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Sensitivity Analysis of Marine Reserves and Its Implications as Fishery Management Tool

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ABSTRACT: *Sensitivity analysis of marine reserves and its implications as a fishery management tool was examined. A fishing free zone was established analytically which led to stability of the fishing free zone by employing linearization, first order differential equation and Jacobian matrix and unstable fishing free zone was obtained. We then apply some protection in the fishing zone and obtain a stable environment with the same techniques. Finally, the use of MATLAB helps us to discover that fishing effort has significant influence on fish biomass. The result reveals that as fishing effort increases, both the fraction of fish stock in the fishing zone and fraction in the reserves zone approaches their respectful extinction limits. It was equally examined that when the biomass in the open zone is heavily fished then there is a significant disparity in stock density between the two zones. An increased flow of transfer from the reserved zone to the fishing zone result in the catch reaching its maximum level as the transfer reaches its peak. Finally, there is predator interference which may lead to extinction of the prey, meanwhile, the protected zone does not allow predator and thus the fishing environment has no implications.*

KEY WORDS: Sensitivity, analysis, stability, marine, protected areas, zone, fishing, MATLAB

INTRODUCTION

Marine reserves are areas closed to exploitation and are noticed as an additional management tool that could control fishing mortality [1]. [2] Then examined the contribution of fully protected tropical marine reserves to fishery enhancement by modeling marine reserve fishery linkages. The impact of reserve establishment on the long run equilibrium fish biomass and fishery catch levels are evaluated and observed that marine reserves are important component of sustainable tropical fisheries management and reserves will be most effective when coupled with fishing effort controls in adjacent fisheries. But [3] investigated marine reserves and its consequences as a fisheries management tool and indicated that prey-predator interactions do matter when the implementation of reserved is considered. [4] Then proposed a stage-structured prey-predator fishery model in the presence of toxicity with taxation as a control parameter of harvesting effort. Despite the fact that scientists and researcher consider the increasing range of closed areas for conservation of marine biodiversity [5, 6]. The use of marine protected areas is directed towards ecosystem functioning and ecosystem services, [7] then studied marine reserve and its consequences in a predator-prey system for ecotourism and fishing and observed that the optimal reserve size corresponding to fisheries management is more than the size of reserve when both the ecotourism and fishery tent and looked into but [8] investigated the negative impact of marine reserves on coral fish, then [9] indicated that marine protected areas can protect the discounted economic rent from the fishery if the habitat is likely to face a shock and fishers have a high discount rate. See also [10]. Besides, [11] investigated the effect of positive marginal monitoring and enforcement costs, 'policing cost', on the optimal exploitation of a fishery under the management of a marine protected area and shown that with positive marginal policing cost, the objective of maximum economic yield is no longer optimal and that some dissipation of economic rent is socially optimal. See also [12] and [13] showed the conceptual issues imperative to marine harvest refuges with application from temperate reef fishes. Though [14] proposed and studied a non-linear differential equation model for the survival of biological species affected by toxicants present in the environment. Their analysis showed that as the emission rate of toxicants in the environment increases, the density of biological species decreases. They emphasized that the biological species may even become extinct if the rate of emission of pollutants increases continuously. [15] The advantage associated with marine protected areas have been investigated widely and is an interesting area of research in theoretical ecology. If equally investigated the economic optimality of implementing a marine protected area to obtain more informative data about fish population, thereby a better management strategy.

Through [2, 10, 13, 17, 18, 19, 20] opined that the use of marine protected areas is pointed towards ecosystem functioning where ecosystems are easily disrupted by fishing efforts, reserves could be more appropriate option. Marine reserve areas do also have the powers to provide a margin of safety and to enhance productivity of some fisheries. These are ecological importance which are long run sustainability of the fishery in the habitat. See also [21, 22, 23]. But [24] considered numerical method in predicting the biodiversity loss and gain of industrial assets due to the variation per growth rate for industry in dealing with normal agriculture on biodiversity scenario. Real life problems are often structured and analyzed with the help of ordinary differential and partial differential equation [25, 26]. Meanwhile in this paper, we considered sensitivity analysis of marine reserves and its implications as a fisher management tool.

MATHEMATICAL FORMULATION

In this paper, we formulate a system of differential equation in line with the work Kar and Kuna (2009)

$$\frac{dx}{dt} = rx(1 - x - y) - M - mxz$$

$$\frac{dy}{dt} = ry(1 - x - y) - M - nyz$$

$$\frac{dz}{dt} = sz\left(1 - \frac{rz}{x + y}\right)$$

$$\frac{h}{E}\alpha y \Rightarrow h = qEy$$

Where q is constant of proportionality and $M = K\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right)$

VARIABLES AND PARAMETER OF THE PROPOSED MODEL

x	-	Prey in free zone
y	-	Predator in free zone
z	-	Biomass density of predictor in free zone
x_1	-	Prey in protected zone
y_1	-	Predictor in protected zone
z_1	-	Biomass density of predictor in protected zone
α	-	Protected zone

$1-\alpha$	-	Free zone
m	-	Relative increase in free zone
n	-	Relative increase in protected zone
q	-	Constant of proportionality (catchability coefficient)
E	-	Effort applied for harvesting the fish
r	-	Intrinsic growth rate
k	-	Net transfer
s	-	Growth rate of predator species
t	-	Time

METHOD OF SOLUTIONS

The fishing free zone point is where there are no implication

3.1 **Determination for the Fishing Free/ Protected Zone**

$$\frac{dx}{dt} = rx(1-x-y) - M - mxz \tag{3.1}$$

$$\frac{dy}{dt} = ry(1-x-y) - M - nyz \tag{3.2}$$

$$\frac{dz}{dt} = sz\left(1 - \frac{rz}{x+y}\right) \tag{3.3}$$

$$\frac{h}{E}\alpha y \Rightarrow h = qEy$$

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Where q is constant of proportionality y and $M = K\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right)$

Hence, equation (3.1), (3.2) and (3.3) becomes

$$\frac{dx}{dt} = rx(1-x-y) - K\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right) - mxz \quad (3.4)$$

$$\frac{dy}{dt} = ry(1-x-y) + K\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right) - nyz - qEy \quad (3.5)$$

$$\frac{dz}{dt} = sz\left(1 - \frac{rz}{x+y}\right)$$

$$\frac{dx}{dt} = 0, \frac{dy}{dt} = 0, \frac{dz}{dt} = 0$$

From (3.4)

$$rx(1-x-y) - K\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right) - mxz = 0$$

$$rx - rx^2 + rxy - \frac{kx}{\alpha} + \frac{ky}{1-\alpha} - mxz = 0$$

$$rx - rx^2 + rxy - \frac{kx}{\alpha} - mxz = -\frac{ky}{1-\alpha}$$

$$x\left(r - rx + ry - \frac{k}{\alpha} - mz\right) = -\frac{ky}{1-\alpha} \quad (3.6)$$

When x=0, y=0

But from $\frac{dz}{dt} = sz\left(1 - \frac{rz}{x+y}\right)$

$$\therefore sz = 0, \quad \therefore z = \frac{0}{s} = 0$$

Thus, the fishing free zone $x, y, z = (0,0,0)$

3.2 **Stability of the Fishing Free Zone**

We will determine the stability of the fishing free zone by linearizing the system of the differential equations by obtaining the Jacobian at fishing free zone state.

$$J = \begin{pmatrix} \frac{dx}{dx} & \frac{dx}{dy} & \frac{dx}{dz} \\ \frac{dy}{dx} & \frac{dy}{dy} & \frac{dy}{dz} \\ \frac{dz}{dx} & \frac{dz}{dy} & \frac{dz}{dz} \end{pmatrix} \tag{3.7}$$

But

$$\frac{dx}{dx} = r(1 - x - y) + rx(1 - 1 - y) - \frac{k}{\alpha} - mz \tag{3.8}$$

$$\frac{dx}{dy} = -rx^2 \tag{3.9}$$

$$\frac{dx}{dz} = -mx \tag{3.10}$$

$$\frac{dy}{dx} = -ry^2 + \frac{k}{\alpha} \tag{3.11}$$

$$\frac{dy}{dy} = r(1-x-y) - rxy - \frac{k}{1-\alpha} - nz - qE \tag{3.12}$$

$$\frac{dy}{dz} = -ny \tag{3.13}$$

$$\frac{dz}{dx} = \frac{sz^2r}{(x+y)^2}, \frac{dz}{dy} = \frac{sz^2r}{(x+y)^2}, \frac{dz}{dz} = \frac{(x+y)(2srz)}{(x+y)^2} = \frac{2srz}{x+y} \tag{3.14}$$

But $x, y, z = (0,0,0)$

$$J = \begin{pmatrix} -\frac{k}{\alpha} - \lambda & 0 & 0 \\ \frac{k}{\alpha} & \left[\left(r - \frac{k}{1-\alpha} - nz - qE\right) - \lambda\right] & 0 \\ 0 & 0 & -\lambda \end{pmatrix} \tag{3.15}$$

Hence, the stability will be obtained by using the Jacobian matrix at fishing free zone.

Thus,

$$J-\lambda I=0=\begin{pmatrix} -\frac{k}{\alpha}-\lambda & 0 & 0 \\ \frac{k}{\alpha} & \left[\left(r-\frac{k}{1-\alpha}-nz-qE\right)-\lambda\right] & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{3.16}$$

$$\left(-\frac{k}{\alpha}-\lambda\right)\left[\left(r-\frac{k}{1-\alpha}-nz-qE\right)(-\lambda)\right]=0$$

$$-\frac{k}{\alpha}-\lambda=0 \quad \Rightarrow \quad \lambda=-\frac{k}{\alpha}$$

Thus, $\lambda=\frac{-k}{\alpha},\left(r-\frac{k}{1-\alpha}-nz-qE\right)(-\lambda)=0 \quad \Rightarrow \quad \lambda=0$ (3.17)

$$\therefore \lambda=0,-\frac{k}{\alpha}$$

Which indicates that this is an unstable zone.

The Protected Zone

We can now determine the non-trivial steady state solution of the protected zone Z at time t. Here the fishing $x \longrightarrow x$, $y=y_1$ and $z=z_1$, then equation (3.4), (3.5) and (3.3) becomes

$$rx_1\left(1-x_1-y_1\right)-k\left(\frac{x_1}{\alpha}-\frac{y_1}{1-\alpha}\right)-mx_1z_1=0 \tag{3.18}$$

$$ry_1(1-x_1-y_1)+k\left(\frac{x_1}{\alpha}-\frac{y_1}{1-\alpha}\right)-ny_1z_1-zEy_1=0 \tag{3.19}$$

$$sz_1\left(1-\frac{rz_1}{x_1+y_1}\right)=0 \tag{3.20}$$

From equation (3.19), we have

$z_1 \neq 0$, similarly

$$1-\frac{rz_1}{x_1+y_1}=0$$

$$\therefore x_1+y_1-rz_1=0$$

$$\therefore x_1+y_1=rz_1$$

$$\therefore z_1=\frac{x_1+y_1}{r} \tag{3.21}$$

From equation (3.17), we have $x_1, y_1, z_1 \neq 0$

$$rx_1-x_1^2-x_1y_1-\frac{kx_1}{\alpha}-\frac{ky_1}{1-\alpha}-mx_1z_1=0$$

$$-x_1^2-x_1\left(r-y_1-\frac{k}{\alpha}-mz\right)+\frac{ky_1}{1-\alpha}=0$$

i.e., $$x^2-x_1\left(r-y_1-\frac{k}{\alpha}-mz\right)-\frac{ky_1}{1-\alpha}=0 \tag{3.22}$$

From equation (3.18), we have $x_1, y_1, z_1 \neq 0$

$$ry_1 - x_1y_1 - y_1^2 + \frac{kx_1}{\alpha} - \frac{ky_1}{1-\alpha} - ny_1z_1 - qEy_1 = 0$$

$$-y^2 + y_1\left(r - x_1 - \frac{k}{1-\alpha} - nz_1 - qE\right) + \frac{kx_1}{\alpha} = 0$$

i.e., $$y^2 - y_1\left(r - x_1 - nz_1 - qE - \frac{k}{1-\alpha}\right) - \frac{ky_1}{\alpha} = 0 \qquad (3.23)$$

Equation (3.21) and (3.22) can be simplify further by mathematical stimulation.

Also, in other words, from equation (3.20) we have

From equation (3.17), we have $x_1, y_1, z_1 \neq 0$

$z_1 = \frac{x_1 + y_1}{r}$ can be simplify as

$x_1 = rz_1 - y_1$ and (3.23.1)

$y_1 = rz_1 - x_1$ as the case may be (3.23.2)

3.3 The Stability of the Protected Zone

We find the stability of the protected zone by the Jacobian method at $x_1, y_1, z_1 \neq 0$ with equation (3.20) in application.

$$J = \begin{pmatrix} \frac{dx_1}{dx_1} & \frac{dx_1}{dy_1} & \frac{dx_1}{dz_1} \\ \frac{dy_1}{dx_1} & \frac{dy_1}{dy_1} & \frac{dy_1}{dz_1} \\ \frac{dz_1}{dx_1} & \frac{dz_1}{dy_1} & \frac{dz_1}{dz_1} \end{pmatrix} \tag{3.24}$$

From equation (3.17), (3.18) and (3.19) we have

$$\frac{dx_1}{dx_1} = -r - \frac{k}{\alpha} - mz \text{ for } z = \frac{x_1 + y_1}{r}$$

$$= -\left(1 + \frac{k}{\alpha r} + m\left(x_1 + y_1\right)\right) \tag{3.25}$$

$$\frac{dx_1}{dy_1} = \frac{k}{1-\alpha} \tag{3.26}$$

$$\frac{dx_1}{dz_1} = -mx_1 \tag{3.27}$$

$$\frac{dy_1}{dx_1} = \frac{k}{\alpha} \tag{3.28}$$

$$\frac{dy_1}{dy_1} = -r - \frac{k}{1-\alpha} - nz_1 - qE \text{ for } z = \frac{x_1 + y_1}{r}$$

$$= -\left(1 + \frac{k}{r(1-\alpha)} + n(x_1 + y_1) + \frac{qE}{r}\right) \tag{3.29}$$

$$\frac{dy_1}{dz_1} = -ny_1 \tag{3.30}$$

$$\frac{dz_1}{dx_1} = -\frac{1}{x_1 + y_1} \tag{3.31}$$

$$\frac{dz_1}{dy_1} = -\frac{1}{x_1 + y_1} \tag{3.32}$$

$$\frac{dz_1}{dz_1} = \frac{-sr}{x_1 + y_1} \tag{3.33}$$

$$\therefore J = \begin{pmatrix} -\left(1 + \frac{k}{\alpha r} + m(x_1 + y_1)\right) & \frac{k}{1-\alpha} & -mx_1 \\ \frac{k}{\alpha} & -\left(1 + \frac{k}{r(1-\alpha)} + n(x_1 + y_1) + \frac{qE}{r}\right) & -ny_1 \\ -\frac{1}{x_1 + y_1} & -\frac{1}{x_1 + y_1} & -\frac{sr}{x_1 + y_1} \end{pmatrix} \tag{3.34}$$

Hence the stability will be obtained by $J - \lambda I = 0$

i.e.

$$J-\lambda I=0\Rightarrow\begin{pmatrix} -\left(1+\frac{k}{\alpha r}+m\left(x_1+y_1\right)\right)-\lambda & \frac{k}{1-\alpha} & -mx_1 \\ \frac{k}{\alpha} & -\left(1+\frac{k}{r(1-\alpha)}+n\left(x_1+y_1\right)+\frac{qE}{r}\right)-\lambda & -ny \\ -\frac{1}{x_1+y_1} & -\frac{1}{x_1+y_1} & -\frac{sr}{x_1+y_1}-\lambda \end{pmatrix} \quad (3.35)$$

$$\therefore\left[-\left(1+\frac{k}{\alpha r}+m\left(x_1+y_1\right)-\lambda\right)\right]\left(\left[-\left(1+\frac{k}{r\left(1-\alpha\right)}+n\left(x_1+y_1\right)+\frac{qE}{r}\right)-\lambda\right]\left[\frac{-sr}{x_1+y_1}-\lambda\right]-\left(\frac{-1}{x_1+y_1}\right)\left(-ny\right)\right)$$

$$-\frac{k}{1-\alpha}\left[\left(\frac{k}{\alpha}\right)\left(\frac{-sr}{x_1+y_1}-\lambda\right)-\left(\frac{-1}{x_1+y_1}\right)\left(-ny\right)\right]$$

$$-mx\left[\left(\frac{k}{\alpha}\right)\times\left(\frac{-1}{x_1+y_1}\right)-\left(\frac{-1}{x_1+y_1}\right)\left(1+\frac{k}{r\left(1-\alpha\right)}+n\left(x_1+y_1\right)+\frac{qE}{r}\right)-\lambda\right]=0 \quad (3.36)$$

By simplify equation (3.36) we will have our λ very large.

Hence $\lambda \rightarrow -\infty$ which indicates that there is stability in the protected zone.

GRAPHICAL RESULTS

Here we present numerical results with the help of Matlab.

SIMULATION WITHOUT PREDATOR INTERFERENCE

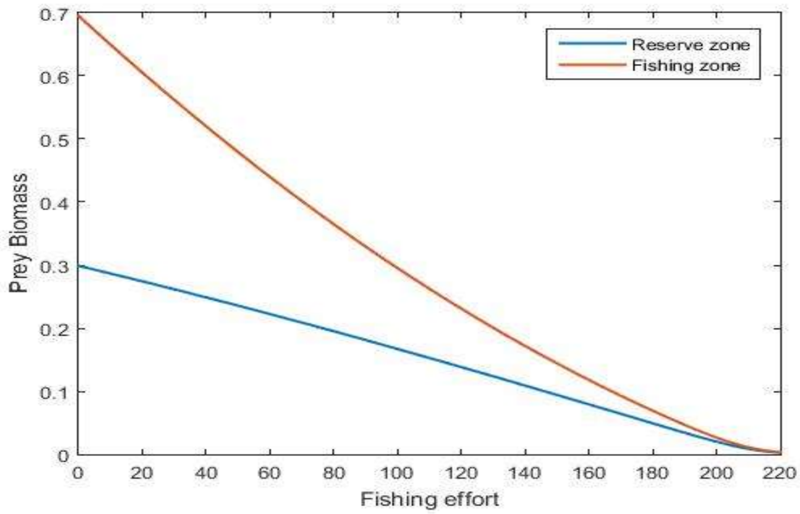

Figure 4.1: Variation of prey biomass with the increasing fishing effort when 30% reserve is taken into consideration

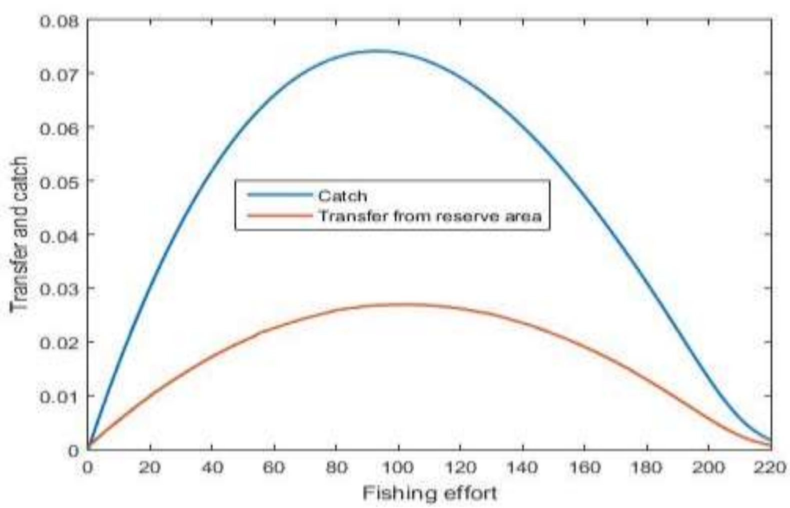

Figure 4.2: Variation of transfer and catch with the increasing fishing effort when 30% reserve is taken into consideration.

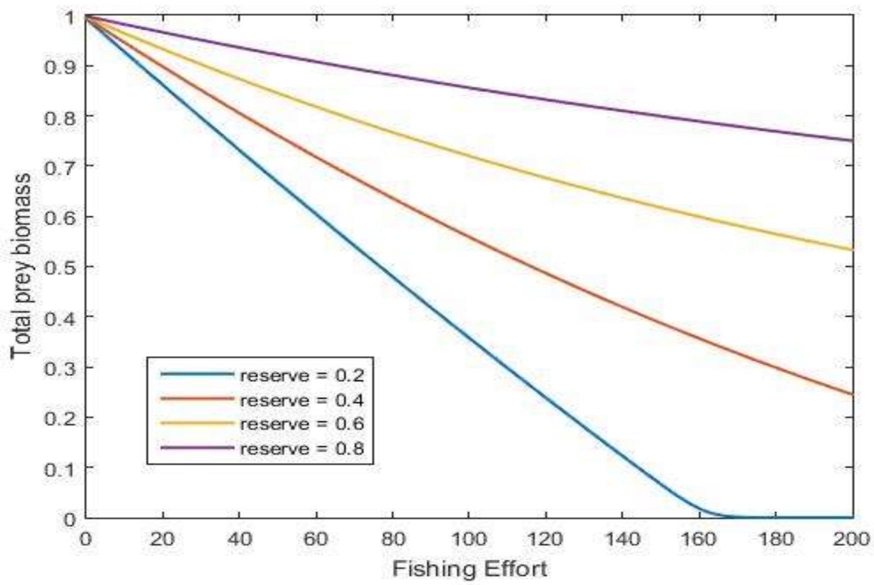

Figure 4.3: Variation of prey biomass with the increasing fishing effort depending on the size of the reserve.

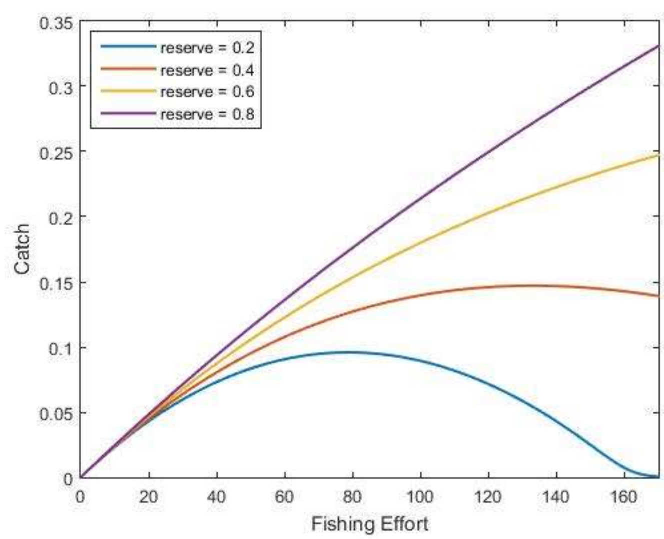

Figure 4.4: Variation of catch with the increasing fishing effort depending on the size of the reserve.

SIMULATION WITH PREY-PREDATOR RELATIONSHIP

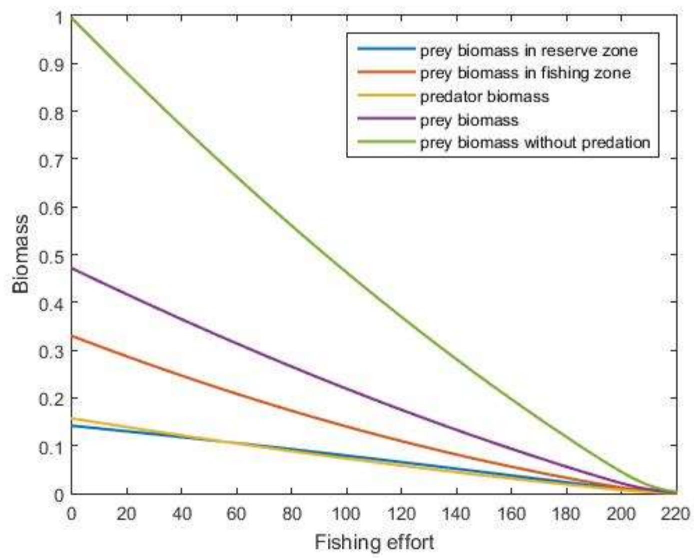

Figure 4.5: Variation of biomass with the increasing fishing effort when 30% reserve is taken into consideration in presence of predators.

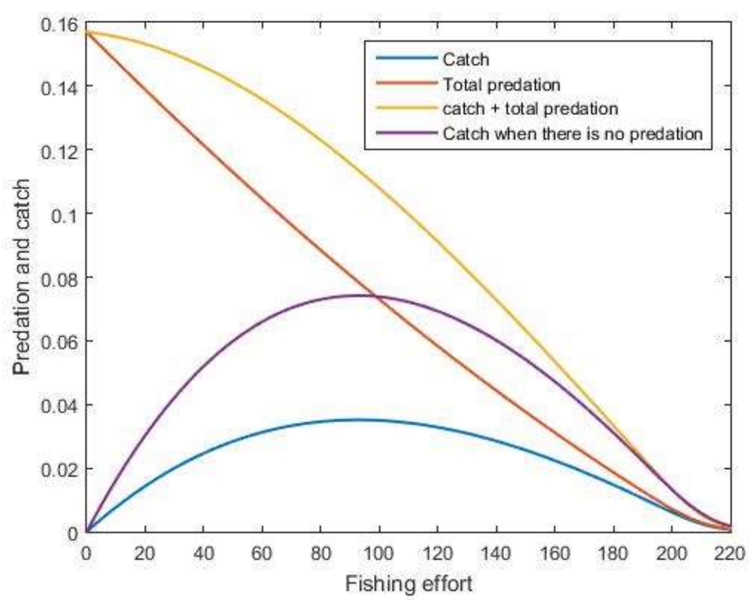

Figure 4.6: Variation of catch and predation with the increasing fishing effort when 30% reserve is taken into consideration.

DISCUSSION

The impact of fishing activity on fish biomass is illustrated in Figure 4.1. It is clear from the figure that fishing effort has a significant influence on fish biomass. As fishing effort increases, both the fraction of the fish stock in the fishing zone and the fraction in the reserve zone tend to approach their respective extinction limits, especially when a 30% reserve is considered for fish biomass. The decrease in the fraction of stock within the reserve zone is primarily attributed to the transfer of biomass from the reserve area to the open area, ultimately affecting harvesting efficiency and contributing to the depletion of the fish stock to its extinction limit. These findings align with the analytical results obtained at the fishing-free equilibrium.

Fig. 4.2 illustrates the relationship between fishing effort, the transfer of fish stock from the reserve zone to the fishing zone, and the catch in the fishing zone. The figure reveals that as fishing effort increases, both the transfer of stock and the catch increase, but only up to a certain point. Beyond this threshold, both the transfer and catch decline and eventually approach zero as fishing effort continues to rise. This phenomenon occurs because, initially, when the biomass in the open zone is heavily fished, there is a significant disparity in stock density between the two zones. This leads to an increased flow of transfer from the reserve zone to the fishing zone, resulting in the catch reaching its maximum level as the transfer flow reaches its peak. However, once this peak is surpassed, during the later stages, when the transfer rate decreases, the catch also decreases and eventually approaches zero as fishing effort intensifies.

In Figure 4.3, the graph illustrates the relationship between the total fish biomass and fishing effort, taking into account the relative size of the reserve. The graph clearly shows that having a portion of the fish stock within the reserve area provides some safeguard against the decline of the overall fish biomass as fishing effort increases. However, it's important to note that it's not feasible to entirely prevent the decline of the entire fish stock within each reserve area, as depicted in the figure. The protection of the fish stock from extinction becomes more effective as the size of the reserve area is enlarged.

The relationship between the catch and fishing effort is depicted in Figure 4.4, and it is evident from the graph that the relative size of the reserve area exerts a significant impact on the catch concerning fishing effort. Upon closer examination of the figures, we observe that in the absence of a reserve area, the catch of the fish stock increases as fishing effort intensifies, eventually reaching its maximum capacity. However, beyond this threshold, the catch starts to decline and ultimately reaches zero because the stock has already collapsed.

Conversely, when the reserve area is expanded, the total fish stock benefits from some protection against extinction in the face of escalating fishing efforts. Consequently, the catch increases in tandem with the rising fishing effort, as more fish migrate from the reserve zone to the open zone due to differences in population densities between the two areas. Nevertheless, the catch diminishes when the portion of the fish stock in the reserve zone collapses, ultimately dwindling to zero as fishing effort continues to increase.

The impact of fishing effort on prey and predator biomass is illustrated in Figure 4.5. The figure reveals that as fishing effort increases, both prey and predator biomass eventually decline to the point of extinction. Notably, even as fishing effort rises, the prey biomass is consistently lower when predators are present compared to when predators are absent, although both ultimately reach extinction under certain fishing effort levels. This intriguing outcome underscores the significance of prey-predator interactions in fishery management. As fishing effort intensifies, the prey biomass diminishes and approaches zero, consequently leading to a reduction in predator biomass, eventually resulting in their extinction due to the absence of available food resources.

The impacts of the exploitation of terms such as catch and predation are illustrated in Figure 4.6 for a specific marine reserve size as fishing effort intensifies. Figure 4.6 provides a clear understanding of how prey biomass is distributed between predators and fishermen. The figure demonstrates that when predators are present, the catch by fishermen is consistently lower than when predators are absent. However, as fishing effort increases, it becomes evident that the overall level of exploitation, including predation and fisherman catch, decreases and eventually approaches zero at a certain level of fishing effort. Additionally, the figure shows that the total predation term also decreases and approaches zero with increasing fishing effort. This is because Figure 4.5 indicates that predator biomass reaches extinction due to competition for food.

CONCLUSION

In this paper, we proposed a mathematical model to investigate the sensitivity analysis of marine reserves and its implications as a fishery management tool. We determine the fishing free zone and protected zone established a stable environment by Routh Hurwitz stability criterion since all eigen values $\lambda_i < 0$ with A, B and C been constants is greater than zero. The non-trivial steady state solution of the protected zone was equally determined; Besides, it was disclosed that fishing effort has a significant influence on fish biomass. It is reported that as fishing effort increases, both the fraction of the fish stock in the fishing zone and the fraction in the reserves zone tend to approach their respective extinction limits especially when a 30% reserve is considered for fish biomass while the decrease in the fraction of stock with in the reserve zone is primarily attributed to the transfer of biomass.

From the reserve area to the open area, ultimately affecting harvesting efficiency and contributing to the depletion of the fish stock to its extinction limit. Those findings are in agreement with the analytical results obtained at the fishing free equilibrium. It was equally found that when the biomass in the open zone is heavily fished, there is a significant disparity in stock density between the wo zones as reported in figure 4.2. This leads to an increased flow of transfer from the reserved zone to the fishing zone resulting in the catch reaching its maximum level as the transfer flow reaches its peak.

Finally, it was equally found that the fishing zone that is not protected is open to predator interference which may lead to extinction of the prey; whereas, the protected zone does not allow predator and hence the fishing environment is reserved.

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