0,1,0,0)$ [55] & [56].

$$\frac{\partial A_\lambda}{\partial x^\mu} = f'(z-t)\begin{pmatrix}0&0&0&0\\1&0&0&-1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$= f'(z-t)\;[\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\-1\\0\\0\end{pmatrix}]$$ or a pair of left-right handed spin-upwards electron spinors

and $$\frac{\partial A_\mu}{\partial x^\lambda} = f'(z-t)\begin{pmatrix}0&0&0&0\\-1&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$= f'(z-t)\;[\begin{pmatrix}0\\-1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\\0\end{pmatrix}]$$ or a pair of right-left handed spin-upwards electron spinors.

$$\begin{pmatrix}\frac{\partial A_\lambda}{\partial x^\mu} & 1\\ \frac{\partial A_\mu}{\partial x^\lambda} & 1\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}^T f'(z-t)\;[\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\-1\\0\\0\end{pmatrix}] + \begin{pmatrix}0\\1\\0\\0\end{pmatrix}^T f'(z-t)\;[\begin{pmatrix}0\\-1\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\\0\end{pmatrix}] + \begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

and hence we may reconstruct the corresponding (quantized) geometric spacetime [57]. According to the implications or proporties of the Dirac (quantum) equation that there should be a pair of electron-positron couple, hence the aformentioned tensor decomposed spinor equation for the positron will be also true as follow:

$$\begin{pmatrix}\frac{\partial A_\lambda}{\partial x^\mu} & 1\\ \frac{\partial A_\mu}{\partial x^\lambda} & 1\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}^T f'(z-t)\;[\begin{pmatrix}0\\0\\1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\-1\\0\end{pmatrix}] + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}^T f'(z-t)\;[\begin{pmatrix}0\\0\\-1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\1\\0\end{pmatrix}] + \begin{pmatrix}0\\0\\1\\0\end{pmatrix}+\begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$$

Actually, we may quantize the potential of the plane wave with reference to the results in [58]. Obviously, the above paired electron-positron spinor equation may be occurred in the boundary of the black hole's even horizon or the quantum entanglement phemonema that may appear. This author's reuslt shows that only the electrons pair will be found under the electromagnetic field of my toy black hole model or the black-hole is swallowing the negative part of the electron-positron pair and losting its mass because of the electron's negative energy. The fact is consistent with the famous Hawking Information paradox's prediction. Actually, the quantum no hair theory may solve such paradox. In the mirror image reverse way, one may compute the metric tensor $g_{\mu\nu}$'s value by the observational counting the number of boundary electrons from my HKLam's statistical linear regression equation model as presented aforementioned. Then we may get the quantized layer of layer (by the Figure 2) of $g_{\mu\nu}$. In reality, we may obtain the optimal value of the above tensors according to the conceptually biological experiments as mentioned in [54].

This author's HKLam statistical model theory computation implies that we may also express the Einstein Gravitational Field Equation in terms of (non-)linear combination/regression model equation (as there is also a non-linear type of electromagnetic energy-stress tensor. At the same time, if the non-linear type of electromagnetic stress-energy tensor can also be quantized [50], then we may have successfully quantized the Einstein Gravitational Field Equation and establish the corresponding anologically equivalent gravitational field equation model according to the quantization of the electromagnetic stress-energy tensor. (This author notes that the non-linear regression may be converted into the linear one by taking the logarithm.) It is also true that the mirror image converse of the above model is also true if we have already known the gravitational field equation model by reversing the above matrix/tensor computation process.

$$\begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) + \frac{1}{c}\left(s_x + s_y + s_z\right) - \delta_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{c}s_x - \sigma_{(xx)} - \sigma_{(xy)} - \sigma_{(xz)} - \delta_2 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{c}s_y - \sigma_{(yx)} - \sigma_{(yy)} - \sigma_{(yz)} - \delta_3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{c}s_z - \sigma_{(zx)} - \sigma_{(zy)} - \sigma_{(zz)} - \delta_4 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2 - \delta_1\right) + \frac{1}{c}\left(s_x + s_y + s_z\right) \\ \left(\frac{1}{c}s_x - \delta_2\right) - \sigma_{(xx)} - \sigma_{(xy)} - \sigma_{(xz)} \\ \left(\frac{1}{c}s_y - \delta_3\right) - \sigma_{(yx)} - \sigma_{(yy)} - \sigma_{(yz)} \\ \left(\frac{1}{c}s_z - \delta_4\right) - \sigma_{(zx)} - \sigma_{(zy)} - \sigma_{(zz)} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \delta_1 & \frac{1}{c}s_x & \frac{1}{c}s_y & \frac{1}{c}s_z \\ \frac{1}{c}s_x - \delta_2 & -\sigma_{(xx)} & -\sigma_{(xy)} & -\sigma_{(xz)} \\ \frac{1}{c}s_y - \delta_3 & -\sigma_{(yx)} & -\sigma_{(yy)} - \sigma_{(yz)} & -\sigma_{(yz)} \\ \frac{1}{c}s_z - \delta_4 & -\sigma_{(zx)} & -\sigma_{(zy)} & -\sigma_{(zz)} \end{pmatrix}$$ by decompositing each of the vector entries's addition terms so as to form the individual entries of the wanted electromagnetic stress-energy tensor. The similar mirror of mirror image processes continue until an optimal value of the modified electromagnetic stress-energy tensor is well obtained. Finally, we may solve the Einstein Field Equation in a novelly new way by repeating the mathematical taylor approximation procedure as shown in the aforemention section. Actually, such an optimal value may be obtained by the gradient descent procedure together with the commercial mathematical software – Matlab for both linear and non-linear kind of the optimization problem. However, such kind of computation may belong to the field of engineering which is out of the focus of the present paper. This author may present the physically computation for the famous Mathematical – Natively Stoke Equation problem when the time or conditions will be available or if there were any interested parties.

Actually, by considering,

$$Z = \frac{1}{\hbar}(i\hbar \frac{\partial A_\mu}{\partial x^\lambda}) \text{ which may give us } Z - \frac{1}{\hbar}(P_\lambda A_\mu) = 0$$

$$\frac{Z}{A_\mu} - \frac{1}{\hbar}P_\lambda = 0$$

$$Z = \Psi\left(\frac{-1}{A_\mu} - \frac{1}{\hbar}i\partial_\lambda\right) = 0$$

$$\frac{-1}{A_\mu} = - \frac{\partial ln(A_\mu)}{\partial x_\lambda} = \frac{i}{\hbar}\partial_\lambda$$

$$ln(A_\mu) = \frac{i}{\hbar}x_\lambda + k_\mu$$

$ln(A_\mu) = x_\lambda i\frac{\theta}{mc^2}t + k_\mu$ which may be transformed into a Diric Equation.

When we take $\hbar = mc^2\frac{t}{\theta}$, then we may get the wanted Diric Equation of the Gravitational plane wave:

$$\Psi_A(t) = e^{\left(\pm i\frac{mc^2}{\hbar}\right)t+k_\lambda}.$$

But according to the Einstein mass-energy equation – E = mc^2, then we may obtain:

$$A_\mu(t) = e^{\frac{x_\lambda}{E_n t}i\theta+k_\lambda}.$$

Obviously, the above equation can be considered as an energy spectrum or a quantization through a suitable Fourier transform as well as a suggestion for the need of a quantization of time.

By reconsidering $\frac{\partial A_\mu}{\partial x^\lambda} = f'(z-t)\begin{pmatrix}0&0&0&0\\-1&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix}$ and $e^{i\pi}$ = -1, then

$\frac{\partial A_\mu}{\partial x^\lambda} = f'(z-t)\begin{pmatrix}0&0&0&0\\-1&0&0&1\\0&0&0&0\\0&0&0&0\end{pmatrix} = f'(z-t)\begin{pmatrix}0&0&0&0\\e^{in\pi}&0&0&e^{i2n\pi}\\0&0&0&0\\0&0&0&0\end{pmatrix}$ which may show the existence of the graviton as the $e^{in\pi}$ appears in the above tensor matrix while $(e^{in\pi})^2$ may be decomposed into two gravitons.

Moreover, with reference to [59], we may have only:
$\{A_\lambda(x,t), \pi_\lambda(y,t)\}D = \delta(x-y)$
and
$\{A_\mu(x,t), \pi_\mu(y,t)\}D = \delta(x-y)$
which is a kind of Dirac Quantization of Free Electrodynamics.

In brief, there may be stress-energy tensor together with the electromagnetic field tensor applied for the mathematics in General Relativity. In this author's opinion, one may apply both of the forward and the mirror image reverse parts to compute these so as for the calculation of the solutions (various types) of the Einstein General Relativity Field Equation. This author have already demonstrated the method in the previous aforementioned and will NOT repeat. Last but not least, for all kinds of tensor, we may apply the HKLam statistical theory (both the forward and the mirror image reverse parts) for them to obtain the linear regression/combination model for a further research or study.

To go ahead step, we may consider:

$$Z = \frac{\Delta u}{c^2}$$

$$x_\mu - x_\lambda = \frac{\Delta u}{c^2}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{g\Delta y}{c^2}$$

$\frac{c^2}{g}\frac{\Delta\lambda}{\lambda} = \Delta y = x_\mu - x_\lambda$

which is just a gravitational redshift and $\frac{\Delta\lambda}{\lambda}$ can be measured by observation and finally get the change in the metric tensor. Hence, we may quantize the space practically for:

$$g_{ij}(x)\frac{\partial x^i}{\partial y^k}\frac{\partial x^j}{\partial y^l}dy^k dy^l$$

In practice, this author suggests that we may reduce the high dimensional tensor by tensor decomposition and apply the gradient descent momentum of the Heavy Ball Method to find a optimized minimum as well as the space quantization like below:

$$x_{t+1} = x_t - \alpha\nabla f(x_t) + \beta(x_t - x_{t-1})$$

Or we may have the following algorithm / steps:
1. Apply the gradient descent to obtain the minimum / optimum metric tensor vector in the sense of data linear regression – an optimization process for the statistical linear regression data;

2. Combine and transform the metric tensor (that observed from the gravitational waves data) into the wanted one;

3. Apply the gradient descent to the optimum transformed metric tensor and find the best solution to the tensor equation A$\underline{x}$ = $\underline{b}$;

4. Decompose the transformed metric tensor into a low dimensional one;

5. Use my HKLam statistical model theory to obtain the corresponding (tensor) linear regression model and get the corresponding quantization or data clustering for the such particular space time from the observed Gravitational waves.;

In fact, by the Stoke's theorem, the line integral of a vector field is equal to the surface integral of the curl of a function over a surface bounded by a closed surface. If in addition, the vector field is a conversative one, then the integral around the closed loop must be zero. As the winding number of the contour integral is the multiply of 2π, then the surface which is fitted by the prescribed bounded curve, the concerned non-trivial zero (say ξ_i) has an undefinited vector field at that zero point ξ_i for the function $\frac{1}{zeta(z)}$. In its mirror image revese way, the contour integral around ξ_i (or ξ_j etc) for the function zeta(z) is zero which is said to be a conservative vector field. Then for a conservative vector field, it is in fact the gradient of some function (say ∇f). When we are letting the gradient of the such function as P = ∇f, then it is known as the scalar potential. If we can find such scalar potential P, then we may get the wanted electric current I = ΔU (or P). In practice, the electric charge itself quantized while each of the contour integral of the non-trivial zeta zeros for the function $\frac{1}{zeta(z)}$ is also quantized by the ratio $\frac{i}{j}$ for some ξ_i , ξ_j etc.

At the same time, there may be a magnetic monopole existence laying between two consecutive ξ_i and ξ_j as the contour integral is always zeros or $\nabla \cdot B = 0$. (In practice, rather than the engineering intepretion in the theoretical black-hole battery feasibility discussion, the above zeta (ξ) function together with $\frac{1}{Zeta(\xi)}$ forms a pair of philosophy or a "Duality" or just academically the primal and dual problem of our simplex method in the subject of operational research [50], [51] & [52]). However,

there is no such kind of the monopole existing in our present known physical world and this may lead to a contradiction. Thus, either the implication of the $\nabla \cdot B = 0$ or
$\oint B \cdot dA = 0$ is wrong – the existence of a momopole or those of the computed Riemann non-trivial zeros are wrong. (N.B. In fact, the electric monopole does exist but NOT the magnetic one. If the magnetic monopole exists, then we may need to rewrite the famous Maxwell equations for its incompleteness). Theoretically, as the high electromagnetic field condition (together with the electrodynamics theory) around the boundary of this author's toy black hole model, there may be a Schwinger effect occuring in-between the two non-trivial zeta roots, say ξ_i and ξ_j. According to [45], the magnetic monopole may also be happened by the dual Schwinger effect under the strong magnetic fields. Hence, in this author's toy black hole model, I have proved the existence or the occurance of the magnetic monopoles in some boundary or area of it. Or we may finally have a unified picture for the Schwinger effect, Hawking radiation, the gauge-gravity relation and the ds-Ads duality issue etc [46]. This author wants to remark that what the Schwinger effect is just the phenomemon under high electromagnetic field where the positron will pair electron to be emitted.
(N.B. With reference to the rectangle of the boundary area of the diagram 3 and the integral form of the Maxwell's equation, we may have:
$\oint E \cdot dA = (1/\varepsilon_0)\int \rho \, dv$ implies
$\oint_{\xi i}$ Pdx +Qdy = $2\pi i$ and
$\oint_{\xi j}$ Pdx +Qdy = $2\pi j$
Hence, $\frac{contourpathintegral\xi_i}{contourpathintegral\xi_j} = \frac{2\pi i}{2\pi j}$ or $\frac{i}{j}$.
Moreover, from the above result and the integral form of Maxwell's equation, around the boundary of any ξ_i within the prescribed rectanglar curvature, there is a space charge or a collection of an excessive electric charge which may be treated as a continuum of charge distributed that particular region of space. Furthermore, the electric charge ratio for the $\frac{q_i fortherectangleof\xi_i}{q_j fortherectangleof\xi_j} =$ $(1/\varepsilon_i)\int \rho_i \, dv$ / $(1/\varepsilon_j)\int \rho_j \, dv$

$$= \frac{contourpathintegral\xi_i}{contourpathintegral\xi_j} = \frac{i}{j}.)$$

In addition, there are also the level curves' layers that are projected from the above mirror inverted cone diagram:

Mathematica Code for generating:

```
ContourPlot3D[x^2 + z^2, {x, -25, 25},
{y, 0, 25}, {z, 0, 25},
 Contours -> {Im[N[ZetaZero[1]]],
Im[N[ZetaZero[11]]],
  Im[N[ZetaZero[25]]]}, AxesLabel ->
{x, y, z}, MaxRecursion -> 2]
```

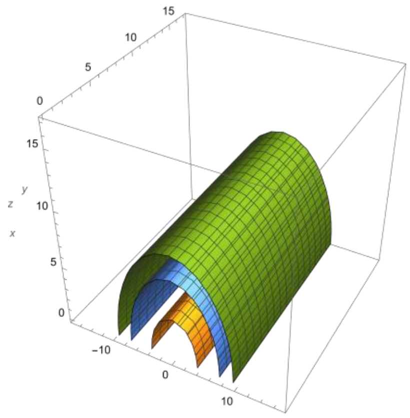

Figure 4: The layered level curves that is projected from the mirror image inverted black hole toy model (or the inverted cone or the layered spherical black hole model) – may be further extended for the quantization of the model as well as the relation to the present (quantum) information entropy theory.

Conclusion – A Disproof to the Unification of Classical Quantum Mechanics & General Relativity

In reality, quantum mechanics and generality relativity are coming from the two different types of views – the classical (Newtonian) absolute time frame of reference and the relative time frame of reference. These two types of reference are actually inconsistent. Indeed, from the classical quantum mechanics, we may crossover with the Galilean Relativity and obtain the wanted (Galilean type) of Quantum Gravity. That is one may go a further step and get the wanted gravity effect to the quantization for low speed particles or non-light speed particles (or the quantum gravity for low speed particles). To be precise, the Galilean Quantum Field Theory may thus be applied as a framework that combines the classical field theory, Galilean Relativity and quantum mechanics. At the same time, there is also a relativistics quantum mechanics for those (near) light speed particles with special relativity. One may also go for a forward step to obtain the relativistics QM for those (near) light speed particles in the curved space and time under the general relativity or gravity's effect to the curved space & time, i.e. the parametrized relativistics quantum mechanics [6] or even go forward to get the quantum gravity for those (near) light speed particles. Quantum Field Theory is tried to act as a framework that combines the classical field theory, special relativity and quantum mechanics. One may thus compare and contrast both of the GQFT and the QFT [11]. In reality, the above results are actually consistent with the fact of Lorentz Transformation (is an essential component which applied in Einstein's (special) Relativity) that [7]:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

or

$$t = \gamma\left(t + \frac{vx'}{c^2}\right)$$

where the above equations will reduce to the Galilean t = t' when x << ct together with v << c. We may assume both of $\frac{ct}{x}$ and $\frac{c}{v}$ equal to some big "M(s)" [9] by employing the technique(s) of business operational research in management or infinity for the Lorentz Transformation, i.e.

$\frac{ct}{x} = M_1$ ----------- (1*) and

$\frac{c}{v} = M_2$ ----------- (2*)

where M_1 and M_2 are some very large numbers [10]. One may solve the above equations (1*) & (2*) into a partial differential equation with variables x and t.

Finally, we may solve the partial D.E. and have:

(1*) divided by (2*) $\frac{tv}{x} = \frac{M_1}{M_2}$

$$\frac{(M_2)(v)}{x} = \frac{M_1}{t}$$

$$\frac{M_2}{x}\frac{\partial x}{\partial t} = \frac{M_1}{t}$$

$$M_2\ \partial ln(x) = M_1\ \partial ln(t)$$

$$x = t^{\frac{M_1}{M_2}} + k \text{ ------------ } (3^*)$$

Then in such a case, the Galilean Relativity may go ahead a step to merge with the wanted classical quantum mechanics equation and obtain the wanted quantum gravity. In fact, M_1 and M_2 are two different large numbers by the application of business operational research management methods as the case in big-M one and two stages one.

Indeed, the equation for Galilean Transformation of coordinates is:

$$v_{\frac{s'}{s}} = \frac{x}{t} = v \text{ ------------ } (4^*)$$

(for x' = 0 and s, s' are two coordinate systems that used in Galilean Relativity).

Substitute (3*) into the (4*), one may obtain:

$$v_{\frac{s'}{s}} = v = t^{\frac{M_1}{M_2}-1} + \frac{k}{t} \text{ ---------- } (5^*)$$

where (5*) may be the elementary conversion equation between the Lorentz Transformation and Galilean Transformation.

Then with reference to [30], we may have the Lorentz factor -- $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. If we apply the Taylor approximation to the above factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, we may get the following home edition licensed Mathematica scripts [30] & [31] together with some computed results:

Case I: For the zero-th ordered approximation,

Script Input:

y = 1/Sqrt[1 - v^2/c^2]
Series[y (x - vt), {v, 0, 0}]
Series[y (t - v/c^2 x), {v, 0, 0}]

Result Output:

(-vt + x) + $O[v]^1$
t + $O[v]^1$

Hence, the result after the substitution may be:

x' = x, t' = t, but x = $t^{\frac{M_1}{M_2}}$ + k = x' for the x-coordinate

Case II: For the first ordered approximation,

Script Input:

y = 1/Sqrt[1 - v^2/c^2]
Series[y (x - vt), {v, 0, 1}]
Series[y (t - v/c^2 x), {v, 0, 1}]

Result Output:

-(vt + x) + O[v]^2
t - (x v)/c^2 + O[v]^2
Hence, the result after the substitution $v = t^{\frac{M_1}{M_2}-1} + \frac{k}{t}$ may be:

t' = t- (x) ($t^{\frac{M_1}{M_2}-1} + \frac{k}{t}$) / c^2 for the t-coordinate.
The aforementioned x' and t' may be the precised or adjusted Taylor approximated x-coordinate and t-coordinate for the Galilean transformation [30]. To think a forward step, we may get the surface area or the electromagnetic flux the the computation of the line contour integral of the transformation by the Green's theorem or the Stoke's theorem. That says by computing the inverse of the above Taylor series [33] and hence obtains the corresponding contour line integral or -((2 c^2 t)/x),
i.e. 2 (c^2) / ($t^{\frac{M_1}{M_2}-1} + \frac{k}{t}$) for $0 < \Theta < \pi$, k is an integral constant. Then by applying the Stoke's theorem, the result is just the wanted surface area or even the electromagnetic flux [32] for the Galilean transformation. In fact, by Stoke's theorem, integrate the summation of all small pieces of these areas provides us the corresponding line contour integral.

One may also observe that the elementary transformed velocity in Galilean Relativity is just the the power indices for the ratio between two big numbers minus 1 or $\frac{M_1}{M_2} - 1$ of the time t.
In fact,

$$x' = \text{x-}v\text{t} = \text{x} - t^{\frac{M_1}{M_2}} + \text{k(for } x \neq t^{\frac{M_1}{M_2}}\text{, otherwise } x' = 0)$$

or

$$\text{x} = x' + v\text{t} = x' + t^{\frac{M_1}{M_2}} + \text{k.}$$

That says, the above paired of equations is just like the famous English poet and painter, Mr. William Blake says that "I am in you, and you in me." Or we may theoretically quantize the gravity according to the Galilean Relativity with reference to the classical quantum mechanics under the absolute time frame reference but NOT quantize the gravity by the General Relativity without any converting equation(s) as the case in the above formula (5*). It is true that Galilean transformations are indeed a good estimation of Lorentz transformations when the particles are at lower speed than light. But the key difference between these two transformations is the absolute Vs relative time reference frame. We therefore CANNOT unify quantum mechanics with general relativity. At the same time, both quantum mechanics and Galilean transformation (or relativity) are actually working well under absolute time frame reference and hence we may get the wanted Galilean Quantum Gravity by overcoming the problem of the incompatability between these two time reference frames. Thus, to be kept in focus with such incompatable situation, the only thing that this author may do is to develop a conversion equation between these two types of time reference frame. Then in such case, we may overcome the gap between these two incompatable time reference frames by getting a past over the bridge of this author's proposed conversion equation(s). Hence, we may transform the Galilean Quantum Gravity formula into the wanted Quantum Gravity (under General Relativity) one. Thus, we do NOT need to focus only on the incompatability for these two time reference frames and the contradictory fact that Galilean transform is a good estimation of the Lorentz transform for those low speed particles less than light. We are just requiring to focus on this author's transformation converting equation(s) by simply employing the business operational management's Big-M method. (N.B. 1. When $M_1 \rightarrow 1$ and $M_2 \rightarrow 1$, where $M_1 \neq M_2$, then the Galilean transformation will change back to the Lorentz transformation. **This author may thus name the ratio $\frac{M_1}{M_2}$ as the Galilean-**

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Lorentz Conversion Power Indices or GLCPI.

2. In fact, for the Einstein's twin paradox, there may be an implicit or hidden (universe or Newton's) absolute time in both of the twins'view. This is because for the sister A who stays on the rocket, the Earth is moving backward and become smaller relatively. But actually the sister A's view, A is stationary or A has her own absolute time relative to the absolute universe or Newton's one. Similarly, for the sister B on the Earth, B is stationary or B has her own absolute time relative to the absolute universe or Newton's one. i.e. Their relative time is: $\frac{\frac{RelTimeA}{AbsoluteUniverseTime}}{\frac{RelTimeB}{AbsoluteUniverseTime}}$ where the absolute universe time may be cancelled in a practical way. Indeed, this author understands that a quantum field theory for time may possibley be the final answer for the present research. However, time and space cannot be framed by the quantities like the Planck's constant for them to be quantized under the present knowledge. Thus, it is NOT feasible to say we must have a unification between quantum mechanics and general relativity as this may be impossible unless we have a breakthrough advancement(s) in the quantization of both space and time.

3. In reality, one may consider the Galilean transforms as the control experiment to the Lorentz transforms or Einstein's General Relativity [20].)

In brief, this author proposes the following algorithm for the quantization to the gravity:

1. Quantize the gravity with the usage in both of quantum mechanics together with Galilean Relativity through GQFT;
2. Convert the Galilean Relativity (transformation) to the Einstein's Special Relativity (Lorentz transformation) by the converting formula as the note mentioned by taking the limit $M_1 \rightarrow 1$ and $M_2 \rightarrow 1$;
3. Obtain the corresponing quantized gravity in Lorentz transformation from the above conversion through Galilean transformation as stated in the step 2 with reference to QFT;
4. Parametrize the computed quantum gravity (for General Relativity) in the curved space and time [6];
5. Finalize the wanted quantum gravity model/equation.

However, if one persists to generalize or unify the classical quantum mechanics with the general relativity or even the theory of everything without any conversion as the case in GLCPI, this author's suggestion is that the feasibility for the such kind of unification may be a zero in reality. This is because there are conflicts between the assumption of absolute time reference and the relative time reference or they are just not practically unified or compatible with such two contradictory kinds of views (or time frame of references). Only if one may develop a completely new kind of view (or the hybrid one between absolute and relative time frame) as the cases cosmic microwave background rest frame or the universal frame etc, may one still have the opportunties to obtain the unification between classical mechanics and general relativity. Or that says, one may well merge both types of absolute & relative time reference(s) for quantum mechanics and general relativity. In such of a case, this author may foresee the kind of new view will be indeed generated a completely new theory or just another story will then be created instead of falling into the mire of the unification to classical quantum mechanics and general relativity.

Limitations and Suggestions

In this paper, the author may propose a way to quantize a toy model of the black hole by using a computer programming code. Hence, it may become easier or as the pioneer for those advanced researchers to quantize a real black hole. As it is well-known that one of the application of quantum gravity (equation) may be the quantization of black hole. Thus, the present paper may be in the mirror reverse way provides a method or background to help those interested scientists seek for the wanted quantum gravity equation(s)/formulae. My suggestion is to make as much as possible observational data (especially from the gravitational wave data of the various black hole(s)) together with the machine learning and data mining methods etc to compute the expected QG equation(s) or model(s) or patterns. That says, one may need to set up a computer & information system in order to achieve advanced data processing for the LIGO data. It is no doubt that under the present technology, we may NOT have the ability to measure, find or capture the expected graviton from the gravitional waves [12]. On the contrary, what we have already known is the relationship between the gravity, gravitional waves and the quantization from the LIGO's observational data as per detected [13]. Then we may compare with the theoretical one that obtained from the algorithm in the aforementioned conclusion section, make the necessary & essential adjustments and hence generate some more accurate or indirect results.

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