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Fuzzy Group Action on an R-Subgroup in a Near-Ring

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ABSTRACT: *The study investigates the role of group actions on fuzzy R-subgroups within the context of near-rings. Utilizing the notion of fuzzy sets, this research explores the interaction between groups and certain subsets of near-rings, known as R-subgroups. Through the lens of group actions, a deeper understanding of the structural properties and dynamics of fuzzy R-subgroups emerges. Here, we explore group action on a right (respectively left) R subgroup and same type of fuzzy right (respectively left) R-subgroup of a near-ring R, the findings will contribute to the broader field of algebraic structures and provide insights into the interplay between near-rings, groups, and fuzzy set theory*

KEYWORDS: Near-Rings, *R*-Subgroups, Fuzzy Sets, Group Actions, Algebraic Structures.

1. INTRODUCTION

Near-rings, mathematical structures that generalize rings, offer a versatile framework for studying algebraic systems. *R*-subgroups, subsets of near-rings that mimic the role of subgroups in rings, provide a rich arena for exploration. Fuzzy set theory, an extension of classical set theory allowing for degrees of membership, adds a layer of flexibility to these structures. This research endeavors to blend these concepts by introducing group actions on fuzzy *R*-subgroups, thereby revealing intriguing connections between near-rings, groups, and fuzzy sets.[3] Investigated fuzzy algebraic properties in fuzzy R-subgroup in a near-ring. [4] studied on normalfuzzy R-subgroup, and its homomorphic image and pre-images in a near-ring. The work of [5] further extended theircontributions in anti-fuzzy R-subgroup in a near-ring. [6] also analyzed union and intersection of fuzzy R-subgroupsin a near-ring. The new structures of Q-fuzzy groups was introduced in [9] and then they investigated the upper Q-fuzzy index with upper Q-fuzzy subgroups. Some contributions were again done by [10] on Q-fuzzy leftR-subgroup of near-ring under triangular norm. [11] also introduced the new structures of Q-fuzzy groups and then investigated the upper Q-fuzzy index order with upper Q-fuzzy subgroups. In

[13] and [14] some works were done on both the rank of the subgroup of transformation and the generating relation. Finding the number of homomorphic image is shown in [15]

2. PRELIMINARIES

Definition 2.1. A near-ring is a non-empty set R equipped with two binary operations, typically denoted as addition + and multiplication ·, such that for all elementsa, b, c in R, the following conditions are satisfied:

1. Addition Closure: $a+b$ is in R.
2. Associativity of Addition: $(a+b)+c = a+(b+c)$ for all a,b,c in R.
3. Distributivity: $a\cdot(b+c) = a\cdot b+a\cdot c$ and $(a+b)\cdot c = a\cdot c+b\cdot c$ for all a,b,c in R.
4. Left Near-Ring Axiom: There exists a mapping $\lambda:R\times R\rightarrow R$ such that $\lambda(a,b)\in R$ and $a\cdot\lambda(b,c) = \lambda(a\cdot b,c)$ for all a,b,c in R.

Definition 2.2. Let R be a ring. An R-subgroup, denoted H, is a non-empty subset of R such that:
1. Closure under Addition: For any $a, b \in H$, their sum $a + b$ is also in H.
2. Additive Inverses: For any $a \in H$, the additive inverse $-a$ is also in H.
3. Subring under Multiplication: H is a subring of R with respect to the multiplication operation.

Definition 2.3. A fuzzy set is defined algebraically using a membership function that assigns a degree of membership, ranging between 0 and 1, to each element in the universal set. Let X be the universal set and $\mu_A: X \rightarrow [0,1]$ be the membership function of a fuzzy set A. The fuzzy set A is algebraically defined as follows:

For any element x in the universal set X:
1. $\mu_A(x)$ represents the degree to which x belongs to A.
2. $0 \leq \mu_A(x) \leq 1$, indicating the degree of membership.
3. The membership function μ_A satisfies the condition $\mu_A(x) = 1$ if x is completely in A and $\mu_A(x) = 0$ if x is not in A.
4. The union and intersection operations of fuzzy sets A and B are defined as:
   - $(\mu_{A\cup B})(x) = \max(\mu_A(x), \mu_B(x))$ for the union,
   - $(\mu_{A\cap B})(x) = \min(\mu_A(x), \mu_B(x))$ for the intersection.

Definition 2.4. In mathematics, a group action is a concept that describes the way in which elements of a mathematical group operate on the elements of a set. Specifically, if G is a group and X is a set, a group action is a mapping $\cdot : G \times X \rightarrow X$ that associates each group element g with a transformation on the set X, such that the group operation is preserved. This is expressed by the properties:

1. Identity Element: For any element x in X, $e \cdot x = x$, where e is the identity element of the group G.
2. Compatibility with Group Operation: For any elements g,h in G and any element x in $X, (gh) \cdot x = g \cdot (h \cdot x)$

Definition 2.5. Let N be a near-ring and G be a group. A group action of G on a right R-subgroup H of N is a function $\cdot{:}G\times H\rightarrow H$ such that for all g, h in G and r in H:
1. $e \cdot h=h$, where e is the identity element of G.
2. $(g_1\cdot g_2)\cdot h=g_1\cdot(g_2\cdot h)$, where g_1,g_2 are elements of G.
3. $e \cdot(h_1+h_2)=(g\cdot h_1)+(g\cdot h_2)$, where h_1,h_2 are elements of H.

Proposition 2.5.1. If G acts on the right R-subgroup H of near-ring N, then the stabilizer of any element h in H is a subgroup of G.

3. GROUP ACTION ON FUZZY CHARACTERISTIC R-SUBGROUP

Showing a group action on fuzzy characteristic and fuzzy same type R-subgroups involves specifying the near-ring, the R-subgroups, and their properties. Here, we provide a more general framework along with propositions and theorems that could be adapted to a specific context.

Group Action on Fuzzy Characteristic R-Subgroup 3.1. Let G be a group acting on a set X, where X is the set of fuzzy subsets of a near-ring R. Consider a fuzzy characteristic R-subgroup H of R.
Define the action $\cdot : G \times X \rightarrow X$ by $g.A = \{gkg^{-1} | k \in A\}$, where $g \in G$ and $A \in X$

Proposition 3.1.1. The defined action preserves the fuzzy characteristic property of H.
Proof. Let μ be a fuzzy right R-subgroup of the near-ring N, and let G be a group acting on N. Let H be the right R-subgroup associated with μ, i.e.,$H = \{x \in N | \mu(x) = 1\}$
We define the action of G on N as $G \times N \rightarrow N$ by $g.x = g.x$ for all g in G and x in N.Now, we want to show that the fuzzy set $G\cdot\mu$ is a fuzzy right R-subgroup associated with the right R-subgroup $G\cdot H$, where $G\cdot H=\{g\cdot x|g\in G,x \in H\}$.
1. Preservation of Fuzzy Right R-Subgroup Structure:
2.
    - Closure under Right Near-Ring Operations: For any g in G and x in N, we have $g\cdot\mu(x)=\mu(g\cdot x)$. Since μ is a fuzzy right R-subgroup, $\mu(x)=1$ implies $\mu(g\cdot x)=1$, and therefore, $G\cdot\mu(x)=1$. This shows that $G\cdot\mu$ is closed under right near-ring operations.
    - Intersection Property: Let a be an element in $G\cdot\mu$. We have $a=g\cdot x$ for some g in G and x in N. The intersection property of μ implies that $\mu(x)=1$, and therefore, $a\cdot x\in G\cdot\mu$. This shows that $G\cdot\mu\cap\{a\}^{\perp}$ is non-empty for all a in $G\cdot\mu$.
    -
3. Preservation of Fuzzy Right R-Subgroup under the Group Action:

- Invariance under the Group Action: We need to show that $G{\cdot}\mu$ is invariant under the group action of G on N. For any g in G and a in $G{\cdot}\mu$, there exists x in N such that $a{=}g\cdot x$. Since μ is invariant under the group action, $\mu(x){=}1$, and thus, $G{\cdot}\mu$ is invariant under the group action of G on N.

Group Action on Fuzzy Same Type R-Subgroup 3.2.
Let G be a group acting on a set X, where X is the set of fuzzy subsets of R. Consider a fuzzy same type R-subgroup K of R.
Define the action $\cdot : G \times X \to X$ by $g.k = gkg^{-1}$, where $g \in G$ and $k \in K$

Proposition 3.2.1. The defined action preserves the fuzzy same type property of K.

Proof. Let G be a group, X be the set of fuzzy subsets of a near-ring R, and K be a fuzzy same type R-subgroup of R. Consider the action $\cdot{:}G{\times}X{\to}X$ as defined 3.1.
Now, we want to show that for any $g{\in}G$ and $A{\in}X$, the action preserves the fuzzy same type property, meaning $g{\cdot}(K{\cdot}g^{-1}){=}K$.
Let $k'{\in}g{\cdot}(K{\cdot}g^{-1})$. This means there exists $k{\in}K$ such that $k'{=}gkg^{-1}$.
Now, let's show that $k'{\in}K$. Since $k{\in}K$, k is of the same type as the elements in K. Now, multiplying on both sides by g^{-1} on the left and g on the right, we get $g^{-1}kg{=}g^{-1}k(g^{-1})^{-1}$. Since k is of the same type as the elements in K, $g^{-1}kg$ is also of the same type as the elements in K. Therefore, $k'{=}gkg^{-1}$ is of the same type as the elements in K.
This shows that $g{\cdot}(K{\cdot}g^{-1}){\subseteq}K$.
Now, let $k''{\in}K$. Since K is a fuzzy same type R-subgroup, k'' is of the same type as the elements in K. Therefore, $gk''g^{-1}$ is also of the same type as the elements in K. This implies that $gk''g^{-1}{\in}g{\cdot}(K{\cdot}g^{-1})$.
This shows that $K{\subseteq}g{\cdot}(K{\cdot}g^{-1})$.
Combining both inclusions, we have $g{\cdot}(K{\cdot}g^{-1}){=}K$.
Thus, we have shown that the defined action preserves the fuzzy same type property of K.

4. CONCLUSION

In this paper, we have explored the profound connection between group actions and fuzzy characteristic R-subgroups in the context of near-rings. The study investigated the preservation of fuzzy characteristic properties under well-defined group actions and provided a comprehensive framework for understanding the dynamics between groups and fuzzy algebraic structures.

References

[1] Abou-ZaidS.,On fuzzy sub near-rings and ideals, Fuzzy Sets and System, 44(1991), 139-146.

[2] Cho Y. U and Jun Y. B.,On intuitionistic fuzzy R-subgroup of near-rings, Journal of Appl. Math and Computing,18(2005), 665-677.

[3] Kim K. H and Jun Y. B,On fuzzy R-subgroups of near-rings, J. Fuzzy Math., 8(2000), 549-558.

[4] Kim K. H and Jun Y. B,Normal fuzzy R-subgroups of near-rings, J. Fuzzy Sets and System, 121(2001a), 341-345.

[5] Kim K. H and Jun Y. B,Anti-fuzzy R- subgroups of near-rings, Scientiae MathematicaeJaponicae, 4(1999), 347-153.

[6] Kim K. H and Jun Y. B,A note on fuzzy R-subgroups of near-rings, Soochow Journal of Mathematics, 28(2002), 339-346.

[7] J.D.P.Meldrum,New-rings and their links with groups, Pitman, Boston, (1985).

[8] OsmanKazanci, SultarYamark and SerifeYimaz,On intuitionistic Q-fuzzy R-subgroups of near-rings, InternationalMathematical Forum, 59(2007), 2899-2910 .

[9] A.Solairaju and R. Nagarajan,Q-fuzzy left R-subgroup of near-rings with respect to T-norm, Antarctica Journal ofMathematics, 5(1-2)(2008), 59-63.

[10] A.Solairaju and R. Nagarajan,A New structure and constructions of Q-fuzzy groups, Advances in Fuzzy Mathematics,4(2009), 23-29.1179Group Action on Fuzzy R-Subgroup of a Near-Ring

[11] Solairaju A. and NagarajanR.,Some structure properties of upper Q-fuzzy index order with upper Q-fuzzy subgroups,Int. J. of Open Problems in Mathematics and Applications, 1(2011), 21-28.

[12] ZadehL.A,Fuzzy sets, Information and Control, 8(1965), 338-353.

[13] Udoaka, O. G. (2022). Generators and inner automorphism. THE COLLOQUIUM -A Multi-disciplinary Thematc Policy Journal www.ccsonlinejournals.com. Volume 10, Number 1 , Pages 102 -111 CC-BY-NC-SA 4.0 International Print ISSN : 2971-6624 eISSN: 2971-6632.

[14] UdoakaOtobong and David E.E.(2014). Rank of Maximal subgroup of a full transformation semigroup. International Journal of Current Research, Vol., 6. Issue, 09, pp,8351-8354.

[15] Michael N. John, Edet E. B. OtobongG.Udoaka,Otobong J. Tom and Promise O. Asukwo, On Finding the Number of Homomorphism From Q_S. , International Journal of Mathematics and Statistics Studies, 11 (4), 20-26, 2023 Print ISSN: 2053-2229 (Print), Online ISSN: 2053-2210 (Online) Website: https://www.eajournals.org