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Applications of First Order Ordinary Differential Equations to Real Life Systems

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Abstract: *This article discussed applications of first order ordinary differential equations to real life systems, various types of differential equations with examples are presented. When a dead body is discovered somewhere, Police and detective security agency are eager to identify the time of death and what caused the death. In this article, Newton's Law of cooling is used to estimate time of death of a dead body discovered in the midnight with the aid of information about the body's surrounding temperature. It is discovered in this research that Newton's law of cooling only works when temperature of the body's surrounding is kept constant. The study also considered other applications of first order differential equations such as population growth model and radioactive decay of radioactive isotopes and illustrative example are given in each case.*

Keywords: applications of differential equations, population growth model, radioactive decay, type of differential equations, newton's law of cooling, time of dearth of a dead body. physical mode.

INTRODUCTION

Many physical phenomena in the field of science, engineering and technology when developed mathematically yield differential equation. This set of equation is called model. The most important aspect of applied mathematics that has gained striking attention is the formulation of mathematical model using set of differential equations to address-real life phenomenon [1] some useful applications of differential equations are found in the areas of population growth and decay, distribution of drug in human body, carbon dating, wave in composite media, aerodynamics, casting of materials, electromagnetics analysis for detection of bar radar, rocket launch trajectory analysis, motion of a space vehicle, heat transfer and temperature problems which apply Newton's law of cooling [5,6]. A differential equation is an equation involving the derivatives of an independent variables with respect to one or more dependent variables. When it contains a single dependent variable, it is called an ordinary differential equation which is our major focus in this study. It is known as partial differential equation if it involves several independent variables and one or more dependent variables. Frigon and Pouso [2] studied theory and applications of first order ordinary differential equations which transformed the usual derivatives by Stielties derivatives. Rahan [4] investigated first order differential equation and Newton's law of cooling. Some relevant works in the field of differential equations are found in [3, 7, 8]. Tar-Ran [9] also studied applications of first order ordinary differential equations in engineering analysis.

Preliminaries

Basic Steps in Building a Differential Equation that Describes Real Life System (Model)

The procedures blow help to build a model.

1. Identify the real life problem correctly
2. State the mathematical assumptions on which the model will be developed, considering the variables
3. Describe fully the variables and parameters of the model
4. Apply the assumptions (in step 2) to formulate differential equations considering parameters and variables described (in step 3)
5. Solve the formulated model (in step 4) and carry out real life interpretation of the model for further application

Type of Differential Equations with Examples

Ordinary Differential Equation (ODE):

Given a function f *and* x, y and derivatives of y, the general form of ordinary differential equation is written as $f(x, y, y', y'', ., ., ., ., y^{(n)}) = 0$

Examples of ODEs: $\frac{dy}{dx} = 5tanx$, $\frac{dy}{dx} = x^3$

Partial Differential Equation (PDE)

A partial differential equation is an equation involving an unknown function of two or more variables and its partial derivatives with respect to this variables [10]

Examples of PDEs: $\frac{\partial^2 z}{\partial x^2 y} = e^y sinx$, $\frac{\partial^2 z}{\partial x^2 y} - 2xy^2 = 0$

Order and Degree of a Differential Equation.

The order of a differential equation is the highest derivative present in the equation whereas the degree of a differential equation is the power to which the highest derivative is raised.

For example, the differential equation

$\frac{\partial^2 y}{\partial x^2} = \left(\frac{dy}{dx}\right)^4 + 2xy = 0$ is of order 2 and degree 1, whereas the differential equation

$\left(\frac{\partial^2 y}{\partial x^2}\right)^4 + \left(\frac{dy}{dx}\right)^5 + 2xy = 0$ is of order 2 and degree 4

Linear and Non-linear Differential Equation.
A linear differential equation is one that can be expressed in the form $y' + p(x)y = q(x)$ whereas a non-linear differential equation is one that is not a linear equation in the unknown function and its derivative which cannot be expressed in the form $y' + p(x)y = q(x)$ but can be expressed as $y' + p(x)y = \alpha(x)y$.

Some Applications of First Order Differential Equation to Real Life Systems.
These are numerous real life applications of first order differential equations to real life systems. In this study we shall discuss the following

- Population growth and decay
- Newton's law of cooling
- Radioactive decay

Population Growth and Decay
Population growth involves a dynamic process which can be developed using differential equation. The exponential growth model or natural growth model is known as Mathus' model [12]. This model is based on the assumption that the rate of change of the population is proportional to the existing population itself. If p(t) represents the total population at time t, the above assumption can be written as

$$\frac{dp}{dt} = kp(t) \qquad (3.1)$$

Where k is the proportionality constant.
The above model (3.1) can also be used in financial institute for example, when saving money in the bank, the balance in savings account with interest compounded continuously exhibits natural growth provided no withdrawal and in this case the constant k represents the annual rate of interest, group of animal populations grows exponential provided size is not affected by environmental factors, in this case k is known as the productivity rate of population and it can also used in migration.
To obtain solution of (3.1) we multiply the equation with e^{-kt}, the integrating factor

$$e^{-kt}\frac{dp}{dt} = kpe^{-kt}$$

$$e^{-kt}\frac{dp}{dt} - kpe^{-kt} = 0$$

$$\frac{dp}{dt}[pe^{-kt}] = 0$$

$$\int \frac{dp}{dt}[pe^{-kt}] = \int 0$$

$$pe^{-kt} = C \text{ or } p - ce^{kt}$$

Suppose the initial population is p_0 then $p(0) = p_o$ and $c = p_0$

$$p(t) = p_o e^{kt} \qquad (3.2)$$

When k > 0 the population grows and when k < 0, the population decays

Example 1.
Suppose the population of a certain community is known to increase at a rate proportional to the number of people living in the community at time t, the population has doubled after 7years, how long would it take to triple?. If it is known that the population of the community is 12,000 after 5years, determine the initial population and predict the population in 40years.

Solution
Let p_0 denotes initial population of the community and $p(t)$ the population of the community at any time t, then from (3.1) we have

$\frac{dp}{dt} = kp$

$p(t) = p_0 e^{kt}$

From (3.2) given that

$p(7) = 2p_o$

$e^{7k} = 2$

$k = \frac{0.6931}{7} = 0.0990$

The solution of the model becomes

$$p(t) = p_o e^{0.0990t} \quad (3.3)$$

Let t, be the time taken for the population to triple then

$3p_0 = p_o e^{0.0990t}$

$e^{0.0990t} = 3$

0.0990t = ln3

$t = \frac{1.0986}{0.0990} = 11.0970 \approx 11 years$

Applying $p(5)$ = 12,000

$12{,}000 = p_0 e^{0.0990 \times 5}$

$p_o = \frac{12{,}000}{e^{0{,}450}} = 7{,}315$

Hence the initial population of the community was $p_o \approx 7{,}315$

Therefore, solution of the model is

$p(t) = 7315e^{0.0990t}$

So that the population in 40years is

$p(40) = 7315e^{40(0.0990)}$

$p(40) = 7315e^{3.960}$

$p(40) = 7315(52.4573)$

$p(40) \approx 383{,}725$

RADIOACTIVE DECAY

In physics and chemistry, a radioactive element disintegrates when it emits energy in form of ionizing radiation. Substances that emit ionizing radiation are known as radionuclides. When a radioactive substance decays, a radionuclide transforms into different atom-a decay product. The atoms keep transforming to new decay products until a state is reached and are no longer radioactive. The radioactive law states that the probability per unit time that a radioactive substance will decay is a constant and independent of time, which means that the amount of nuclei

undergoing the decay per unit time is proportional to the total number of nuclei in the given substance [11].
The mathematical expression of the radioactive law is.

$$\frac{dA}{dt}\ \alpha\ A$$

$$\frac{dA}{dt} = kA \qquad (3.4)$$

Where A(t) is the amount of substance and k is constant of proportionality.
Suppose the initial amount of the substance is A_o then
$A(0) = A_o$ and solving (3.4) using initial condition we have

$$A(t) = A_o e^{kt} \qquad (3.5)$$

Equation (3.5) is the solution of (3.4) where the constant k can be obtained from half-life of the radioactive substance.
The half-life of a radioactive material can be defined as the time it take for one-half of the atom in an initial amount (A_o) to transform into atoms of new element. The half-life determines stability of a radioactive element. The half-life of a radioactive substance is directly proportional to its stability.
Let T be the half-life of a radioactive element, then

$$A(T) = \frac{A_o}{2} \qquad (3.6)$$

Applying (3.5) and (3.6) we have,

$$\frac{A_0}{2} = A_o e^{kt}$$

$$T = \frac{-ln}{k} \qquad (3.7)$$

Example 2
If the half-life of a radioactive element is 18days and we have 40g at the end of 30days. Determine the amount of radioactive element present initially
Solution
Let A(t) represent the amount present at time t and Ao the initial amount of the element.

$$\frac{dA}{dt} = kA$$

A(40) = 30
Solving the IVP, yields.

$$A(t) = A_0 e^{kt}$$

But from (3.7)

$$k = \frac{-ln2}{T}$$

$$k = \frac{-ln2}{18} \qquad (3.8)$$

Applying A(40) = 30

$$40 = A_o e^{30k}$$

$$A_o = 40e^{-30k} \qquad (3.9)$$

Using (3.8) we have

$$A_o = 40e^{\frac{30ln2}{18}}$$

$$A_o = 127g$$

Newton's Law of Cooling

Another important real life application of differential equation is Newton's law of cooling. Sir Isaac Newton developed huge interest in quantitative finding the loss of heat in a body and a formula was derived to represent this phenomenon. The law states that the rate of change of temperature of a body is directly proportional to the difference is solid object and surrounding environment at a given instant of time.

$\frac{dT}{dt}\alpha(T_0 - T_5)$

$\frac{dT}{dt} = k(T_0 - T_5)$ (3.10)

Where T(0) = T_0

$T_0 =$ Temperature of the body

$T_5 =$ Temperature of surrounding

K = Constant of proportionality

$\int \frac{dT}{T_0 - T_5} = \int kdt$

$ln|T_0 - T_5| = kt + c$

$T_0 - T_5 = e^{kt+c}$

Applying T(0) = T_0 yields

$C = T_0 - T_5$

$T(t) = T_5 + (T_0 - T_5)e^{kt}$

Suppose the temperatures at t_1 and t_2 are given we have

$T(t_1) - T_5 = (T_0 - T_5)e^{kt_1}$

$T(t_2) - T_5 = (T_0 - T_5)e^{kt_2}$

So that

$\frac{T(t_1)-T_5}{T(t_1)-T_5} = e^{k(t_1-t_2)}$ (3.11)

Example 3

A police man discovered a dead body at the midnight in a room when the temperature of the dead is 90°F, the body temperature of the room was kept constant at 70°F. Three hours later the temperature of the body dropped to 85°F. Determine the time of death of the victim

Solution

$T(0) = 98.6^oF \ \ (37^0c) = T_o$ and $T_5 = 70^oF$

Provided the victim was not sick

$\frac{dT}{dt} = k(T_0 - 70),\ T(0) = 98.6$

But

$T(t) = T_5 + (T_0 - T_5)e^{kt}$

So that

$\frac{T(t_1)-T_5}{T(t_2)-T_5} = e^{k(t_1-t_2)}$

$T(t_1) = 90^oF\ and\ T(t_2) = 85^oF$

$\frac{90-70}{85-70} = e^{3k}$
$t_1 - t_2 = 3hours$
$k = \frac{1}{3} ln \frac{4}{3} = 0.0959$
Let t_1 and t_2 represent the times of death and discovery of the dead body then
$T(t_1) = T(0) = 98.6^o F$
And $T(t_2) = 90^o F$
The time of death (t_3) = $t_1 - t_2$ and from (3.11) we have

$$\frac{T(t_1)-T_5}{T(t_2)-T_5} = e^{kt_3} \qquad (3.12)$$

$$\frac{98.6-70}{90-70} = e^{kt_3}$$

$$t_3 = \frac{1}{k} ln \frac{28.6}{20} = \frac{1}{0.0959} ln \frac{28.6}{20} \approx 3.730$$

Therefore, the person died at about 8:18pm

CONCLUSION

Many physical problems in the field of science, economics, engineering and technology remain meaningless without application of differential equations to transform them to models. By this article real life problems are solved with the aid of differential equations. basically, our concentration is on population growth and decay, radioactive decay and Newton's law of cooling. The physical growth and decay of any population which well discussed in this article is of great concern to humanity. We used the population growth model to predict population of a given community in 40years time, this means that population growth model can be used to predict population of a country in future when some facts about the country are known. This can actually help the government of such country to plan ahead of time and equally embark on population control measures when necessary. Radioactive decay is of great important for nucleus as the decay transforms it the a stable state, many of these modern technologies are product of radioactive decay, a large amount of energy can be generated using decay in nuclear rector which is then converted to electrical energy for use in various form, in medical science, radioactive isotopes which can undergo radioactive has a great application because these isotopes are referred to as tracers and are injected into the body of a patient, in the body, the tracers gives off radiation which is harmless though may be detected through device and through this detection, scientists (physicians) are able to investigate blood flow to specific organs and evaluate organ function or bone growth [11]. There are numerous such applications of differential equations such as drug distribution in human body, survivability with HIV/AIDS, under water acoustic signal processing, crystal growth, transportation and distribution of chemicals through the body, radio interferometry, seismic wave propagation in the earth (earthquake), aircraft landing field length. It article showed that differential equations have numerous applications in real life systems.

Conflict of Interests
The authors declare that they have no conflict of interest.

REFERENCES

[1]. Mersha A.E (2015). Some application of first order differential equations to real word system. MSC Graduate seminar

[2]. Frigon, M. and Pouso, R.L (2017): theory and application of first order Systems of stielties differential equation. Advances in Nonlinear Analysis 6(1): 13-36

[3]. Simons, G.F. (2016). Differential equations with applications and historistirical not CRC pres

[4]. Rehan, Z. (2020) application of first order differential equations to heat convection in a fluid Journal of applied mathematics and physics, 8, 1456-1462

[5]. Hassa, A and Zakan Y (2018) application of first order in differential equation in temperature problem

[6]. Karlikeya, N and Jararaya, A (2016) application of first order differential equation to heat transferring in solids

[7]. Ziv .D. (2013): A first source in differential equations with modeling application. Books/cole, cengage learning

[8]. Shior, M.M, Odo, C.E, Agbata B.C, Ezugorie, I.G, Arivis, S.S (2022). Numerical solution of initial value problem of ordinary differential equations by Adams- Moulton predictor-corrector method. IJM MS 7(3):17-30

[9]. Tai-Ram. Hus (2018). Applications of first order differential equation in engineering analysis, applied engineering analysis

[10]. Brent J. L. Andrew A.P (2022) Partial differential equation advance mathematics for engineering students. https://www. Science direct.com /topics/mathematics/partial-differential equations.

[11]. Harideo. C (2013) Application of differential equation to population growth: Tribhuran university journal 24(2):75-80.

[12]. Jitender, S (2022). Newton's law of cooling formula solved problem: https://www.concepts-of-physics.com/thermodynamics

[13] Mahata, G, Raut, D.S, Parida, C., Baral, S. Mandangi (2022) Applications of first order differential equations *IJEST 6(5), 23-33*