Fitting Alternative Autoregressive and Moving Average Models to Nigeria Crude Oil Prices

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ABSTRACT: *The need to compare the efficiency between the Autoregressive Integrated moving average (ARIMA (p,d,q)) models when modelling Crude oil price is the motivation behind this research. The research focuses on different orders of Autoregressive Integrated moving average models. The trend Analysis of the original series were plotted and was observed that crude oil prices were not stationary. The data were transformed by taking a natural log and the series becomes stationary after first differenced. The ACF and PACF of the stationary time series were also plotted which were the basis for the suggested ARIMA models. Error variances for the suggested ARIMA (p,d,q) models were derived and estimated as the basis for model performance comparison. Empirically, Crude Oil Price data spanning from January 2006 to July 2023 were used for the analysis. Findings from the study has revealed that, ARIMA (2,1,1) with the least error variance outperformed the other suggested models. The study further stated the estimated models for forecast of the future value of the crude oil price. The study recommends the use of error variance as a criterion for best model suggestion and ARIMA (2,1,1) was selected as the best model for modelling Nigeria Crude oil price.*

KEYWORDS: error variance, error variance of ARIMA (p,d,q), stationary time series, autoregressive integrated moving average (ARIMA) model

INTRODUCTION

Nigeria is a country with diverse culture and ethnic groups with 36 states and the Federal Capital Territory. United State Agency for International Development (2021), ranked Nigeria as the 31st-largest economy in the world in terms of nominal GDP and the largest in Africa. In late 2023, Nigeria GDP dropped and it was ranked the 32nd – largest in the World. Nigeria Economy is highly dependent on crude oil production which makes the country a mono-economy nation. The crude oil industry is the largest and most significant sector in Nigeria Economy, contributing significantly to the country's Gross Domestic Product (GDP) and providing a major source of energy for both Nigeria and the world [15]. Due to its importance, the oil industry has a significant influence on Nigeria's economic and political landscape. In 1956, crude oil was discovered in Nigeria with full operation starting in the late 1950s. Prior to the establishment of the Nigerian National Petroleum Company (NNPC) in 1977 with the aim to regulate the price of crude oil and participate in the country's oil business, oil exploration was only open to foreign companies due to insufficient facilities and skilled personnels in oil sector. The oil industry grew constantly with time to become a global giant. Nowadays, Nigeria is Africa's main oil producer with 18 operating pipelines and an average daily production of some 1.8 million barrels in 2020, Nigeria is the eleventh largest oil producer worldwide. The petroleum industry accounts for about nine percent of Nigeria's GDP and for almost 90 percent of all export value [7].

Crude oil has been the engine of Nigeria economy for decades and has also played a key role in its development and success. It plays a significant role in the Nigerian economy as the largest contributor in terms of total government revenue and also as the overall contributor in her export composition. It accounted for about 82.1% of total government revenue during the oil boom in 1974 before reducing to a share of 64.3% in 1986 which was as a result of the rapid decline in of crude oil price at the international market. The share of oil revenue in total government revenue still remains substantial as evidenced by the attainment of 85.6% and 86.1% in 2004 and 2005 respectively. Currently, Nigeria tops oil-producing countries in Africa, and the country depends heavily on the oil sector. In the year 2000, crude oil exports accounted for about 83% of the Federal Government's revenue and about 98% of export earnings. It also generated more than 14% of Nigeria's Gross Domestic Product (GDP), provided about 65% of government budgetary revenues and 95% of foreign exchange earnings. Nigeria's proven oil reserves were estimated to be between 16 to 22 billion barrels in the year 1997 by the United States Energy Information Administration (EIA) [8]. In 2010, about 10% of United States (U.S.) overall oil imports were provided by Nigeria; and among all the countries exporting oil to the U.S, Nigeria was ranked as the fifth-largest source. In July 2014, the oil supply declined due to the alternative use of shale production in America. Currently, the largest consumer of Nigerian Oil is India [5].

Prior to the discovery of crude oil in Nigeria, Nigeria strongly relied on agricultural exports such as palm produce, cocoa, cotton, timber, groundnut and rubber to improve the economy of the country [10] [14]. The agricultural sector was the mainstay of the Nigeria economy contributing about 95% of her foreign exchange earnings, generating over 60% of her employment capacity and approximately 56% of her Gross Domestic Product [17]. As an oil-exporting developing country, Nigeria's economic growth has been unstable due to its dependence on the fluctuating international oil market. The country's over-independence on oil exports has made it helpless to oil price shocks from the international market, which is evident from the significant decline in non-oil exports over the past three decades as reported in a study by [11].

Between 1999-2004, the oil market has experienced predictable seasonal fluctuations. Oil prices were relatively stable, averaging between USD20 and USD30 per barrel between 1999 and early 2004, although it reached as high as USD147 per barrel in July 1998. However, the global financial crisis in September 2008 led to a sharp drop in oil prices, with an average of around USD53 per barrel by the end of 2008. From 2014 to 2016, there was another sharp drop in oil prices, from an average of USD110 in June 2014 to as low as USD29 per barrel in January 2016, due to increased oil production in the United States. The present fluctuations in oil price situation in the international market, including the Saudi Arabia-Russia price war and the COVID-19 pandemic, has resulted in a significant downward trend in the oil price at the international market, thereby creating fear and uncertainty for oil producers and a country like Nigeria whose major source of revenue is the crude oil. This information was reported in works of [12], [1], [15] and [9].

In recent times, crude oil price has been one of the major economic challenges confronting most likely all the oil producing countries in the world especially countries whose Gross Domestic Product (GDP) is highly dependent on oil export like Nigeria [2]. The crude oil price from January 2009 to February 2016 follows an upward trend and between March 2013 and February 2016 the price gets a downward trend. From March 2016 to 2021, the price trend has been so unstable with the greatest price shock between March and April of 2020 due to COVID-19 Pandemic. In 2022, the trend starts moving upward again with a sharp rise between March to July and experiences a fall from August till July 2023. These fluctuating price of oil at the international market has affected significantly Nigeria economic whose economy is highly dependent on crude oil export thereby subjecting the country to an unstable economy and increase in the country's inflation rate. These has poised a great challenge to Nigeria government and Oil Industries on how to design a best model for modelling crude oil price so as to guide against fluctuating economy as a consequence of an oil price shock from the international market. Many researchers over the years have applied different criteria to select the best model for modelling Nigeria crude oil price with no application of the Error Variance approach. This work seek to derive and estimate error variances of the suggested Autoregressive Integrated Moving Average (p,d,q) models, formulate an estimated models for modelling crude oil price and to forecast the future value of the crude oil price.

LITERATURE REVIEW

[16], in their studies used ARIMA and GARCH models to modelled crude oil price in Nigeria spanning from January, 1998 to September, 2013 and applied AIC, HQC, and SIC criteria for model selection. The model with the smallest values of the criteria were considered as the best model. Hence ARIMA (3, 1, 1) and GARCH (2, 1) were found to be the best model for forecasting the crude oil price data series.

[14], in their work used ARIMA model and applied Box-Jenkins four-step iterative methodology comprising of model identification, model fitting, diagnostic and forecasting in selecting the best model for modelling the Nigeria crude oil prices data. Two optimal time series models were selected namely; ARIMA (2,1,1) and ARIMA (3,1,1) based on the three-information criteria used; AIC, BIC and HQC. Thus, based on the criteria of mean square error; root mean square error; mean absolute error; it was observed that ARIMA (3,1,1) model was the best model that fits the data with minimum values of predictive measures.

[6], demonstrated in their studies modelling of the Nigerian Bonny Light Crude oil prices using a classical statistical model; Autoregressive Integrated Moving Average (ARIMA), two machine learning model; Artificial Neural Network (ANN) and Random Forest (RF) and Fuzzy Time Series (FTS) Model. These models were compared in modeling the Nigerian Bonny Light crude oil price data for the periods from January, 2006 to December, 2020. The performance measures used for the comparison were the modified Diebold-Mariano test, the Root Mean Square Error (RMSE), the Mean Absolute Percentage Error (MAPE) and Nash-Sutcliffe Efficiency (NSE) values. Based on their findings of the performance measures, ANN (4, 1, 1) and RF performed better than ARIMA (1, 1, 0) model but FTS model using Chen's algorithm outperformed every other model suggested. The results from the studies recommend the use of FTS model for forecasting future values of the Nigerian Bonny Light Crude oil price.

METHODOLOGY

Crude oil price can be studied and predicted using a stochastic modeling approach that captures the time dependent structure embedded in the time series crude oil price data. The Autoregressive Integrated Moving Average (ARIMA) model, popularly known as Box-Jenkins Methodology (G. P. E. Box and G. M. Jenkins (1978) procedure for fitting Autoregressive Integrated Moving Average (ARIMA) model is used in this study to model monthly average data of Nigeria Crude Oil Price from January 2006 to July 2023. The Trend Analysis of the original series were plotted and it was observed that crude oil prices are not stationary. The series were therefore transformed by taking a natural logarithm and after first difference the series became stationary. Trend analysis of the stationary series was plotted, the ACF and PACF of the stationary time series data were also plotted and used for models' suggestion. The error variances of the suggested ARIMA (p,d,q) models are derive and estimated. The model with the smallest error variance is considered the best model for modelling the crude oil price. The estimated model for the best model is formulated for forecast of the future prices of the crude oil.

Let $\psi_p(\beta)(1-\beta)^d \nabla Y_t = \theta_q(\beta)\varepsilon_t$ (1)

Where Y_t is the time series data, ψ_p and θ_q are the parameters of autoregressive and moving average processes respectively, $(1-\beta)^d$ is the regular differencing to remove the stochastic trend (if any) in the series and the value of d is determined by the number of regular differencing required to completely isolate the trend from the series.
∇Y_t is the backshift operator.

Using the backshift operator AR(p) can be presented as

$$\left(1-\psi_1\beta-\psi_2\beta^2-\psi_3\beta^3-\cdots-\psi_p\beta^p\right)Y_t=\varepsilon_t \quad (2)$$

Similarly, MA(q) can be represented as

$$\left(1+\theta_1\beta+\theta_2\beta^2+\theta_3\beta^3+\cdots+\theta_q\beta^q\right)X_t=\varepsilon_t \quad (3)$$

This implies that, ARIMA (p, d, q) can be written as

$$\nabla Y_t = \mu + \psi_1 Y_{t-1} + \psi_2 Y_{t-2} + \cdots + \psi_p Y_{t-p} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} + \varepsilon_t \qquad (4)$$

Procedures for Model Specification

i. **Time series plot**: a time series plot is the series of discrete numerical points indexed (or graphed) in time line order. Time series are sequences of successive equally spaced points in a line chart. A time series plot or graph reveals the trend exhibited by the data, the seasonal and cyclic pattern of the data set.

ii. **Differencing**: Differencing is use to make a time series process which was originally non -stationary to become stationary by differencing the time series process at different lag until it becomes stationary.

iii. **Stationary Trend Analysis Plot:** A stationary time series plot is the graph of the time series process after differencing.

iv. **The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF):** The ACF and the PACF are two important tools for determining the nature of the underlying mechanism of a stochastic process. In particular, they can be used to determine the orders of the underlying AR and MA processes respectively. The analysis of ACF and PACF to identify appropriate models is known as the Box & Jenkins methodology Box and Jenkins (1976). Given a set of observations $Y_1, \ldots Y_t$, we need to decide what the appropriate model might be. The estimated ACF and PACF are the tools which can be used to suggest the models. If the ACF exhibits slow decay and the PACF cuts off sharply at certain lags, we would identify the series as AR model. If the PACF shows slow decay and the ACF shows a sharp cut-off at certain lags, we would identify the series as being MA model. If both the ACF and PACF show cut-offs, we would identify the series as being mixed ARMA. A pure autoregressive model is suggested if ACF decays exponentially and PACF exhibits a significant cut-off mostly within the two lags. But, when PACF decays exponentially, and ACF exhibits a significant cut-off at initial lags, the MA model is suggested. An ARIMA model is suggested if there is a cut-off at both the ACF and PACF of the difference series, and it is called Autoregressive Integrated Moving Average, or ARIMA (p, d, q) [18]. The word Integrated comes in because of the differencing in order to make the series stationary before fitting ARIMA model to the data.

v. **Error Variance:** Error variances of the suggested ARIMA (p,d,q) models are derive and the model with least error variance is considered best.

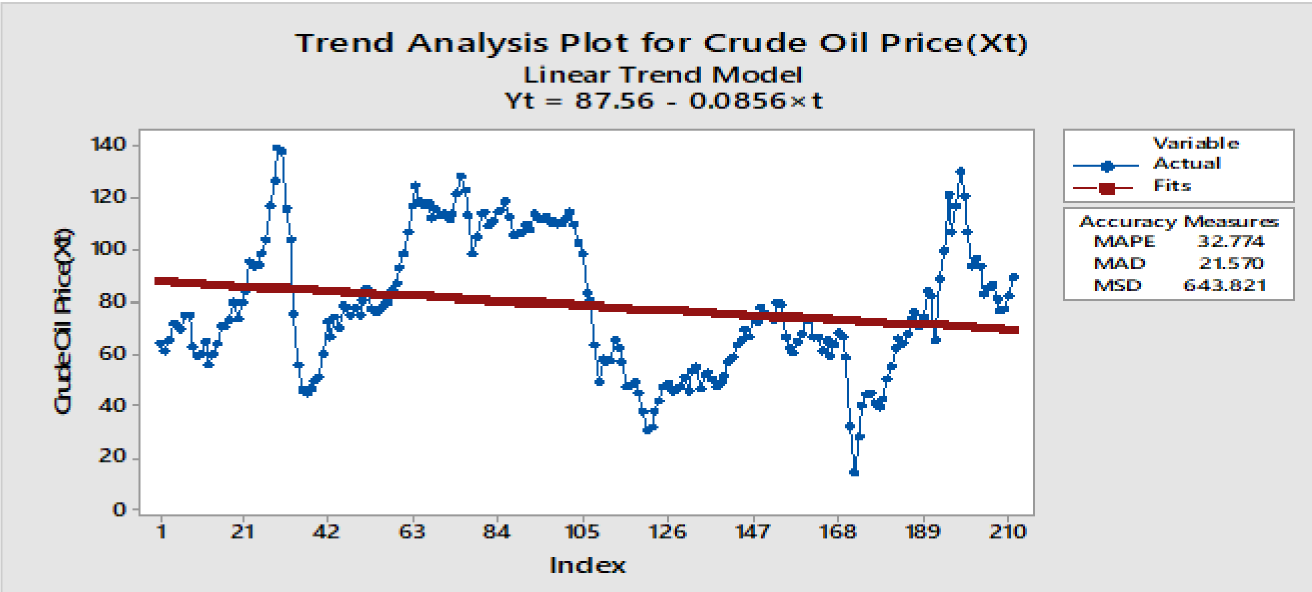

Figure1: Trend Analysis plot for Nigeria Crude Oil Price

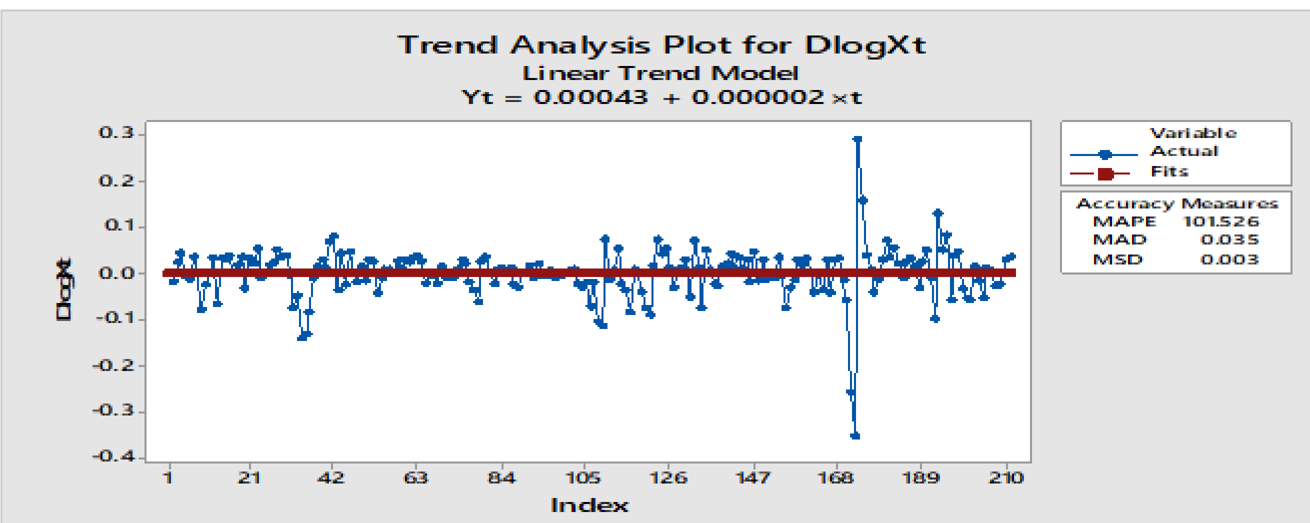

Figure2: Stationary Time Series Plot for Nigeria Crude Oil Price

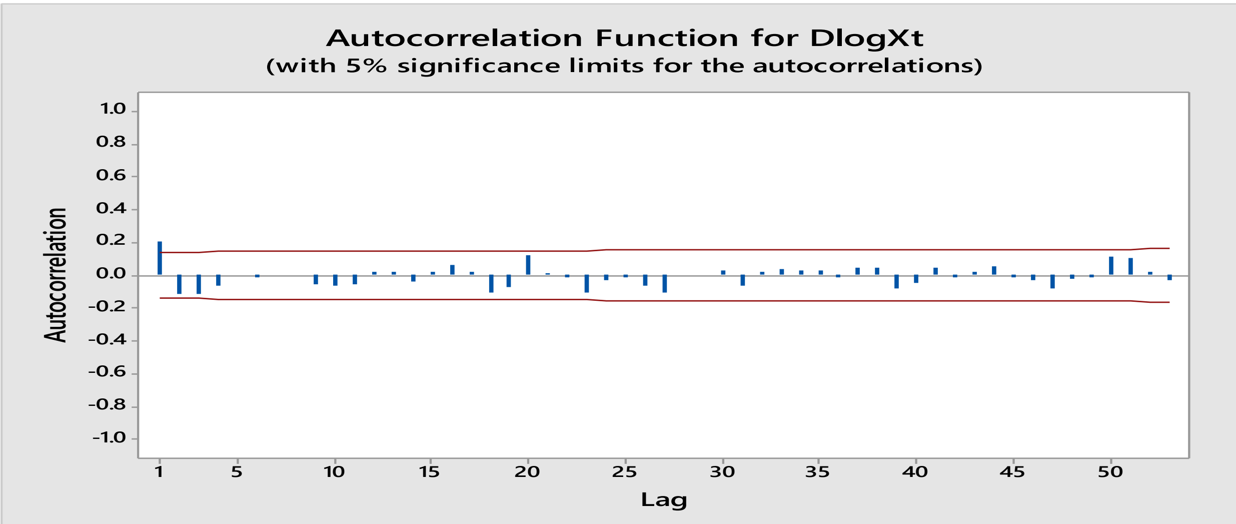

Figure3: ACF of a Stationary Time Series Plot for Nigeria Crude Oil Price

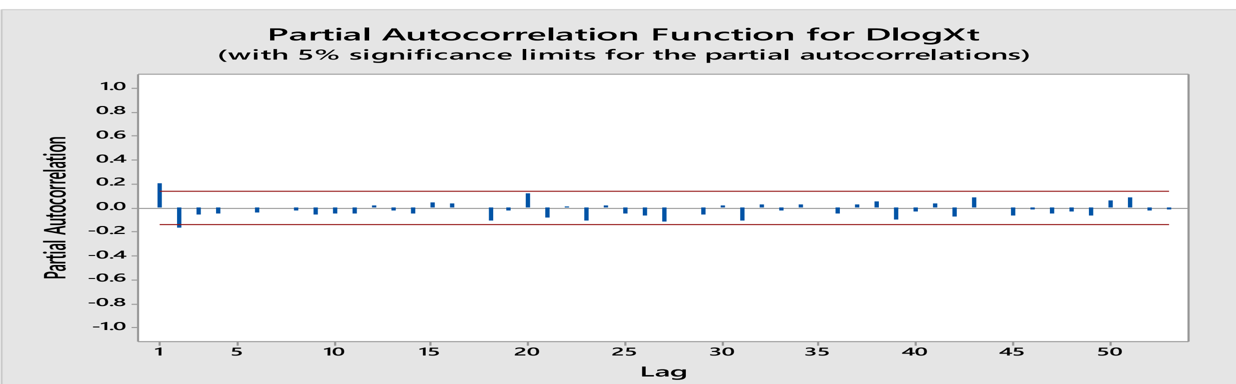

Figure 4: PACF of a Stationary Time Series Plot for Nigeria Crude Oil Price

From figure (3) and (4) , the following three models were suggested for the Nigeria Crude Oil price data; ARIMA (0,1,2) , ARIMA (1,1,1) and ARIMA (2,1,1).

Derivation of Error Variances of the Suggested ARIMA (p,d,q) Models

Error Variance of ARIMA (0,1,2) model

Recall that;

$$Var(Y_t) = E(Y_t^2) - E(Y_t)^2 \tag{5}$$

But $E(Y_t) = 0$ *since its a white noice process*

Thus, $Var(Y_t) = E(Y_t^2)$ (6)

From (4), ARIMA (0,1,2) model is given by

$$Y_t = \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2} \quad (7)$$

Where, $Y_t = the\ timeseries\ process,\ \varepsilon_{t-1} and\ \varepsilon_{t-2}\ are\ first\ and\ second\ lagged\ terms\ of\ the\ process\ with\ parameters,\ \theta_1\ and\ \theta_2 respectively, and\ \varepsilon_t\ is\ the\ error\ term.$

From (6), Multiply equation (5) by itself and take expectation

$$(Y_t)(Y_t) = (\varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2})\ (\varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-2}) \quad (8)$$

$$E(Y_t^2) = E\big(\varepsilon_t^2 - \theta_1\varepsilon_t\varepsilon_{t-1} - \theta_2\varepsilon_t\varepsilon_{t-2} - \theta_1\varepsilon_t\varepsilon_{t-1} + \theta_1^2\varepsilon_{t-1}^2 + \theta_1\theta_2\varepsilon_{t-1}\varepsilon_{t-2} - \theta_2\varepsilon_t\varepsilon_{t-2} + \theta_1\theta_2\varepsilon_{t-2}\varepsilon_{t-2} + \theta_2^2\varepsilon_{t-2}^2\big) \quad (9)$$

$$\delta_{Yt}^2 = \delta_{\varepsilon t}^2 + \theta_1^2\delta_{\varepsilon t}^2 + \theta_1^2\delta_{\varepsilon t}^2 \quad (10)$$

$$\delta_{Yt}^2 = \delta_{\varepsilon t}^2(1 + \theta_1^2 + \theta_2^2) \quad (11)$$

$$\delta_{Yt}^2 = \delta_{\varepsilon t}^2\left(1 + \sum_{i=1}^{2}\theta_i\right) \quad (12)$$

$$\delta_{\varepsilon t}^2 = \frac{\delta_{Yt}^2}{(1 + \sum_{i=1}^{2}\theta_i)} \quad (13)$$

Equation (13) is the error variance of ARIMA (0,1,2)

Error Variance of ARIMA (1,1,1) model

ARIMA (1,1,1) is the combination of AR(1) and MA(1) with first order of differencing.

From (4), the model is given by

$$Y_t = \psi_1 Y_{t-1} - \theta_1\varepsilon_{t-1} + \varepsilon_t \quad (14)$$

From (6), Multiply (14) by itself and take expectations

$$(Y_t)(Y_t) = (\psi_1 Y_{t-1} - \theta_1\varepsilon_{t-1} + \varepsilon_t)\ (\psi_1 Y_{t-1} - \theta_1\varepsilon_{t-1} + \varepsilon_t) \quad (15)$$

$$E[Y_t^2] = E[\psi_1^2 Y_{t-1}^2 - \psi_1\theta_1 Y_{t-1}\varepsilon_{t-1} + \varepsilon_t\,\psi_1 Y_{t-1} - \psi_1\theta_1 Y_{t-1}\varepsilon_{t-1} + \theta_1^2\varepsilon_{t-1}^2 - \varepsilon_t\,\theta_1\varepsilon_{t-1} + \varepsilon_t\,\psi_1 Y_{t-1} + \varepsilon_t\,\theta_1\varepsilon_{t-1} + \varepsilon_t^2] \quad (16)$$

$$\delta_{Yt}^2 = \psi_1^2\delta_{Yt}^2 + \theta_1^2\delta_{\varepsilon t}^2 - 2\psi_1\theta_1\delta_{\varepsilon t}^2 + \delta_{\varepsilon t}^2 \quad (17)$$

$$\delta_{Yt}^2 = \frac{\delta_{\varepsilon t}^2 \ (1+\theta_1^2 - 2\psi_1\theta_1)}{1-\psi_1^2} \quad (18)$$

$$\delta_{\varepsilon t}^2 = \frac{\gamma_0(1-\psi_1^2)}{1+\theta_1^2 - 2\psi_1\theta_1} \quad (19)$$

Equation (19) is the error variance of ARIMA (1,1,1) model.

Error Variance of ARIMA (2,1,1) model

This model is the combination of AR (2) and MA (1) with first order differencing

Thus, from (4) the model is given by

$$Y_t = \psi_1 Y_{t-1} + \psi_2 Y_{t-2} + \varepsilon_{t1} + \varepsilon_{t2} - \theta_1\varepsilon_{t-1} \quad (20)$$

Let $\varepsilon_{t1} + \varepsilon_{t2} = \alpha_t$ (21)

$$Y_t = \psi_1 Y_{t-1} + \psi_2 Y_{t-2} - \theta_1\varepsilon_{t-1} + \alpha_t \quad (22)$$

Equation (22) is the model for ARIMA (2,1,1).

Multiply (22) by itself and take expectations

$$(Y_t)(Y_t) = (\psi_1 Y_{t-1} + \psi_2 Y_{t-2} - \theta_1\varepsilon_{t-1} + \alpha_t)(\psi_1 Y_{t-1} + \psi_2 Y_{t-2} - \theta_1\varepsilon_{t-1} + \alpha_t) \quad (23)$$

$$\begin{aligned} E(Y_t^2) = E(&\psi_1^2 Y_{t-1}^2 + \psi_1\psi_2 Y_{t-1}Y_{t-2} - \psi_1\theta_1 Y_{t-1}\varepsilon_{t-1} + \alpha_t\psi_1 Y_{t-1} + \psi_1\psi_2 Y_{t-1}Y_{t-2} + \psi_2^2 Y_{t-2}^2 \\ &- \psi_2\theta_1 Y_{t-2}\varepsilon_{t-1} + \alpha_t\psi_2 Y_{t-2} - \psi_1\theta_1 Y_{t-1}\varepsilon_{t-1} - \psi_2\theta_1 Y_{t-2}\varepsilon_{t-1} + \theta_1^2\varepsilon_{t-1}^2 \\ &- \alpha_t\theta_1\varepsilon_{t-1} + \alpha_t\psi_1 Y_{t-1} + \alpha_t\psi_2 Y_{t-2} - \alpha_t\theta_1\varepsilon_{t-1} + \alpha_t^2) \end{aligned} \quad (24)$$

$$\delta_{Yt}^2 = \psi_1^2\delta_{Yt}^2 + \psi_2^2\delta_{Yt}^2 - 2\psi_1\theta_1\delta_{\varepsilon t}^2 + \theta_1^2\delta_{\varepsilon t}^2 + \delta_{\varepsilon t}^2 \quad (25)$$

$$\delta_{Yt}^2 = \frac{\delta_{\varepsilon t}^2(1+\theta_1^2 - 2\psi_1\theta_1)}{(1-\psi_1^2-\psi_2^2)} \quad (26)$$

$$\delta_{\varepsilon t}^2 = \frac{\gamma_0(1-\psi_1^2-\psi_2^2)}{(1+\theta_1^2 - 2\psi_1\theta_1)} \quad (27)$$

Equation (27) is the error variance of ARIMA (2,1,1) model.

RESULTS / FINDINGS

Error Variances of the suggested ARIMA models for Nigeria Crude Oil Price Data

For ARIMA (0,1,2)

The parameters estimate as obtained from Minitab software are $\theta_1 = -0.2254,\ \theta_2 = 0.0931, \gamma_0 = 0.003087$

Estimated model for ARIMA (0,1,2), from (7) is given by

$$\hat{Y}_t = 0.2254\varepsilon_{t-1} - 0.0931\varepsilon_{t-2} \tag{28}$$

From equation (13), the error variance of ARIMA (0,1,2) model is

$$\delta^2_{\varepsilon t} = \frac{0.003087}{1 + 0.0508 + 0.0087} = \frac{0.003087}{1.0595}$$

$$\delta^2_{\varepsilon t} = 0.0029$$

For ARIMA (1,1,1)

The parameters estimate as obtained from Minitab software are $\psi_1 = -0.2074, \theta_1 = -0.4518, \gamma_0 = 674.6495$

Estimated model for ARIMA (1,1,1), from (3.62) is given by

$$\hat{Y}_t = -0.2074Y_{t-1} + 0.4518\varepsilon_{t-1} \tag{29}$$

From equation (19), the error variance of ARIMA (1,1,1) model is

$$\delta^2_{\varepsilon t} = \frac{0.003087(1-0.0430)}{1+0.02041-0.1874} = \frac{0.0037}{1.0167}$$

$$\delta^2_{\varepsilon t} = 0.0036$$

For ARIMA (2,1,1)

The parameters estimate are; $\psi_1 = 0.6867, \psi_2 = -0.2523,\ \theta_1 = 0.4631,\quad \gamma_0 = 674.6495$ respectively.

Estimated model for ARIMA (2,1,1), from (22) is given by

$$\hat{Y}_t = 0.6867Y_{t-1} - 0.2523Y_{t-2} + 0.4631\varepsilon_{t-1} \tag{30}$$

From (27), the error variance of ARIMA (2,1,1) model is given by

$$\delta^2_{\varepsilon t} = \frac{0.003087(1 - 0.4716 - 0.0636)}{1 + 0.2145 - 0.6360} = \frac{0.0014}{0.5785}$$

$$\delta_{\varepsilon t}^2 = 0.0024$$

DISCUSSION OF THE ANALYSIS RESULT

Figure (1) shows the time series plot of the original process. The series which were not originally stationary were transformed by taking a natural log of the series and thereafter the first differenced the series becomes stationary as showed in figure (2). The ACF and PACF of the stationary process were plotted as presented in figure (3) and (4) respectively and were used to suggest ARIMA (0,1,2), ARIMA (1,1,1) and ARIMA (2,1,1) models respectively. Error variances of the three suggested models were estimated using the derived equations in (13),(19) and (27) to be 0.0029 for ARIMA (0,1,2), 0.0036 for ARIMA (1,1,1) and 0.0024 for ARIMA (2,1,1) respectively. The estimated model for each suggested ARIMA models were presented in (28), (29) and (30) for forcasting of the future value of the crude oil price. Comparing the efficiency of the the models using the error variance, ARIMA (2,1,1) model having the least value of the error variance outperformed the other two suggested models and is considered the best model.

CONCLUSION

This work uses error variances to compare the performance of the suggested ARIMA (p,d,q) models for Nigeria Crude Oil Price data spanning from January 2006- July 2023. The result from the findings revealed that ARIMA(2,1,1) having the smallest error variance is the best model for modelling Nigeria Crude Oil Price. It is recommended that error variance should be consider as one of the best criterion for sellecting the best model when modelling any time series data.

REFERENCES

[1] Adamu, I., Ogbonna, C.J., Adamu, Y., & Zakari, Y. (2021). "Modelling Coronavirus Pandemic Using Univariate and Multivariate Models". The Nigerian Perspective. *Asian Journal of Probability and Statistics*, 15(4), pp.134-143

[2] Ayadi, O.F. (2005) "Oil Price Fluctuations and the Nigerian Economy". OPEC Review: pp.199-217.

[3] Box, G. E. P., and Jenkins, G. M. (1976), Time Series Analysis: Forecasting and Control, revised edition, San Francisco: Holden Day.

[4] Central Bank of Nigeria, CBN (2014)): "The Structure of the Nigerian Economy and Implications for Development", Realm Communication Ltd, Lagos.

[5] Chika, I. (2014) US Shut Its Door on Nigeria's Oil Exports. http://grandmotherafrica.com/u-s-shuts-its-door-on- nigerias-oil-exports/

[6] Desmond Chekwube Bartholomew, Ukamaka Cynthia Orumie, Chukwudi Paul Obite, Blessing Iheoma Duru, Felix Chikereuba Akanno (2021), "Modeling the Nigerian Bonny Light Crude Oil Price: The Power of Fuzzy Time Series" *Open Journal of Modelling and Simulation, 9, pp.370-390*

[7] Doris Dokua Sasu (2023) Contribution of oil sector to GDP in Nigeria 2018-2023

[8] EIA (1997) Country Analysis Brief: Nigeria, Independent Statistics and Analysis, U.S. Energy Information Administration.

[9] Ellwanger, R., Sawatzky, B. and Zmitrowicz, K. (2017). Factors behind the 2014 oil price decline.

[10] Frynas, J.G. (1999) Oil in Nigeria: Conflict and Litigation between Oil Companies and Village Communities. Lit Verlag, Munster.

[11] Huntington, H. G. (2014). Oil Price Forecasting in the 1980s: What Went Wrong? *The Energy Journal, 15 (2), pp.1-22*.

[12] Maugeri, L. (2009). Understanding oil price behavior through an analysis of a crisis. Review of Environmental Economics and Policy, 3(2), pp.147-166.

[13] Mujtaba Suleiman, Isah Muhammad, Adamu Zakar Adamu, Yahaya Zakari, Rufai Ali Muhammad, Ibrahim Adamu and Mannir Abdu (2023) "Modelling Nigeria Crude Oil Prices using ARIMA Time Series Models" NIPES *Journal of Science and Technology Research* 5(1), pp.230-241, ISSN-2682-582

[14] Nwanne, N.C. and Eze, O.R. (2019) "Effect of Government Oil Revenue on Agricultural Sector Growth: Evidence from Nigerian Economy."

[15] Ramat, M. H., Zakari, Y., Usman, M., Muhammad, I., Falgore, J.Y. & Dikko, H. G. (2022). "The Effect of Exchange Rate on Gross Domestic Product (GDP) on the Nigerian Economy using ARDL-ECM approach". *International journal of Communication in Physical Sciences, 8(2): pp.211-221.*

[17] Suleiman, S., Alabi, M. A., Issa, Suleman, U. Usman, Umar Adamu (2015) "Modelling and Forecasting the Crude Oil Price in Nigeria "*International Journal of Novel Research in Marketing Management and Economics* Vol. 2, Issue 1, pp:1-13.

[18] Usoro A. E (2020). On the stationarity of multivariate time series analysis. Benin Journal of Statistics.Vol.3. pp160-169.

[19] World Bank (2013), "Nigeria Economic Report". No. 1, World Bank, Washington DC.

Appendix

Monthly data of Nigeria Crude Oil Price from Jan.2006 – July.2023

	Year								
2006	2007	2008	2009	2010	2011	2012	2013	2014	2015
63.85	55.57	94.26	44.95	77.62	97.96	113.81	115.24	110.19	48.81
61.33	59.97	98.15	46.52	75.06	106.57	121.87	118.81	110.83	58.09
65.00	64.28	103.73	49.70	80.27	116.56	128.00	112.79	109.47	56.69
72.09	70.46	116.73	51.16	85.29	124.49	122.62	105.55	110.41	57.45
71.18	70.40	126.57	60.02	77.54	118.43	113.08	106.00	111.90	65.08
69.32	73.28	138.74	72.24	75.79	117.03	98.06	106.06	114.60	62.06
75.13	79.76	137.74	66.52	77.18	117.86	104.62	109.78	109.63	57.01
75.15	73.76	115.84	74.00	78.67	111.99	113.76	107.84	102.33	47.09
62.97	79.76	103.82	70.22	79.45	115.73	114.36	113.59	98.27	48.08
59.49	83.86	75.31	78.25	84.42	113.12	108.92	112.29	83.50	48.86
59.81	95.05	55.51	78.11	86.71	113.92	111.05	111.14	80.42	44.82
64.70	93.40	45.87	75.11	92.79	111.46	114.49	112.75	63.28	37.80
	Year								
2016	2017	2018	2019	2020	2021	2022	2023		
30.66	55.01	69.68	60.39	66.68	54.87	88.71	84.78		
31.70	46.39	66.67	64.89	58.45	62.48	99.64	86.04		
37.76	52.13	74.72	67.67	32.29	65.62	121.23	81.10		
41.59	52.94	72.37	73.08	14.28	64.30	106.51	76.91		
47.01	50.57	77.64	73.65	27.90	67.83	116.72	76.97		
48.46	47.42	75.38	66.74	40.30	73.46	130.1	82.27		
45.25	49.01	74.72	66.24	44.10	75.93	120.54	89.30		
46.15	51.64	73.35	61.05	45.06	70.72	106.34			
47.43	56.79	79.59	65.27	40.85	74.55	93.25			
50.94	58.46	79.18	59.10	39.74	84.11	96.57			
45.25	63.56	66.59	63.56	42.70	82.16	93.36			
53.48	65.11	62.00	68.56	50.33	65.41	82.5			