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System of Non-Linear Stochastic Differential Equations with Financial Market Quantities

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ABSTRACT: *In this paper, two systems of modified stochastic differential equations were considered. The variable coefficient problem was solved using Ito's theorem to obtain an analytical solutions which was used to generate various behaviors of asset values which shows as follows: (i) increase in α when μ and σ are fixed increases the value of asset returns. (ii) a little increase on time when return rates and stock volatility are fixed increases the value of assets.(iii) an increase in the volatility parameter increases the value of asset pricing and α parameter shows the various levels of long term investment plans, (iv) increase in rate of mean-reversion parameter reduces the value of asset. (v) An increase in the volatility parameter decreases the value of asset pricing (vi) The goodness of fit probability QQplots are not statistically significant and besides do come from a common distribution which has a vital meaning in the assessment of asset values for capital market investments. Nevertheless, the Tables 1,2 and 3 are best in comparisons with Tables 4,5 and 6 in terms of predictions for capital investments. The governing investment equations are unique and therefore are found to be satisfactory.*

KEY WORDS: asset value, normality test, financial market and stochastic analysis.

INTRODUCTION

Typically, investments are economic enterprises linked up to risk. Investors take risk for the survival of investments. Therefore, risk plays a pivotal role in the efficient administration of investments' portfolios because it enhances the determination of changes in stock returns and portfolios. This helps the investor with a mathematical basis for investment decisions [1] Bonds, stocks, property, etc, are all examples of the risk linked to a security.

However, as a result of the risk involved in the management of investments' portfolios, the idea of the insurance of lives and properties, etc, is introduced by insurance companies. In reality, insurance companies share third party in their responsibility of financial outcomes or results. Risk transfer or risk sharing is the procedure where by insurance company on financial out of its coverage duty in many ways with risk transfer agreement, risk among various insurance companies globally will be divided. Hence, in the phenomenon of huge lose from financial situation as insurance companies will not face risk, explicitly, reinsurance refers to division and distribution of risk. In general, risk is a prevalent factor inasmuch as humans are concerned, since we adequately secure risky or riskless assets. The best way of modelling these factors is as the trajectory or path of a diffusion process defined on basic probability space, with the geometric Brownian motion, used as the established reference model [2]. Modelling financial concepts cannot be overstated, this is as a result of its several applications in science and technology.

For instance, [3] considered the maximization of the exponential utility and the minimization of the ruin probability and the results demonstrated the same or kind of investments scheme or approach for zero interest rate [4] Considered an optimal reinsurance and investment problem for insurer with jump diffusion risk process. [5] studied the risk reserved for an insurer and a reinsurer to follow Brownian motion with drift and applied optimal probability of survival problem under proportional reinsurance and power utility preference. Similarly, [7] considered the excess loss of reinsurance and investment in a financial market and obtained optimal strategies. [8] engaged a problem of optimal reinsurance investment for an insurer having jump diffusion risk model when the asset price was control by a CEV model. [9] studied strategies of optimal reinsurance and investment for exponential utility maximization under different capital markets. [10] examined investment problem with multiple risky assets. [1] examined an optimal portfolio selection model for risky assets established on asymptotic power law behaviour where security prices follow a Weibull distribution. The research of [11] assessed the stability of stochastic model of price fluctuation on the floor of the stock market, where precise steps were derived, which aided the determination of the equilibrium price and growth rate of stock shares. [12] studied the unstable property of stock market forces, making use of proposed differential equation model. [13] did a stochastic analysis of stock prices and their characteristics and obtained results which showed efficiency in the use of the proposed model for the prediction of stock prices. Similarly, [14] considered the stochastic formulations of some selected stocks in the Nigerian Stock Exchange (NSE), and the drift and volatility measures or quantities for the stochastic differential equations were obtained and the Euler-Maruyama technique for system of SDEs was applied in the stimulation of the stock prices. [8] produced the geometric Brownian motion and assessment of the correctness or exactness of the model, using detailed analysis of stimulated data. Furthermore, [15] looked at stochastic problem of unstable stock market prices obtained conditions for determining the equilibrium price, required and adequate conditions for dynamic stability and convergence to equilibrium of the growth rate of the valued function of stocks. All the same, [16] looked at a stochastic problem of unstable prices at the floor of the stock market. From their evaluation, the equilibrium price and the market growth rate of shares were found out. A lots of scholars has done extensively on stock prices such as [18-19] and [21-22] etc.

Earlier studies have therefore investigated similar problems but did not consider the effects of some stochastic variables in assessing asset values. In particular, some studies, for instance [14] ,[15] and [20] etc.

In this study we considered system of Stochastic Differential Equations (SDEs) with some key stock market quantities. Apart from correctly posing the models for the assessment of asset values, we also solved in details by the method of Ito's to obtain two different solutions which gave various behaviors of asset values. Goodness -of -fit test of Quantile-Quantile (QQ) plot comparisons was used to identify a class of probability distributions, the random process to show if the asset values comes from a common distributions. This paper extends the work of [20] by in corpora ting more stock quantities in the assessment of asset values. To this end, our novel contribution is unique in this area of mathematical finance.
This paper is set as follows: Section 2.1 presents the Mathematical framwork, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

MATHEMATICAL FRAMWORK

For proper understanding of this paper we therefore present few intricate definitions covering this dynamic area of study:
Definition 1: Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set T, $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.
Definition 2: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).
Definition 3: Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k \qquad (1)$$

where X_k are independent and identically distributed random variables.
where X_k are independent and identically distributed random variables.
Definition 4: (Differential Equation): is an equation which has functions and their derivatives. In reality the functions is associated to real quantities whereas the derivatives denotes rate of change. Example of differential equation is follows

$$\frac{dS(t)}{dt} = \mu S(t) \qquad (2)$$

$$S(0) = S_0 \tag{3}$$

where $S(t)$ represent asset price, μ rate of return, $\frac{dS(t)}{dt}$ is the rate of change of asset price and S_0 is the initial stock price; (2) and (3) can be obtained using variable separable which gives:

$$S(t) = S_0 e^{\mu t} \tag{4}$$

Therefore μ is not known completely which is subject to environmental effects. Therefore (2) can be written as

$$dS(t) = \mu S(t)\,\mathrm{dt} + \sigma S(t)\,\mathrm{dz(t)} \tag{5}$$

Where σ is the volatility, dZ is the Brownian motion or Wieners process which is random term, the stochastic term added to (2) gives (5) which makes it stochastic differential equation

Definition 5: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), ..., W_m(t))^T, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T,$$

$$X(0) = x_0,$$

where $T > 0$, x_0 is an n-dimensional random variable and coefficient functions are in the form $f : [0, \mathrm{T}] \times \mathbb{R}^n$ *and* $g : [0, T] \times \mathbb{R}^n \to \times \mathbb{R}^{n \times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S)$$

Where dX, dZ are terms known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$,[22].

Theorem 1.1:(Ito's lemma). Let $f(S,t)$ be a twice continuous differential function on $[0, \infty) \times \mathrm{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0\ ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2} (dS_t)^2 + \text{higer order terms } (h.\mathrm{o,t})\ ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$dF_t = \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2} \frac{\partial^2 F}{\partial S_t^2}(a_t dt + bdz(t))^2$$

$$= \frac{\partial F}{\partial S_t}\left(a_t dt + bdz(t)\right) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2 dt,$$

$$= \left(\frac{\partial F}{\partial S_t}a_t + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}b_t^2\right)dt + \frac{\partial F}{\partial S_t}b_t dz(t)$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies and hence, the function $F(S,t)$,Ito's lemma gives:

$$dF = \left(\mu S\frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 F}{\partial S^2}\right)dt + \sigma S\frac{\partial F}{\partial S}dz(t)$$

FORMULATION OF THE PROBLEM

Stochastic quantities or variables are considered in the assessment of asset values .Assuming α *and* β measures returns and stock volatility. Also we assume small positive parameter ε which denotes the inverse of rate of mean-reversion of stock returns [23]; thereby incorporating the following stock quantity $\frac{v\sqrt{2}}{\sqrt{\varepsilon}}$ to measure stock volatility level of asset values with a finite time investment horizon $T > 0$. Therefore, we have the following system of stochastic differential equations below:

$$dX_1(t) = \alpha\mu X_1(t)dt + \beta\sigma X_1(t)dZ^{(1)}(\mathrm{t}) \tag{6}$$

$$dX_2(t) = \frac{1}{\varepsilon}\mu X_2(t)dt + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma X_2(t)dZ^{(1)}(\mathrm{t}) \tag{7}$$

with the following initial conditions:

$$\left.\begin{aligned} X_1(0) = X_0, t > 0 \\ X_2(0) = X_0, t > 0 \end{aligned}\right\} \qquad (7)(8)$$

where, $X_1(t)$ *and* $X_2(t)$ are asset prices, The expression dz , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion where μ is an expected rate of returns on stock, α measures the rate at which the drift (return) reaches its long term investments plans, β is a constant, σ is the volatility of the stock , dt is the relative change in the price during the period of time, v is volatility of volatility while ε denotes the inverse of rate of mean-reversion of stock returns.

METHOD OF SOLUTION

The model (6) –(7) is stochastic differential equations whose solutions are not trivial. We implement the methods of Ito's lemma in solving for $X_1(t)$ *and* $X_2(t)$ To grab this problem we note that we can forecast the future worth of the asset with sureness.

From (6) Let $f(X_1,t)=\ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_1}=\frac{1}{X_1},\ \frac{\partial^2 f}{\partial X_1^2}=-\frac{-1}{X_1^2},\ \frac{\partial f}{\partial t}=0 \tag{9}$$

According to Ito's gives

$$df(X_1,t)=\sigma X_1\frac{\partial f}{\partial X_1}dZ(t)+\left((\mu X_1(t)\frac{\partial f}{\partial X_1}+\frac{1}{2}\sigma^2X_1^2\frac{\partial^2 f}{\partial X_1^2}+\frac{\partial f}{\partial t}\right)dt \tag{10}$$

Substituting (6) and (8) into (10) gives

$$=\beta\sigma X_1\frac{1}{X_1}dZ(t)+\left((\alpha\mu X_1(t)\frac{1}{X_1}+\frac{1}{2}\sigma^2X_1^2(-\frac{1}{X_1^2})+0\right)dt$$

$$=\beta\sigma dZ(t)+\left((\alpha\mu-\frac{1}{2}\sigma^2\right)dt$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_1=\int_0^t df(X_u,u)=\int\left((\alpha\mu-\frac{1}{2}\sigma^2\right)du+\beta\int_0^t\sigma dZ(t)$$

$$\ln X_1-\ln X_0=\left((\alpha\mu u-\frac{1}{2}\sigma^2u\right)|_0^t+\beta(\sigma Zu)|_0^t \tag{11}$$

$$\ln\left(\frac{X_1}{X_0}\right)=\left((\alpha\mu-\frac{1}{2}\sigma^2\right)t+\beta\sigma Z(t)$$

Taking the ln of the both sides and applying the initial condition in (8) gives:

$$X_1(t)=X_0\exp\left(\alpha\mu-\frac{1}{2}\sigma^2\right)t+\beta\sigma Z(t) \tag{12}$$

From (7) Let $f(X_2,t)=\ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_2}=\frac{1}{X_2},\ \frac{\partial^2 f}{\partial X_2^2}=-\frac{-1}{X_2^2},\ \frac{\partial f}{\partial t}=0 \tag{13}$$

According to Ito's gives

$$df(X_2,t)=\sigma X_2\frac{\partial f}{\partial X_2}dZ(t)+\left((\mu X_2(t)\frac{\partial f}{\partial X_2}+\frac{1}{2}\sigma^2X_2^2\frac{\partial^2 f}{\partial X_2^2}+\frac{\partial f}{\partial t}\right)dt \tag{14}$$

Substituting (7) and (9) into (14) gives

$$= \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma X_2 \frac{1}{X_2} dZ(t) + \frac{1}{\varepsilon}\left(\mu X_2(t)\frac{1}{X_2} + \frac{1}{2}\sigma^2 X_2^2\left(-\frac{1}{X_2^2}\right) + 0\right)dt$$

$$= \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma dZ(t) + \frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)dt$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_2 = \int_0^t df\left(X_2, u\right) = \int \frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)du + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\int_0^t \sigma dZ(t)$$

$$\ln X_2 - \ln X_0 = \frac{1}{\varepsilon}\left((\mu u - \frac{1}{2}\sigma^2 u\right)\Big|_0^t + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\left(\sigma Z u\right)\Big|_0^t \qquad (15)$$

$$\ln\left(\frac{X_2}{X_0}\right) = \frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)t + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma Z(t)$$

Taking the ln of the both sides and applying the initial condition in (8) gives:

$$X_2(t) = X_0 \exp\frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)t + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma Z(t) \qquad (16)$$

3. 1 Results and Discussion

This Section presents result whose solutions are in (12) and (16), Hence we have the following:

$X_1\left(t\right) = X_0 \exp\left(\alpha\mu - \frac{1}{2}\sigma^2\right)t + \beta\sigma dz\left(t\right)$, where, $dz = 1, t = 1$ and $\beta = 0.25$

Table 1: The effects of α on the assessment of asset values through stock returns.

Initial stock prices (X_0)	Returns (μ)	α	Volatility (σ)	*Asset values* $X_1\left(t\right)$
5.00	0.5	0.1	0.2	5.2023
	0.5	0.2	0.2	5.4664
	0.5	0.3	0.2	5.7441
	0.5	0.4	0.2	6.0361
6.00	0.9	0.5	0.2	9.2736
	0.9	0.6	0.2	10.1423
	0.9	0.7	0.2	11.0926
	0.9	0.8	0.2	12.1325
5.20	0.95	0.25	0.2	6.5134
	0.95	0.35	0.2	7.1576
	0.95	0.45	0.2	7.8659
	0.95	0.55	0.2	8.6448

Table 1 labels the impact of α on the assessment of asset values. It is understandable that increase in α when μ *and* σ are fixed increases the value of asset returns. This also shows the various levels of long term investment plans. The changes of asset values are very instructive to investors.

Table 2: The effects of time t influences over asset values changes .

$$X_1(t) = X_0 \exp\left(\alpha\mu - \frac{1}{2}\sigma^2\right)t + \beta\sigma dz(t), \text{where, } dz = 1 \text{ and } \beta = 2$$

	Time (t)	Returns (μ)	α	Volatility (σ)	*Asset values* $X_1(t)$
5.00	0	0.5	0.1	0.2	0.4
	1	0.5	0.2	0.2	5.8164
	2	0.5	0.3	0.2	11.7883
	3	0.5	0.4	0.2	18.3583
6.00	0	0.9	0.5	0.2	0.4
	1	0.9	0.6	0.2	10.4922
	2	0.9	0.7	0.2	22.4852
	3	0.9	0.8	0.2	36.6476
5.70	0	0.95	0.25	0.2	0.4
	1	0.95	0.35	0.2	8.1910
	2	0.95	0.45	0.2	17.5348
	3	0.95	0.55	0.2	28.6637

Noticeably it can be seen the impacts of time on asset values; an increase on time when return rates and stock volatility are fixed: 0.1-0.4, 0.5-0.8,0.25-0.55 and 0.2 increases the value of assets. This implies that time is a crucial factor on asset values; lots of assets appreciates and depreciates in value over time.

Table 3: The effects of stock volatility (σ) on the assessment of asset values changes

$$X_1(t) = X_0 \exp\left(\alpha\mu - \frac{1}{2}\sigma^2\right)t + \beta\sigma dz(t), \text{ where, } dz = 1, t = 1 \text{ and } \beta = 2$$

Initial stock prices (X_0)	Returns (μ)	α	Volatility (σ)	*Asset values* $X_1(t)$
5.00	0.5	0.1	0.1	5.4301
	0.5	0.1	0.2	5.5523
	0.5	0.1	0.3	5.6251
	0.5	0.1	0.4	5.6522
6.00	0.9	0.2	0.5	7.3392
	0.9	0.2	0.6	7.2
	0.9	0.2	0.7	7.0224
	0.9	0.2	0.8	6.8162
5.70	1.0	0.3	0.91	6.9056
	1.0	0.3	0.92	6.8793
	1.0	0.3	0.93	6.8529
	1.0	0.3	0.94	6.8264

Table 3 represents the various levels of stock volatility on the assessment of asset values. An increase in the volatility parameter increases the value of asset pricing. This is substantially dependable because volatility causes significant changes on the price history of asset over time. It reads the degree of price changes in the life of asset values
.

Table 4: The effects of ε on the assessment of asset values through stock returns

$$X_2(t) = X_0 \exp\frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)t + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma dz(t), \text{where, } dz = 1, t = 1 \text{ and v=0.3}$$

Initial stock prices (X_0)	Re*turns* (μ)	ε	Volatility (σ)	*Asset values* $X_2(t)$
5.00	0.5	0.1	0.2	607.8204
	0.5	0.2	0.2	55.3056
	0.5	0.3	0.2	24.9201
	0.5	0.4	0.2	16.7348
6.00	0.9	0.5	0.2	34.9946
	0.9	0.6	0.2	26.1181
	0.9	0.7	0.2	21.1936
	0.9	0.8	0.2	18.1199
5.70	0.95	0.25	0.2	235.3768
	0.95	0.35	0.2	81.3998
	0.95	0.45	0.2	45.1477
	0.95	0.55	0.2	31.0336

Table 4 showed the effect of rate of mean-reversion of returns over trading days. Clearly , increase in rate of mean-reversion parameter reduces the value of asset. This is obvious, because it used as part of a statistical analysis of market conditions and can be part of an overall trading strategy; which applies well to the ideas of buying low and selling high, by hoping to identify abnormal activity that will, theoretically revert to a normal pattern.

Table 5: The effects of $Time(t)$ on the assessment of asset values .

$$X_2(t) = X_0 \exp\frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)t + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma dz(t), \text{ where, } dz = 1, \varepsilon = 25 \text{ and v=0.25}$$

Initial stock prices (X_0)	Time (t)	Re*turns* (μ)	Volatility(σ)	*Asset values* $X_2(t)$
5.00	0	0.5	0.2	0.0141
	1	0.5	0.2	5.1111
	2	0.5	0.2	10.2080
	3	0.5	0.2	15.3049
6.00	0	0.9	0.2	0.0141
	1	0.9	0.2	6.2291
	2	0.9	0.2	12.4441
	3	0.9	0.2	18.6590

5.70	0	0.95	0.2	0.0141
	1	0.95	0.2	5.9302
	2	0.95	0.2	11.8462
	3	0.95	0.2	17.7622

Undoubtedly it can be seen the influences of time on asset values; an increase on time when return rates and stock volatility are fixed: 0.5- 0.95-0 and 0.2 increases the value of assets. This suggests that time is a vital factor on asset values; lots of assets appreciates and depreciates in value over time. This informs investors on how to effectively manage their investments; see Table 5.

Table 6: The effects of $Volatility(\sigma)$ **on the assessment of asset values** .

$X_2(t) = X_0 \exp \frac{1}{\varepsilon}\left(\mu - \frac{1}{2}\sigma^2\right)t + \frac{v\sqrt{2}}{\sqrt{\varepsilon}}\sigma dz(t)$, where, $dz = 1, t = 1$ and v=0.25

Initial stock prices (X_0)	Re*turns* (μ)	ε	Volatility(σ)	*Asset values* $X_2(t)$
5.00	0.5	0.1	0.1	705.9866
	0.5	0.1	0.2	607.7757
	0.5	0.1	0.3	473.4975
	0.5	0.1	0.4	333.8789
6.00	0.9	0.2	0.5	289.4915
	0.9	0.2	0.6	220.0638
	0.9	0.2	0.7	159.2128
	0.9	0.2	0.8	109.6773
5.70	1.0	0.3	0.91	40.7780
	1.0	0.3	0.92	39.5771
	1.0	0.3	0.93	38.4000
	1.0	0.3	0.94	37.2465

Table 6 describes the several levels of stock volatility on the valuation of asset prices. An increase in the volatility parameter decreases the value of asset pricing. This is really reliable because volatility causes significant changes on the price history of asset over time. It shows levels of changes in asset prices.

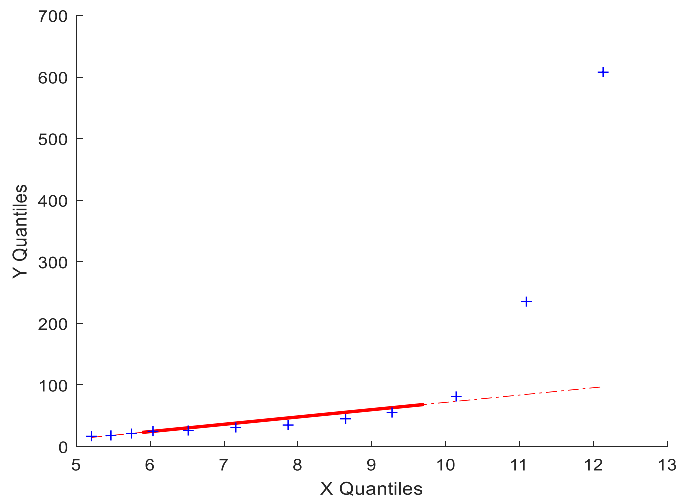

Figure 1: Quantile-Quantile (QQ) plot comparisons on two asset values of Tables 1 and 4

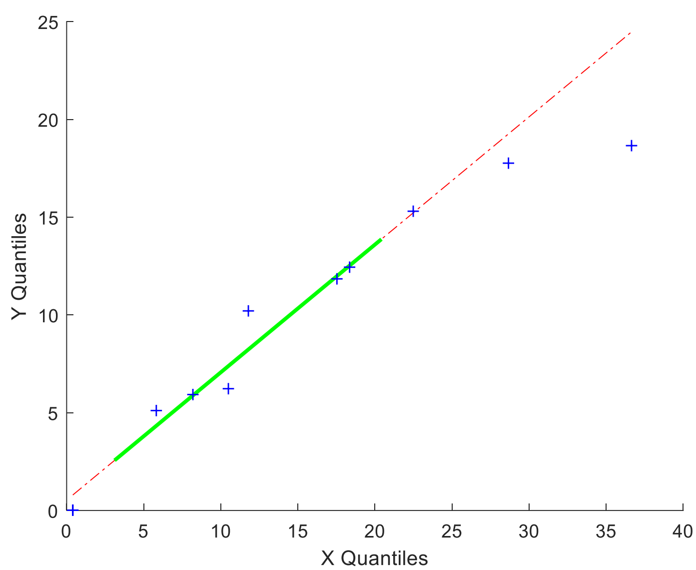

Figure 2: Quantile-Quantile (QQ) plot comparisons on two asset values of Tables 2 and 5

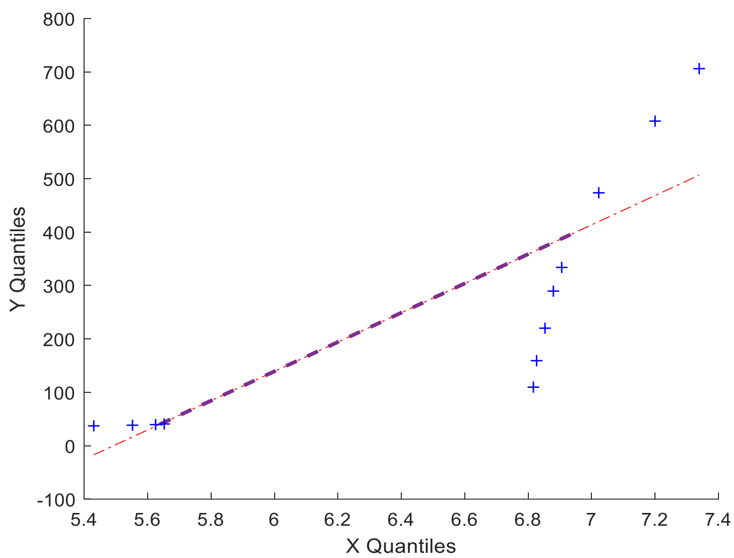

Figure 3: Quantile-Quantile (QQ) plot comparisons on two asset values of Tables 3 and 6

It can be seen in Figures 1, 2 and 3 QQ plot indicates that the two of Tables :1 and 4, 2 and 5, 3 and 6 do not comes from a common distributions. They are statistically in-significant, not correlated and have different levels of financial remunerations hence it trades around secular trend in capital market investments.

CONCLUSION

This study, considered two modified system of stochastic differential equations which showed discrepancies on various stochastic quantities over asset prices. The asset values were obtained which shows different behavior of asset values: (i) increase in α when μ *and* σ are fixed increases the value of asset returns. (ii) a little increase on time when return rates and stock volatility are fixed increases the value of assets.(iii) an increase in the volatility parameter increases the value of asset pricing and α parameter shows the various levels of long term investment plans, (iv) increase in rate of mean-reversion parameter reduces the value of asset. (v) An increase in the volatility parameter decreases the value of asset pricing (vi) The goodness of fit probability QQplots are not statistically significant and besides do come from a common distributions which has a vital meaning in the assessment of asset values for capital market investments.

In all, the Tables 1,2 and 3 are best in comparisons with Tables 4,5 and 6 in terms of predictions for capital investments. We shall be looking at the numerical approximations of the two proposed systems in the next study.

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