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The Quantized Constants with Remmen's Scattering Amplitude to Explain Riemann Zeta Zeros

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ABSTRACT: *Riemann Hypothesis has been proposed by Bernhard Riemann since year 1859. Nowadays, there are lots of proof or disproof all over the internet society or the academic professional authority etc. However, none of them is accepted by the Clay's Mathematics Institute for her Millennium Prize. In the past few months, this author discovered that there may be a correlation exists between the real and imaginary parts of Riemann Zeta function for the first 10 non-trivial zeros of the Riemann function etc. Indeed, when one tries to view the correlation relationship as a constant like the Planck's one. Then we may show that Riemann Zeta zeros are indeed discrete quantum energy levels or the discrete spectrum as electrons falling from some bound quantum state to a lower energy state (or Quantum Field Theory). That may be further explained by Remmen's scattering amplitude or the S-matrix. We may approximate the S-matrix by applying the HKLam theory to it and predict the scattering amplitude or even the Riemann Zeta non-trival zeros etc.*
By the way, the key researching equations or formula in the following content will be around the Taylor expansion of the Riemann Zeta function, their convergence etc. In additional, I will also investigate the ('') as shown below:*

$$\prod_{i=1}^{\infty}(z - z_i) = \xi(0.5 + i*t) = \sum_{n=1}^{\infty} 1/n^{(0.5+i*t)} = \prod_{j=1}^{\infty}\left(1 - 1/p_j^{(0.5+i*t)}\right)^{-1} \quad \text{------------} \; (*'')$$

as we may find the existence of some constants like the Planck's one.
For the application of the aforementioned scholarly outcome, it is well-known that if one can find the pattern of the appearance to the prime number and hence break the public key cryptography in the everyday usage of information technology security etc.
KEYWORDS: quantized constants, Remmen's scattering amplitude, Riemann Zeta Zeros

INTRODUCTION

Riemann Hypothesis was established by the Bernhard Riemann in the year 1859. Over the years, many mathematicians do their best to solve it and hope to win one of the Millennium Prize offered by the Clay's Mathematics Institute. However, until recently, there is still no news about who has gotten the prize although lots of proof or disproof may appear. This author with the help of the newest programming software — Mathematica, U.S.A. has already done some works in the topic and may present those results in this paper. However, before I try to write the outcomes formally, it is necessary to first verify whether all of the non-trivial zeros located on the Riemann Control Strip by the same software together with some of the complex analysis theorems etc. With reference to my another twined paper [1] in complex analysis topic for the Riemann conjecture, this author propose the following algorithm to compute (or prove) those non-trivial zeros for the Riemann zeta function:

1. Compute the contour of inverse zeta function over a closed path (without singularity) [4];
2. The resulted complex value NOT equals to zero implies the existence of zeta roots;
3. By applying the Argument Principle and (/ or) the Rouche's Theorem to the function, we may find the number of zeros (and poles) inside the closed contour region if the function is holomorphic [4].
4. If furthermore, one may zoom out the convergence radius (or conceptually the vice versa of the analytic continuation extending), one may gradually approach those singularities (both poles and zeros) by substituting circumference's numerical values (from 0 to 2 pi) of the convergent circle into (the integral of) the function to get either infinity answer (implies a pole) or zero answer (implies a root) through suitable programming code;
5. The function may then be expressed in the form of Euler product with both poles and zeros; Finally, we may then extend the squared closed path by analytic continuation (& regularization) from the whole zeta critical strip and repeat step 1 to 5 until filling the whole complex plane.

In the following background section, I will first review how historically correlation may be related to the distribution of primes. In the result section, I will present what are my main outcomes getting from the software Mathematica, U.S.A. In the discussion section, I will show how does the idea of the correlation that I have mentioned in the abstract section originate and how its results may relate to the computation of the suggested Jacobin matrix for the transformation etc. Last but not least, I will include some applications of the present discovery in the private key cryptography and hence the information technology security etc.

Some Computational Analytic Number Theory

Outcomes (Proof & Derivation)

In this section, I will try to use the software Maple (Soft, ver2022) to compute the Taylor approximation of the Riemann Zeta function. When the above computer calculated result works together with the outcome from the youtube video [1], we may get a (quantum) Fourier transform or the (quantum) harmonic oscillator for the phase shift of the harmonic function (-- equation v) while the inverse of the Fourier transform (to such equation -- v) may imply some quantum state circuits or the possibility of quantum computing etc. Practically, there were lots of academic scriptures in computational algebraic number theory [9], [10] but a little or hardly find one in the computational analytic number theory [11], this author wants to act as a trailblazer or just blaze a trail in such mathematical researching area.

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The following are some of my computational analytic number theory results:

1. To find the real part of the non-trivial zeros of the Riemann Zeta function, we just start by applying the equation to Maple (meaning – using Taylor expansion series for the function $\frac{1}{x^s}$ where s = u + v*i at the point x = k with k = 1 to infinity and $u, v \in \mathbb{R}$:

$$g := taylor\left(\frac{1}{x^{u+vi}}, x = k\right)$$

It will give you a Taylor series expansion like the following:

$$g_1 = \frac{1}{e^{[(u+v*i)*ln(k)]}} - \frac{u+v*i}{k*e^{[(u+v*i)*ln(k)]}} *(x-k) + \frac{\left(\frac{(u+v*i)^2}{k^2}\right)-\left(\frac{i^2*v^2+2*i*u*v-v*i+u^2-u}{2*k^2}\right)}{e^{[(u+v*i)*ln(k)]}} * (x-k)^2 + \frac{1}{e^{[(u+v*i)*ln(k)]}} [\left(\frac{(i^2*v^2+2*i*u*v-v*i+u^2-u)*(u+v*i)}{2*k^3}\right) - \left(\frac{(i^3*v^3+3*i^2*u*v^2-3*i*u^2*v-6*i*u*v+u^3+2*v*i-3*u^2+2*u)}{6*k^3}\right) - \left(\frac{(i^2*v^2+2*i*u*v+v*i+u^2)*(u+vi)}{2*k^3}\right)]* (x-k)^3 + \frac{1}{e^{[(u+v*i)*ln(k)]}} [-\frac{i^4v^4+4i^3uv^3-6i^3v^3+6i^2u^2v^2-18i^2uv^2+4iu^3v+11i^2v^2-18iu^2v+u^4+22iuv-6u^3-6vi+11u^2-6u}{24k^4} + \frac{(i^3v^3+3i^2uv^2-3i^2v^2+3iu^2v-6iuv+u^3+2vi-3u^2+2u)(vi+u)}{6k^4} - \frac{(i^2v^2+2iuv+vi+u^2+u)(i^2v^2+2iuv-vi+u^2-u)}{4k^4} + \frac{(i^3v^3+3i^2uv^2+3i^2v^2+3iu^2v+6iuv+u^3+2vi+3u^2+2u)(vi+u)}{6k^4}]*(x-k)^4 + \frac{1}{e^{[(u+v*i)*ln(k)]}} [-\frac{i^5v^5+5i^4uv^4-10i^4v^4+10i^3u^2v^3-40i^3uv^3+10i^2u^3v^2+35i^3v^3-60i^2u^2v^2+5iu^4v+105i^2uv^2-40iu^3v+u^5-50i^2v^2+105iu^2v-10u^4-10}{120k^5} + \frac{(i^4v^4+4i^3uv^3-6i^3v^3+6i^2u^2v^2-18i^2uv^2+4iu^3v+11i^2v^2-18iu^2v+u^4+22iuv-6u^3-6vi+11u^2-6u)(vi+u)}{24k^5} - \frac{(i^2v^2+2iuv+vi+u^2+u)(i^3v^3+3i^2uv^2-3i^2v^2+3iu^2v-6iuv+u^3+2vi-3u^2+2u)}{12k^5} + \frac{(i^3v^3+3i^2uv^2+3i^2v^2+3iu^2v+6iuv+u^3+2vi+3u^2+2u)(i^2v^2+2iuv-vi+u^2-u)}{12k^5} - \frac{(i^4v^4+4i^3uv^3+6i^3v^3+6i^2u^2v^2+18i^2uv^2+4iu^3v+11i^2v^2+18iu^2v+u^4+22iuv+6u^3+6vi+11u^2+6u)(vi+u)}{24k^5}]* (x-k)^5$$

+ $O((x-k)^6)$ ----------------------- equation (*) where in this case x NOT equals to k as by direct substitution of x = k may eliminate all terms with $(x-k)^n$. Thus, in such case, we may need to approach such substitution by x = k $\pm \Delta k$.

Indeed, the full Taylor series extension for the Riemann Zeta function is: G:= sum(taylor(1/x^(i*v + u), x = k), k = 1 .. infinity) or just by running the Maple Soft to give :

$$G(x) = \sum_{k=1}^{\infty}. \{ \frac{1}{e^{[(u+v*i)*ln(k)]}} - \frac{u+v*i}{k*e^{[(u+v*i)*ln(k)]}} *(x-k) + \frac{\left(\frac{(u+v*i)^2}{k^2}\right)-\left(\frac{i^2*v^2+2*i*u*v-v*i+u^2-u}{2*k^2}\right)}{e^{[(u+v*i)*ln(k)]}} * (x-k)^2 + \frac{1}{e^{[(u+v*i)*ln(k)]}} [\left(\frac{(i^2*v^2+2*i*u*v-v*i+u^2-u)*(u+v*i)}{2*k^3}\right) - \left(\frac{(i^3*v^3+3*i^2*u*v^2-3*i*u^2*v-6*i*u*v+u^3+2*v*i-3*u^2+2*u)}{6*k^3}\right)$$

$$-\left(\frac{(i^2*v^2+2*i*u*v+v*i+u^2)*(u+vi)}{2*k^3}\right) \quad]* \quad (x-k)^3 \quad + \quad \frac{1}{e^{[(u+v*i)*ln(k)]}} \quad [$$

$$-\frac{i^4v^4+4i^3uv^3-6i^3v^3+6i^2u^2v^2-18i^2uv^2+4iu^3v+11i^2v^2-18iu^2v+u^4+22iuv-6u^3-6vi+11u^2-6u}{24k^4}$$

$$+\frac{(i^3v^3+3i^2uv^2-3i^2v^2+3iu^2v-6iuv+u^3+2vi-3u^2+2u)(vi+u)}{6k^4}$$

$$-\frac{(i^2v^2+2iuv+vi+u^2+u)(i^2v^2+2iuv-vi+u^2-u)}{4k^4}$$

$$+\frac{(i^3v^3+3i^2uv^2+3i^2v^2+3iu^2v+6iuv+u^3+2vi+3u^2+2u)(vi+u)}{6k^4}]*(x-k)^4+\frac{1}{e^{[(u+v*i)*ln(k)]}}[$$

$$-\frac{i^5v^5+5i^4uv^4-10i^4v^4+10i^3u^2v^3-40i^3uv^3+10i^2u^3v^2+35i^3v^3-60i^2u^2v^2+5iu^4v+105i^2uv^2-40iu^3v+u^5-50i^2v^2+105iu^2v-10u^4-10}{120k^5}$$

$$+$$

$$\frac{(i^4v^4+4i^3uv^3-6i^3v^3+6i^2u^2v^2-18i^2uv^2+4iu^3v+11i^2v^2-18iu^2v+u^4+22iuv-6u^3-6vi+11u^2-6u)(vi+u)}{24k^5}$$

-

$$\frac{(i^2v^2+2iuv+vi+u^2+u)(i^3v^3+3i^2uv^2-3i^2v^2+3iu^2v-6iuv+u^3+2vi-3u^2+2u)}{12k^5}$$

$$+$$

$$\frac{(i^3v^3+3i^2uv^2+3i^2v^2+3iu^2v+6iuv+u^3+2vi+3u^2+2u)(i^2v^2+2iuv-vi+u^2-u)}{12k^5} -$$

$$\frac{(i^4v^4+4i^3uv^3+6i^3v^3+6i^2u^2v^2+18i^2uv^2+4iu^3v+11i^2v^2+18iu^2v+u^4+22iuv+6u^3+6vi+11u^2+6u)(vi+u)}{24k^5}]*(x-k)^5$$

$$+ O((x-k)^6) \}$$

2. By considering the Taylor series terms up to degree 2, we may have the following approximation:

$$G_2(x) = \sum_{k=1}^{\infty}. \{ \frac{1}{e^{[(u+v*i)*ln(k)]}} - \frac{u+v*i}{k*e^{[(u+v*i)*ln(k)]}}*(x-k) + \frac{\left(\frac{(u+v*i)^2}{k^2}\right)-\left(\frac{i^2*v^2+2*i*u*v-v*i+u^2-u}{2*k^2}\right)}{e^{[(u+v*i)*ln(k)]}}*(x-k)^2 + O((x-k)^3) \}$$

Or using the Maple solve function,

$$solve\left(\frac{1}{e^{(u+vi)ln(k)}}-\frac{u+vi}{ke^{(u+vi)ln(k)}}(x-k) + \frac{-\frac{u^2+2uvi+vi^2-u-vi}{2k^2}+\frac{(u+vi)^2}{k^2}}{e^{(u+vi)ln(k)}}(x-k)^2, u\right)$$

we may get the result like below:

$$\frac{-vik+vix-\frac{3k}{2}+\frac{x}{2}+\frac{\sqrt{k^2-6xk+x^2}}{2}}{-x+k}, \frac{-vik+vix-\frac{3k}{2}+\frac{x}{2}-\frac{\sqrt{k^2-6xk+x^2}}{2}}{-x+k}$$

3. By considering the real part of the above equation and taking limit as k tends to infinity, we may get:

$\frac{(-3k+x)}{2(-x+k)}$ = [1 - $\frac{k+x}{-2x+2k}$] = 1 – R where the residue R tends to $\frac{1}{2}$ as k tends to infinity. Or we may prove that the non-trivial zeros are actually laying on the control strip line -- 0.5 of the Riemann Hypothesis. For the checking the convergence of the above Taylor Expansion (up to the quadratic terms) series, we may consider the term $O((x-k)^3)$ or

$$\frac{1}{e^{[(u+v*i)*ln(k)]}}\left[\left(\frac{(i^2*v^2+2*i*u*v-v*i+u^2-u)*(u+v*i)}{2*k^3}\right) - \left(\frac{(i^3*v^3+3*i^2*u*v^2-3*i*u^2*v-6*i*u*v+u^3+2*v*i-3*u^2+2*u)}{6*k^3}\right) - \left(\frac{(i^2*v^2+2*i*u*v+v*i+u^2)*(u+vi)}{2*k^3}\right)\right]*(x-k)^3$$

i.e. $|G(x) - G_2(x)| \leq \left|\frac{G_2^{(3)}(\xi)}{n!}(x-k)^3\right|$ for some 1 < ξ < infinity (by Mean Value Theorem)

When x is approaching to k by a delta amount, i.e. $k \pm \Delta k$, then we may have:

$$|G(x) - G_2(x)| \leq |M(\Delta k)^3|$$
$$|G(x) - G_2(x)| \leq \varepsilon$$

In other words, $|G(x) – G_2(x)|$ is bounded and closed, hence the Taylor Expansion series (up to the quadratic terms) for Riemann Zeta function must be converged.

4. Next, according to [1], we may find the imaginary part of the non-trivial zeros of the Riemann Zeta function by considering the Harmonic series :

$$H_n = \sum_{k=1}^{n}\frac{1}{k} \text{ or } H_x = \sum_{k=1}^{x}\frac{1}{k} \quad \text{where x tends to infinity}$$
$$= cot(x) \quad \text{for some } x \in \mathbb{R}$$

Now, consider the Riemann Zeta function with complex index [3],

$$\xi(\sigma) = \sum_{k=1}^{n}\frac{1}{k^{(u+vi)}} \quad \text{where n tends to infinity, } \sigma = u+vi \text{ for some } u, v \in \mathbb{R}$$

Hence in practice, what the differences between H_n and ξ(σ) is just by a rotation and a zoom in or out. After somer direct computation, we may find that v = $\frac{H_k}{-ln(k)}$ or v = $\frac{-cot(k)}{ln(k)}$. If we employ the approximation $ln(k)$ = $\frac{2(x-1)}{(x+1)}$, then v = $-\frac{(x+1)}{2(x-1)}cot(x)$ or v = $\frac{2cot(x)}{(2x-1)}$. By the Fundamental Theorem of Calculus, v = $\frac{d}{dx}\frac{ln(x)}{tan(ln(x))}$. (as $\frac{d}{dx}\frac{ln(x)}{tan(ln(x))} = \frac{1}{tan(x)ln(x)} = \frac{-cot(k)}{ln(k)}$).

When we set the above differential to zero, then The function $tan(z) = 1$, attains its maximum/minimum at z = -i $ln\frac{\sqrt{16+(\pi+4n\pi)}}{4-i(\pi+4n\pi)}$ if we take z = $ln(x)$ and $tan(z)$ = 1 for $\frac{ln(x)}{tan(ln(x))}$ = constant.

5. By applying the Maple soft together with the $ln(k) = \frac{2(x-1)}{(x+1)}$ to $\frac{ln(x)}{tan(ln(x))}$, we may have:

$$diff\left(\frac{\left[\frac{2(x-1)}{(x+1)}\right]}{tan\left[\frac{2(x-1)}{(x+1)}\right]}, x\right) =$$

$$\frac{\left[\frac{2}{x+1}-\frac{2(x-1)}{(x+1)^2}\right]}{tan_{\frac{2(x-1)}{x+1}}} - \frac{[0]}{tan^2_{\frac{2(x-1)}{x+1}}},$$

$$solve\left(\left(\left(\frac{4}{(x+1)^2}\cdot cot\left(\frac{2(x-1)}{(x+1)}\right)\right), x\right) =$$

$-\frac{\pi+4}{\pi-4}, -\frac{\pi-4}{\pi+4}$ or

$$solve\left(\left(\frac{4}{(x+1)^2}\cdot cot\left(\frac{2(x-1)}{(x+1)}\right)\right), x, explicit, allsolutions\right) =$$

$$-\frac{4\pi\_Z1\sim+\pi+4}{4\pi\_Z1\sim+\pi-4}, -\frac{4\pi\_Z2\sim-\pi+4}{4\pi\_Z2\sim-\pi-4}$$

Actually, the above result (the optimized point or maximum/minimum) should be:
$-\frac{4n\pi+\pi+4}{4n\pi+\pi-4}$ or $-\frac{4m\pi-\pi+4}{4m\pi-\pi-4}$ where n,m are integers,
For checking the convergence of the function v = $\frac{4}{(x+1)^2}cot\left(\frac{2(x-1)}{(x+1)}\right)$, we may use the first ten non-trivial zeros of the Riemann Zeta function to form the equation as:
$(0.0004319\cdot x^9-0.02216\cdot x^8+0.4869\cdot x^7-5.986\cdot x^6+45.1\cdot x^5-214.5\cdot x^4+638.4\cdot x^3-1134\cdot x^2+1080\cdot x-395.2)$ -------- (*')
then by computing the difference between v and the above equation (*'), we may have

$$fsolve\left(\left(\left(\frac{4}{(x+1)^2}\cdot cot\left(\frac{2(x-1)}{(x+1)}\right)\right) - (0.0004319\cdot x^9-0.02216\cdot x^8+0.4869\cdot x^7-5.986\cdot x^6+45.1\cdot x^5-214.5\cdot x^4+638.4\cdot x^3-1134\cdot x^2+1080\cdot x-395.2)\right), x, 0..\infty\right)$$

= 0.7948976085
which thus imples v is the best approximation to those imaginary part of the non-trivial zeros to Riemann Zeta function as the radius of convergence is smaller than one.

In brief, this author has just proved that for those non-trivial zeros of the Riemann Zeta function, we may have:
1. Real part equals to 0.5;
2. Imaginary part equals to v = $\frac{4}{(x+1)^2}cot\left(\frac{2(x-1)}{(x+1)}\right)$ or the mathematical model equation for it,

That is the mathematical model equation for all non-trivial zeros of the Riemann Zeta Function:
0.5 +/- $\frac{4}{(x+1)^2}cot\left(\frac{2(x-1)}{(x+1)}\right)$i or 0.5 +/- i*($\frac{4}{(x+1)^2}cot(ln(x))$).

MAJOR DISCOVERIES & RESULTS

The major discovered outcome of my present research is that there may be a correlation (or a direct proportional) relationship between the real part and the imaginary part of the Riemann zeta function for the Re(s) = 0.1 gradually increasing to the Re(s) =1 plus the first 10 non-trivial zeros correspondingly.

In other words, the real and imaginary parts of the following have a correlation relationship [12], [13]:

First correlation (or a direct proportional) relationship line between Imaginary part Vs Real part – Mathematica Script: Evaluate[ReIm[Zeta[0.X + 14.1347i]]]
(Zeta (0.1 + ZetaZeros (first non-trivial zeros)))
.
.
(Zeta (1 + ZetaZeros (first non-trivial zeros)))

Second correlation (or a direct proportional) relationship line between Imaginary part Vs Real part – Mathematica Script: Evaluate[ReIm[Zeta[0.X + 21.0220i]]]
(Zeta (0.1 + ZetaZeros (second non-trivial zeros)))
.
.
(Zeta (1 + ZetaZeros (second non-trivial zeros)))

.
.
.
.
.
.
.

10th correlation (or a direct proportional) relationship line between Imaginary part Vs Real part – Mathematica Script: Evaluate[ReIm[Zeta[0.X + 49.7738i]]]
(Zeta (0.1 + ZetaZeros (10th non-trivial zeros)))
.
.
(Zeta (1 + ZetaZeros (10th non-trivial zeros)))

Table 1: The relationship pseudo formula script for the first 10 correlation straight line with real part as the inner looping variable (from 0.1 to 1).

In fact, the above idea comes mainly from the spectral graph of [1], where the graph of the Re(s) = 0.1 shows that Re(ξ(s)) meets Im(ξ(s)) at ξ(s) with a negative value for the first non-trivial zeros. Next for another graph, Re(ξ(s)) meets Im(ξ(s)) at ξ(s) = 0 for the first non-trivial zeros in the graph Re(s) = 0.5.Then Re(ξ(s)) meets Im(ξ(s)) at ξ(s) with a positive value for the first non-trivial zeros in the graph Re(s) = 0.75. If we connect these changing values (from negative through zero to positive) and views as a line, it is just equivalent when one looks through these graphs (0.1 — Figure 2, 0.5 — Figure 3, 0.7 — Figure 4) [5]

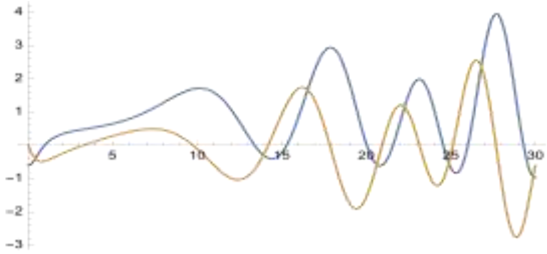

Figure 2: A plot of Zeta function 0.1 + i*t with real against imaginary part
where they meet at non-trivial zeros with negative values.

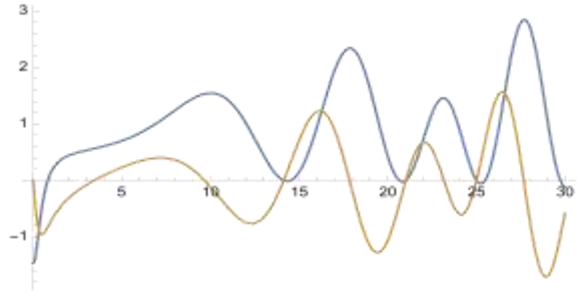

Figure 3: A plot of Zeta function 0.5 + i*t with real against imaginary part where
they meet together at those non-trivial zeros with zeros or roots.

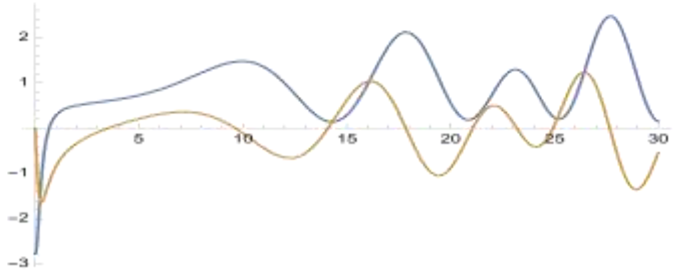

Figure 4: A plot of Zeta function 0.7 + i*t with real against imaginary part where they meet together with non-trivial zeros with some positive values.in a multidimensional array mode or a 3-dimensional perspective with the linked line of these changing values located in an additional Z-axis plane etc. Hence, when we substitute back numerical values into both zeta and zeta-zero functions of the software Mathematica (Ver 13.1, Home Edition), U.S.A., save the output values into a text file and import them into the spreadsheet software, one may then get the wanted correlation relationship straight line that pass through the origin coordinate (0,0) between the real and imaginary parts (shown in Figure 5 & 6 for example) for each time running of the double looping variables in real (from 0.1 to 1) and then the imaginary parts (first to the 10th non-trivial zeros) as shown in the aforementioned Table 1 etc.

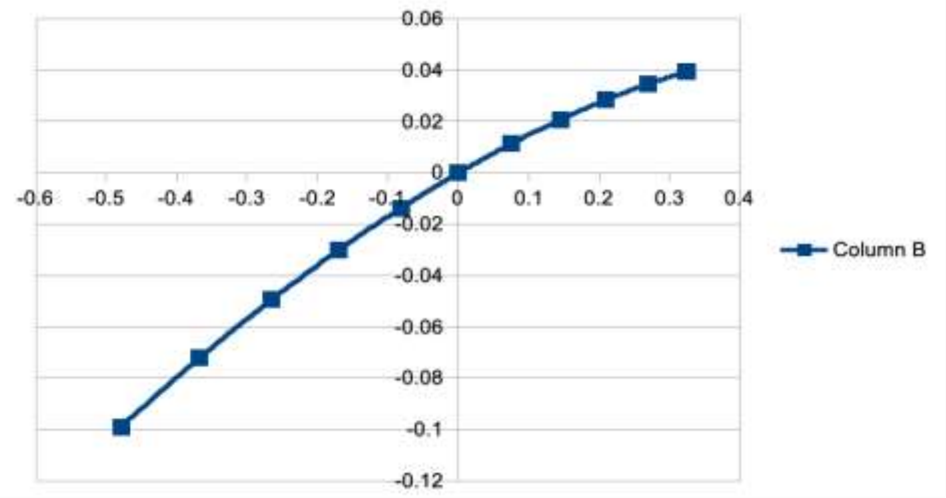

Figure 5: The positive correlation shows between the double looping variables of real part (0.1 to 1) with the first non-trivial zeros (fixed) as the imaginary part from the spreadsheet software — Libra Office, Calc.

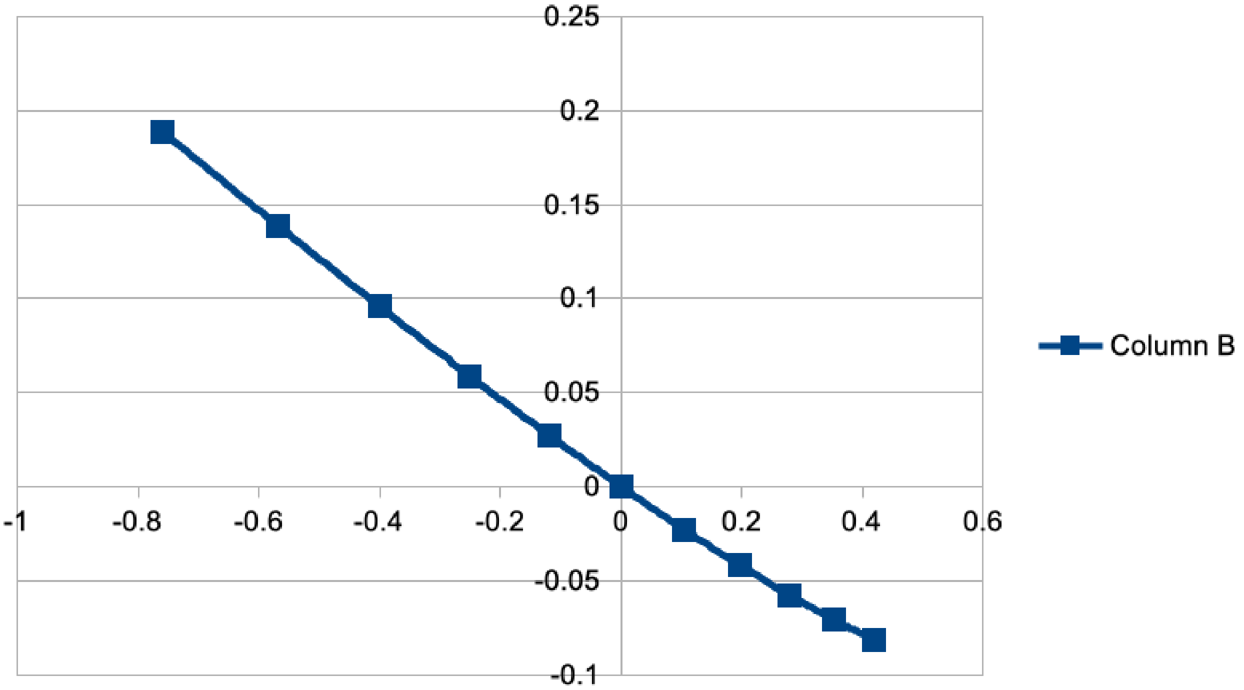

Figure 6: The negative correlation shows between the double looping variables of real part (0.1 to 1) with the second non-trivial zeros as the imaginary part (fixed) from the spreadsheet software — Libra Office, Calc.

This author notes that from the straight line as shown in figure 5 and 6, the lines pass through the origin ((0,0) coordinate) for the first two non-trivial zeros. This is also true for the first 10 non-trivial zeros as find by programming script and may be extended for all of the non-trivial zeros. In other words, Re(ξ(s)) meets Im(ξ(s)) at ξ(s) = 0 whenever Re(s) = 0.5 for all of the non-trivial zeta zeros. In a vice versa way, for whenever we discover Re(ξ(s)) meets Im(ξ(s)) at ξ(s) = 0, then Re(s) = 0.5 is true together with the existence of non-trivial zeros. Then from the explicit formula of Riemann, the prime number counting function π(n) can be expressed in terms of the computed non-trivial zeros and hence predict the prime distribution [2]. Hence, for the imaginary part = non-trivial zeros, zeta(0.5 + non-trivial zeros) equals to (0,0).

Indeed, the most significant discovery for this section is that there may be a direct proportion laying between the real part and the imaginary part of the Zeta non trivial zeros. In other words, Im(ξ(s)) = K*Re(ξ(s)) or a general equation y = Kx. Take for a case study, from figure 5 & 6, the computed constants K are 0.16124064 and -0.2290361 (or the slope of the straight line in fig 5 & 6) for the first and second non-trivial zeros respectively. If we list the constant K for the first 10 non-trivial zeros, we may get the following table:

1st zero	0.16124064	2nd zero	-0.2290361
3rd zero	0.37439402	4th zero	-0.6512619
5th zero	0.72556664	6th zero	-0.3234603
8th zero	0.86464811	7th zero	-0.2313555
10th zero	0.61678987	9th zero	-1.4066589

List 7: The constant K(s) for the first 10 Riemann non-trivial zeros.

The above list for these non-trivial zeros for the Riemann Zeta functions shows that these constant K(s) are indeed discrete just like the case in Planck's constant in quantum mechanics. We may divide these constant K(s) into two categories with one have some positive values while the rest has the negative one. Detailed discussions will be shown in the section below.

DISCUSSIONS

As shown in the section – major discoveries & results, we find a direct proportional relationship between the imaginary part of the zeta (0.X + nth non-trivial zeros) with those real part. That is img(ξ(s)) = K Re(ξ(s)) where s = 0.X + nth non-trivial zeros gradually as shown in the table 1. This fact shows that Riemann Zeta zeros are indeed discrete quantum energy level just like the case in Planck's constant for quantum mechanics. Indeed, Planck's constant provides us with the relationship between the energy of a photon and its frequency. In practice, with the energy mass relationship, we may further get the relationship between mass and frequency. Or in other words, we may have:

$$E = \hbar * f \quad \text{------------------}(1)$$
$$E = m * c^2 \quad \text{------------------}(2)$$
i.e. $$m * c^2 = \hbar * f$$
or $$m = \frac{\hbar * f}{c^2}.$$

From the equation (1), it shows there is a direct proportional relationship laying between the energy and the frequency, such constant is named as the Planck's one. Similar case does apply in my research as depicted in table 1 and figure 5 & 6. However, these constant K(s) are indeed discrete and may be explained by the energy levels of an atom such as hydrogen etc.

Next, if one consider those negative values of these constant K(s), one may observe that there is a sudden drops from the seventh zero to the 9th zero, it may imply the presence of a discrete spectrum as electrons sudden fall (or a jump) from some bounded quantum state to a lower energy state. Such kind of phenomenon may be explained by the Quantum Field Theory (QFT). Actually, we may consider the QFT as a kind of reformulation to the infinity of quantum harmonic oscillators [6]. Each of these oscillators has a discrete spectrum. One may further correspond a particle to each level of the discrete spectrum. In reality, one may view the source of the harmonic potential as a string or a one-dimensional field theory where the next-neighbor couplings lead to many or even an infinity coupled oscillators. Indeed, the spectra of these oscillators are continuous before quantization. When the quantization occurs, the spectra becomes discrete, the actual quantization happens in the oscillators'displacement from equilibrium (but NOT the constant K(s)) or the real dynamical variables [6].

In order to have an in-depth explaination for the Riemann Zeta function, some scientists tries to relate it with the quantum mechanics. One of them is the Scientist --Remmen [7]. Indeed, what a quantum field theory actually is that it consists of various tools for the scientists to depict the pictures about any kind of interactions between the particles. What the scientist Remmen has done is tried to encodes the quantum mechanical probability that particles may interact with each other. In practice, the scattering amplitudes seem to be worked well with the complex plane and have the properties that the scattering amplitudes are analytic around every point except a set of poles. At the same time, these scattering amplitudes lay along a line. All these properties fit well the one of Riemann Zeta function's zeros. Hence, the significance of my present research's major discoveries and results (the discrete constant K(s) between imaginary and real parts as shown in Table 1, Figure 5 & 6, List 7) are now providing a perfect fit (in quantum mechanics together with QFT) evidence for the proof of Remmen's suggestions to Scattering Amplitudes in the explaination of Riemann Zeta function etc.

In physics, what do we mean by scattering amplitudes? It is actually the probability amplitude of the outgoing spherical wave relative to the incoming plane wave in a stationary-state scattering process. Thus, to describe the scattering process, one may use the S-matrix or the scattering matrix to relate the initial state and the final state of a physical system undergoing such kind of scattering process. The elements in the S-matrix are known as scattering amplitudes while the matrix is closely related to the transition probability amplitude in quantum mechanics and to cross sections of various interactions. Thus, this author proposes that one may apply my self-developed HKLam theory to approximate the S-matrix and establish the corresponding statistical model for the scattering process physical system etc. Then, one may further predict the undergoing scattering process together with a suitable calibration of the model after a well-performed feed back.

CONCLUSION

There may be a relationship already established between the Euler product form of Zeta function that expressed in terms of prime number and the products of zeros [14] visually that obtained from the graph [1] or from its computational results etc. In other words, each of the prime number can be expressed in terms of the zeta function's non-trivial zeros. To go a step, one may even continue to guess (or predict) the next feasible primes through the zeta function's non-trivial zeros product from computing. Recursively, each prime number can be expressed in terms of the products of zeta zeros from the graph [5] or from computing. In a vice versa way, for any given product of the zeta zero roots computed from [5], one may get the corresponding prime number. This may help us further improve the present cryptography and hence revolute a change in the present information security technology.

In brief, the following mathematical equation may give you an idea for any prime number in terms of the roots that find in [5]:

$$\prod_{i=1}^{\infty}(z - z_i) = \xi(0.5 + i*t) = \sum_{n=1}^{\infty} 1/n^{(0.5+i*t)} = \prod_{j=1}^{\infty}\left(1 - 1/p_j^{(0.5+i*t)}\right)^{-1} \text{ ------------ } (*)$$

The contradiction finds for the above equation when we replace s = 0.5 + i*t by s = 1. Indeed, by [8] and the prime number theorm [15], we may show that $g(p_n) < ep_n$ for $n>n_0$. That is, $g(p_n) = P_n - P_{n-1} < eP_n$. Hence, we may find a bounded values for prime numbers $p_{j+1} - p_j <= x$ which contradicts to the fact there is NO known bound on the prime gaps. Hence, the equation (*) only holds for the case s = 0.5 + i*t. As we may see from the figure 2,3,4, for all of the other cases s = k + i*t where 0 < k/{0.5} <= 1 never have values ξ(s) = 0. Any assumption of zeros Z_I s like the formula shown on the most left hand side of (*) in these cases will lead to the similar bounded prime gap contradiction just like the case when s = 1. Since there is no other zeros for the cases s = k + i*t where 0 < k/{0.5} <= 1, we may conclude that there is a need for the shift of x = Re(s) = 0.5 to x = Re(s) = 0.

Or the Riemann hypothesis is thus proved – "All non-trivial zeros of ξ(s) lay on the control strip of x = Re(s) = 0.5 for $s \in \mathbb{C}$. Indeed, the Riemann Zeta function can be expained by:
1. Riemann Zeta Zeros are discrete quantum energy levels;
2. Discrete spectrum such that electrons may fall from some bound quantum state to a lower energy state implies the quantum field theory (QFT);
3. Scientist Remmen's explaination by scattering amplitude (may be proved by my evidences as shown in table 1, figure 5 & 6, list 7)
4. S-matrix is just the collection of scattering amplitude that forms the matrix;
5. One may approximate the S-matrix by HKLam theory and hence establish the corresponding statistical model for prediction together with the calibration of a better model etc.

In reality, my computed Mathematical model equation for the Riemann Zeta function through Maple soft (Ver 2022, Home Edition) is:

$$0.5 \text{ +/- } \frac{4}{(x+1)^2} cot\left(\frac{2(x-1)}{(x+1)}\right) i \text{ or } 0.5 \text{ +/- } i*\left(\frac{4}{(x+1)^2} cot\left(ln(x)\right)\right).$$

Certainly, there are still some limitations of the present research such as the cross over between Laplace or Fourier transform together with the radius of convergence etc. These in-depth study may then be left to my future papers or those who feel interested to investigate in the topic of computational analytic number theory.

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