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A Verification of Riemann Non-Trivial Zeros by Complex Analysis by Matlab™ Computation

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ABSTRACT: *With my most recent paper, I tried to prove the Riemann Hypothesis by catching out those contradictory parts of the non-trivial zeros. In the present paper, I will try to verify these known values of Riemann nontrivial zeros by first using U.S.A. Matlab coding with a list of well-organized complex analysis theories. At the same time, as the major core of my verification is just a mono-direction one (i.e. there may be a possibility of the missing non-trivial zeros although the residue value is zero), hence this author try to solve such problem by assuming that there are some other zeros existing between the two known zeros but the contradiction arises – as singularity implies the residue has a value with a multiple of 2πi. In addition, this author also apply the ingenious design (or a hybrid skill) with Feynman technique and Integration by parts to solve a special zeta function integral. Next, this author finds that one may consider those non-trivial zeros as a Fourier transform (or an impulse) between other normal complex numbers. The result is consistent with my previous papers in quantum physics [23], [25] for the electron jumps or reverse. Hence, we may get the (dirac) delta equation for Riemann Zeta. Then we may formulate our quantum circuit & computer. Finally, this author concludes all findings with an algorithm for searching, finer and checking the non-trivial zeros like below:*

***Step 1**: Use the computer software with some suitable program codes for an elementary search of feasible non-trivial zeta values among the closed real-complex plane interval – Method Matlab Simulation for searching zeta zeros;*

***Step 2**: Substitute back the values laying in the contour interval for zeta as found in Step 1 into the limit of* $\frac{ln(zeta(z))}{(zeta\prime(z))}$ *in order to adjust the answer in a finer and accurate way (just like the case of Newton's method etc) with more decimal digitals – Method Ingenious Design for finer the zeta zero's values;*

***Step 3**: Employ the Cauchy Residue Theorem for a check and hence confirm the previous found non-trivial zeta roots' uniqueness without any zeta zeros laying in between the two consecutive zeta roots – Method Cauchy's Residue for checking those already found zeta zeros.*

KEYWORDS: verification, Riemann non-trivial zeros, complex analysis, matlab™ computation

INTRODUCTION

There are lots of ways to solve the problem of Riemann Hypothesis. In my previous paper [1], I tried to use some algebraic methods and got some self-referenical contradictory results. These outcomes may finally lead to an algorithm for solving the Riemann Hypothesis. In the present paper, this author will mainly apply one of the complex analysis method – analytic continutation by

Taylor series [2] & [3] together with the complex contour integral [4] to find the numerical values of the Riemann Hypothesis non-trivial zeros. Indeed, this author will first implement some U.S.A. Matlab™ (student lisenced 2022 version) programming script to seek for the non-trivial zeros values' approximation. At the same time, the computer that calculates values in between these Riemann non-trivial zeros will give the complex contour integral equal to zeros. However, the result does NOT imply that there are NO non-trivial roots existing as the pole may exist or the residue cancellation may occur [5]. This author will show mathematically that the computed Riemann non-trivial zeros are actually uniquely defined as otherwise a contradiction will occur (or a multiple of 2π). Certainly, one should eliminate the option of residue cancellation from the complex conjugate paired Riemann non-trivial zeros in such case.

A Literatue Review for the well organised (or linked) mathematical theories to guess the non-trivial Zeros

When we want to compute or vertify those Riemann non-trivial zeros through the method of complex analysis, we may need to start from the elementary integration on the real line [7]. Indeed, if we integrate a function f(x) on the real line from the left to right, we may get a positive value in general. On the contrary, if we integrate on the real line from the right to left, then the resulted value will be negative. However, the above situation is NOT true in the case of the complex functions where the path of the contour is actually irrelvant. Thus, for the following contour integral:

$$\oint_c \frac{1}{z^2} dz ,$$

the computed value is in practice path independent or we will get the same calculated value NO matter we select which path(s) for the computation of the complex contour integral. To be precise [5], we have the following theorem:

Actually, by the Fundamental Theorem of Calculus, we have:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

$$\frac{d}{dx} f(x) = f'(x)$$

Indeed, it makes sense for the integral from 1 to 2 on the real line but NOT makes sense for the integral from 1 to i (or $\sqrt{-1}$) . But if we blindly apply Fundamental Theorem of Calculus to the function f'(z) = Z^2, then the

f(z) = $\frac{Z^3}{3}$. If we integrate the contour $\oint_c z^2 dz$ from 0 to (1+i), we may get: [$\frac{(i+1)^3}{3}$ - $\frac{0^3}{3}$] or after simplify, $\frac{-2}{3}+\frac{2}{3}i$ or we have the Principal of Path Independence [5]:

Let f(z) be a function that is analytic throughout a simple connected domain D, and let z1 and z2 lie in D. Then if we use contours lying in D, the value of $\int_{z_1}^{z_2} f\ (z)$ will not depend on the particular contour used to connect z_1 and z_2. The mirror image inverse is also true.

Hence, $\oint_c \frac{1}{z^2}dz$ from z_0 to z_1 = $\frac{z_1^3}{3}-\frac{z_0^3}{3}$. Indeed, the integration of $\int. \frac{1}{z^2}dz = \frac{-1}{z}$ + c. In addition $\oint_c \frac{1}{z^2}dz$ from z_0 to z_1 equals to $\frac{-1}{z_1}$ - $\frac{-1}{z_0}$. However, if one is considering the contour $\int. \frac{1}{z}dz = ln(z)$ + c. If this is a contour from 1 to i, then $\oint_c \frac{1}{z}dz = ln(1) - ln(i)$. It is no doubt that $ln(1) = 0$ but in general, $ln(1)= 2n\pi i$ for $n \in \mathbb{Z}$. Similarly, $ln(i) = (2n+\frac{1}{2})\pi i$ for $n \in \mathbb{Z}$. To sum up: If the single-valued F(z) is the anti-derivative of f(z), both are well defined around curve C with start & end points z_0 & z_1, then

$$\oint_c f(z) = F(z_1) - F(z_0)$$

Actually, $ln(z)$ is NOT a single valued function but a multiple-valued one.

In reality [5], (N.B. One need to employ parametric substitution for the complex contour integration)$\oint_{|z|=r} \frac{1}{z}dz = I \int_0^{2\pi} e^0\, d\theta = 2\pi i$, by [5], we have the famous Cauchy Integral Formula:

$$f(z_0) = \frac{1}{2i\pi}\oint_C \frac{f(z)}{(z-z_0)}dz$$

$$= 0 \qquad \text{if the contour does NOT contain a singularity}$$

$$\text{or} \quad = 2i\pi \qquad \text{if the contour does contain a singularity}$$

In other words, we may find out or verify all zeta zeros along the axis x = 0.5 by the above method. But the main problem is that the computer calculated value equals to zero does NOT imply there will be NO singularity nor non-trivial Zeta zeros. There may be indeed some missing zeros. However, the problem may still be solved by [5]:

1. Fixable Poles;
2. Extension of the Laurent series;

(N.B. The proof is shown in [8])

3. Cauchy's residue theorem

(N.B. The proof is shown in [5])

A Matlab simulation for calculating the non-trivial Zeta Zeros

In order to locate the approximate (or the elementary initial guess of non-trivial zeta roots) position on the x = 0.5 axis, this author have first written the following program segment in Matlab code [10] & [15], [18] as below:

```
fun = @(z) 1./zeta(z);
m = 140;
for j = 40:m
    K = 0.25.*j
    L = 0.25.*(j + 1)
    C = [0.25+K.*1i 0.25+L.*1i 0.75+L.*1i 0.75+K.*1i]
     q2 = integral(fun,0.25+K.*1i,0.25+K.*1i,'Waypoints',C)
end
```

In practice, the main idea or component of the above program code is to loop along the line x = 0.5 (in the middle surrounded) with each side's width of 0.25. Then the computed values are substituted back into the complex contour integral. Hence, one may get the answer with values either equal to zero or a multiple of 2πI. Thus according these two types of answer, one may classify the contour with either "NO non-trivial zeros" (corresponds to a contour integral value equals to zero) – the detailed proof will be provided in the next section or "non-trivial zeros" (corresponds to a contour integral value equals a multiple of 2πI) – the Cauchy's Residue Theorem. Indeed, by the application of analytic continuation for Re(z) > 0.5 to $\frac{1}{\zeta(z)}$ (N.B. One may easily to have a fact check that $\frac{1}{\zeta(z)}$ fullfils the requirements of analytic continuation for all z with Re(z) > 0.5 from either through the text-book or the Internet), one may further extend the above two types of contour integral result into the lower (negative imaginary) part of the positive real-negative complex plane whenever Re(z) > $\frac{1}{2}$. In addition, there will be NO need to amend and repeat the program code segment for the positive real Re(z) > 0.5 but negative (or lower) complex plane to search for the conjugate pair of those non-trivial zeta zeros in the upper part of the real-complex plane.

By the way, for the Re(z) < 0.5, the analytic continuation may NOT work for the $\frac{1}{\zeta(z)}$ function.

Thus, the best way to solve the Riemann Hypothesis problem (or a search for non-trivial zeta zeros) is to amend and modify the above posted Matlab program segment code [11] for the real-complex plane located on the left hand side of the real number line x = 0.5 such as the case below [16] & [17], [19]:

```
fun = @(z) 1./zeta(z);
m = 140;
n = 40;
for s = 1:n
    O = 0.05.*s
    P = 0.05.*(s+2)
        for j = 40:m
            K = O.*j
            L = O.*(j + 1)
            C = [O+K.*1i O+L.*1i P+L.*1i P+K.*1i]
            q2 = integral(fun,O+K.*1i,O+K.*1i,'Waypoints',C)
        end
end
```

In practice, the main concept of the above program segment code is to first loop over the x-coordinate(s) and then loop over the y-coordinates. Hence, with the both pair of (x,y) double looping, the computer software U.S.A. Matlab will compute my proposed complex contour integral formula's answer for the code: O+K.*1i O+L.*1i P+L.*1i P+K.*1i and runs according to the remaining real-complex plane $\{0 < Re(z) < 0.5 \ \& \ Im(z) > 0 \}$. By making some suitable adjustments (this will be left to those interesting parties as it may NOT be a difficult task) to the above program segment code for the $Re(z) < 0$, one may continue the non-trivial zeta zeros searching process [20]. The complex contour integral results as this author may find for $0 < Re(z) < 0.5$ so far is zero which indicates there is NO non-trivial zeta zeros existing in the above Re(z) range and hence with necessary amendments, one may also find the complex contour integral result for the range $\{0 < Re(z) < 0.5 \ \& \ Im(z) < 0 \}$ or the conjugate pair for $\{0 < Re(z) < 0.5 \ \& \ Im(z) > 0 \}$. Certainly, one may also make some necessary amendments and modifications to my program segment code as posted above for the lower (negative imaginary) real-complex plane (or the $Re(z) < 0 \ \& \ Im(z) > 0$ or $Im(z) < 0$), (this will be left to those interesting parties as it may NOT be a difficult task) and thus look for any non-trivial zeta zeros if they actually exist [21].

In brief, by the Area Theorem [21], there may be a comformal mapping induced from the Matlab program codings. The implication is the preservation of the mapping angle together with the area but NOT the perimeters [22].

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A Proof for the Uniqueness of Riemann non-trivial Zeros

Suppose there are other n roots $\{s_1, s_2, \ldots, s_n\}$ that lays on the Riemann non-trivial zeros (say y_1 & y_2) and on the upper complex plane with the corresponding negative complex conjugate at the line Re(z) = 0.5.

Then for any root $s_i \in set(s_1, s_2, \ldots, s_n)$, the residue for ξ(s) at individual s_i is:

$$2\pi I \left|Res\left[\frac{1}{(z-s_1)(z-s_2)\ldots(z-s_n)(z-y_1)\ldots(z-y_n)}, s_i\right]\right|$$

$$= 2\pi I \left| \lim_{z\to z_i} \frac{1}{(z-s_1)(z-s_2)\ldots(z-s_n)(z-y_1)\ldots(z-y_n)} \right|$$

$$= 2\pi I \frac{1}{(z-s_1)(z-s_2)\ldots(z-s_{(i-1)})(z-s_{(i+1)})\ldots(z-s_n)(z-y_1)\ldots(z-y_n)}$$

$. \neq 0$ which is a multiple of 2π.

Then for all of the extra roots that lay between the interval, say $0.5+y_1i$ & $0.5+y_2i$, we have:

$$\oint_c \frac{ds}{\zeta(s)} = \oint_c \frac{dz}{(z-s_1)(z-s_2)\ldots(z-s_n)(z-y_1)\ldots(z-y_n)}$$

$$= -2\pi I\sum \left|Res\left[\left(\frac{1}{(z-s_1)(z-s_2)\ldots(z-s_i)(z-s_{(i+1)})\ldots(z-s_n)(z-y_1)\ldots(z-y_n)}\right), s_i atthecomplexplane\right]\right|$$

$$= -2\pi I\sum \left|\frac{1}{(s_i-s_1)(s_i-s_2)\ldots(s_i-s_{(i-1)})(s_i-s_{(i+1)})\ldots(s_i-s_n)(s_i-y_1)\ldots(s_i-y_n)}\right|$$

$. \neq 0$ and is also a multiple of 2π.

Obviously, the above result is contradicting to the computed contour integral value obtained from the U.S.A. Matlab™ liscened student version computation for the interval between ($0.5+Iy_1$, $0.5+Iy_2$) which is zero (suppose $0.5+Iy_1$, $0.5+Iy_2$ locate on the upper complex plane while the negative plane is just by adding a negative sign to make it positive or take the absolute value without loss of generality).

(N.B. The inverse of the Cauchy's Residue Theorem may seem to be true in such case. This is because if the reside of a fuction is equal to a multiple of π, then the reader may finally prove that the fuction is also analytic.)

In addition, if there may be any **residue cancellation** for the conjugate pair of the complex Riemann Zeta non-trivial roots, we may need to **take the absolute value of the residue** in order to prevent such situation happens. Hence, the contradiction to the computer program's calaulation indicates the assumption of the extra roots $\{s_1, s_2, \ldots, s_n\}$ laying on the interval $\{0.5+Iy_1$ & $0.5+Iy_2\}$ of two consecutive Riemann roots should be wrong. Thus, we may conclude that the known Riemann Zeta's non-trivial zeros are uniquely laying on the real number line x = 0.5. **Or the gap between any two consecutive Riemann Zeta's non-trivial zeros contains NO other zeros. Indeed, this author have already proved the present known Riemann Zeta non-trivial zeros are unique**.

An Intuitive Illusion for the Contour Complex Integral

For the common (ordinary/normal) differentiation and integration over the real number line, we may establish the following derivative-primitive relationship:

A derivative's root (F'(x) = f(x) = 0, i.e. solve f(x) = 0 for x to find the roots of f(x)) is just its primitive function's (i.e. F(x)) optimum (maximum/minimum) points (i.e. substitute x's previous computed values back into F(x) to find its optimum points).

Therefore, according to the above derivative-primitive relationship, for those computer calculated complex contour integral equal to zero, the roots of the complex contour integral will then be equal to its primitive function's optimum points. That is:

$$\int_{a+bI}^{c+dI} . \oint_c \frac{ds}{\zeta(s)} \, ds = (\int_{a+bI}^{c+dI} . f(s)\, ds)$$

$$= [F(s_1) - F(s_2)]$$

= optimium (maximum/minimum) points.

But we are talking about the complex valued function and all of the integrals are evaluated by complex numbers and indeed we cannot compare these complex numbers' dimensions. Thus, there will be NO maximum or minimum for these complex valued numbers. We may only compare those complex values' norm or modulus in a numerical sense (i.e. real-valued numbers) but NOT the complex valued number obtained from the Zeta function etc. Otherwise, the comparison is just an interesting common mistakes or an intuitive illusion.

In reality, the aforementioned complex contour integeral Mean Value Theorem may lead to both of the Maximum/Minimum Modululus Theorem or even the Liouville's Theorem [4] & [5] etc.

However, one more thing that is interesting may be the average modulus of such integral Mean Value Theorem, which gives us for any consecutive interval of two non-trivial Zeta Riemann Zeros with the average value h(z) on the circle | z-z_0 | = r is given by [3]:

$$A(r) = \int_0^{2\Pi} h\,(z_0 + re^{\Theta}) d\Theta$$

Then for $z_1 = z_0 + r_1 e^{\Theta_1}$ & $z_2 = z_0 + r_2 e^{\Theta_2}$ with the parametrization

$\gamma:[0, 2\pi] \rightarrow \mathbb{C}$, t |→ $z_0 + re^{\frac{i*t}{r}}$ which is the arc length, we may have:

$$A(r_1) - A(r_2) = \frac{1}{2\Pi*(r_1-r_2)}\,(r_1 - r_2)\left[\int_0^{2\Pi} h\left(z_0 + r_1 e^{\frac{i*t_1}{r_1}}\right)dt - \int_0^{2\Pi} h\left(z_0 + r_2 e^{\frac{i*t_2}{r_2}}\right)dt\right]$$

$$= \frac{1}{2\Pi*(r_1-r_2)}\left(\frac{r_1}{t_1} - \frac{r_2}{t_2}\right)\int_0^{2\Pi}\left[h\left(z_0 + r_1 e^{\frac{i*t_1}{r_1}}\right) - h\left(z_0 + r_2 e^{\frac{i*t_2}{r_2}}\right)\right]dt$$

where $A = \frac{1}{2\Pi*(r)}\int_0^{2\Pi} h\left(z_0 + re^{\frac{i*t}{r}}\right)dt$ or the average value equals to zero [4].

This result may imply the existence of a Fourier transform over the complex number axis. This is because one may imagine the appearance of the Zeta roots that are located between the normal complex numbers. The situation is just like a sudden impulse for those Zeta zeros. Or even through a suitable (similar to the mirror image kind of inverse) Laplace transform together with the delta function associated equation/formula,

$$\frac{\left(4kDirac(p) - 4Dirac(1,p) + 4cot\left(\frac{2(k-1)}{k+1}\right)^2\left(kDirac(p) - Dirac(1,p)\right) + 3cot\left(\frac{2(k-1)}{k+1}\right)\left(k^2Dirac(p) - 2kDirac(1,p) + Dirac(2,p)\right)\right)}{(k+1)^4}$$
$$\frac{+16\left(1 + cot\left(\frac{2(k-1)}{k+1}\right)^2\right)\left(k^2Dirac(p) - 2kDirac(1,p) + Dirac(2,p)\right)\left(4cot\left(\frac{2(k-1)}{k+1}\right) + 3k + 3\right)}{(k+1)^6}$$
$$\frac{+4\left(-2Dirac(1,p) + Dirac(p)(1+3k)\right)cot\left(\frac{2(k-1)}{k+1}\right)}{(k+1)^3} = 0$$

(by using Canadian Maple-soft, Maple, Version 2022, Student Edition with paid license), one may even establish the corresponding quantum circuit when further investigated from some suitable software(s).

In practice, the aforementioned concept of considering the Riemann non-trivial zeros as a sense of discontinuity (or holes) between those normal and continuous imaginary number along the x = 0.5 may be extended to the topic of algebraic topology. In other words, one may apply the ideas of both homology and homotopy (which may be indeed complementary to each other) together with the corresponding residue theorem etc for a fixed closed chain in a complex plane [9], to find out all of the Riemann non-trivial zeros along the axis x = 0.5. Such problem may then lead to the study of the “stable Homotopy Around the Arf-Kervaire Invariant” etc. At the same time, there is an additional application for both of the homology and homotopy in the algebraic topology in topic of our structural biology. That say, for those infected virus with envelop(es) such as the SARS-CoV-2 one, if we can apply both of the homology and homotopy theories [6]to find out/identify all of holes and have a fully investigation in all of the virus’s structural envelop(es) (or holes) through the respective specially developed computer software, then we may develop the corresponding drugs and hence block all of these virus’ envelop(es). The result is we can finally reduce or even eliminate the spread of such enveloped virus or to be precise, the COVID-19 infection etc.

To sum up, one may consider those zeta zeros as the holes (singularities) or levels of level of cliff in a structural sense (normal complex numbers and then non-trivial zeta zero that repeated forever). At the same time, one may think such kind of structure as the different discrete energy levels of the electron(s) in quantum mechanics or quantum physics. The result is consistent with my previous papers' findings in quantum mechanics [23], [24]. Then there may be a quantum leap(s) or jumping(s) for the electrons. Hence, there may be a Laplace or Fourier transform and the vice versa that gives us the delta dir-ac equation(s)/formula(e) for establishing the corresponding quantum circuit(s) or even the quantum computer [25].

An Ingenious Design to Compute the Zeta Contour Integral

In order, to compute the complex (contour) integral in particular as (0.25+13*I, 0.25+13.25*I, 0.75+13.25*I, 0.75+13*I), one may get (by the Feynman' Integration techniques & the Partial Integration with Integration by Parts method [13] & [14]):

$$\int_{a+b*I}^{c+d*I} \frac{1}{zeta(z)} dz = \int_{a+b*I}^{c+d*I} \frac{\partial ln(zeta(z))}{\partial zeta(z)} dz$$

$$= \int_{a+b*I}^{c+d*I} \frac{1}{zeta\prime(z)} \partial ln(zeta(z))$$

$$= \frac{ln(zeta(z))}{(zeta\prime(z))} - \int_{a+b*I}^{c+d*I} ln(zeta(z))\, \partial \left(\frac{1}{(zeta\prime(z))}\right)$$

$$= \left[\frac{ln(zeta(z))}{(zeta\prime(z))} \right]_{a+b*I}^{c+d*I} - \int_{a+b*I}^{c+d*I} ln(zeta(z)) \frac{zeta\prime\prime(z)}{[-zeta\prime(z)]^2} dz$$

$$= \left[\frac{ln(zeta(z))}{(zeta\prime(z))} \right]_{a+b*I}^{c+d*I} + \int_{a+b*I}^{c+d*I} \frac{ln(zeta(z))}{[zeta\prime(z)]} \frac{\partial ln(zeta\prime(z))}{\partial(zeta\prime(z))} \partial(zeta'(z))$$

$$= \left[\frac{ln(zeta(z))}{(zeta\prime(z))} \right]_{a+b*I}^{c+d*I} + \left[\frac{ln(zeta(z))}{(zeta\prime(z))} \frac{1}{zeta\prime(z)} zeta'(z) \right]_{a+b*I}^{c+d*I} - \int_{a+b*I}^{c+d*I} zeta'(z) \partial \frac{ln(zeta(z))}{[zeta\prime(z)]^2}$$

$$= \left[(n) \frac{ln(zeta(z))}{zeta\prime(z)}\right]_{a+b*I}^{c+d*I}$$

But by considering $\int_{a+b*I}^{c+d*I} \frac{1}{zeta(z)} dz = n \left[\frac{ln(zeta(z))}{(zeta\prime(z))}\right]_{a+b*I}^{c+d*I}$ for the complex contour integral in the case as (0.25+13*I, 0.25+13.25*I, 0.75+13.25*I, 0.75+13*I), one may get the following:

$$\left[\frac{ln(zeta(z))}{(zeta\prime(z))}\right]_{0.75+13*I}^{0.75+13.25*I} + \left[\frac{ln(zeta(z))}{(zeta\prime(z))}\right]_{0.75+13.25*I}^{0.25+13.25*I} + \left[\frac{ln(zeta(z))}{(zeta\prime(z))}\right]_{0.25+13.25*I}^{0.25+13*I} + \left[\frac{ln(zeta(z))}{(zeta\prime(z))}\right]_{0.25+13*I}^{0.75+13*I} = 0$$

With respect to the above ingenious design for the complex contour integral, one may get all of the integral evaluations are cancelled with each other and hence finally we may get a zero. There may be a possibility that some zeta zeros is still inscribed. Hence and obviously, by the above ingenious method, any zeros result calculated may NOT imply there are NO non-trivial zeta zeros inscribed inside the contour like the case (0.75+14.25*I, 0.75+14*I, 0.25+14.25*I, 0.25+14.25*I). Indeed, if there is a non-zero contour integral and suppose $\{s_1, s_2, \ldots s_i, \ldots, s_n\}$ are all non-trivial zeros:

Case I: S are (or approaching to) non-trivial zeta roots S_i,

$$\lim_{s \to s_i} . \frac{ln(zeta(s))}{(zeta\prime(s))}$$

$$= \lim_{s \to s_i} . \frac{\sum ln(s-s_i)}{\sum (s-s_1)\ldots(s-s_{(i-1)})(s-s_{(i+1)})\ldots(s-s_n)}$$

But if we express the logrithmic function in Taylor Series, then one may get:

ln $(s-s_i)$ = $[s-(s_i-1)]$ + s- $\frac{(s_i-1)^2}{2}$ +… = $[s-(s_i-1)]$ (approximately)

as $\lim_{s \to s_i}$.[ln $(s - s_i+1)$] ⟶ 1, when $s - s_i$ ⟶ 0 or s ⟶ s_i, where s_i are zeta non-trivial roots.

Also [12], Zeta'(s) = $-\sum_{n=2}^{\infty} \frac{ln(n)}{n^s}$

$$= - \left\{ \frac{ln(2)}{2^s} + \frac{ln(3)}{3^s} + \ldots \right\}$$

$$= - \frac{ln(2)}{2^s} \text{ (approximately)}$$

Hence, $\lim_{s \to s_i} . \frac{ln(zeta(s))}{(zeta\prime(s))}$ = 1 /[- $\frac{ln(2)}{2^{s_i}}$]

$$=[- \frac{2^{s_i}}{ln(2)}]$$

$$= e^{-2^{s_i} - ln(2)} \text{ (take the exponent for both numerator and de-numerator)}$$

Let y = -2^{s_i}, then $ln(y)$ = $s_i\, ln(-2)$ = $s_i\, ln(2)$ + πI and according to [26], we have:

$$ln(y) = s_i\,(s_i \sqrt[s_i]{2} - s_i) + \pi I$$

$$\lim_{s_i \to \infty} . ln(y) = \lim_{s_i \to \infty} . s_i\,(s_i \sqrt[s_i]{2} - s_i) + \pi I$$

$$\lim_{s_i \to \infty} . ln(y) = \pi I$$

$$y \longrightarrow e^{\pi I} \text{ when } s_i \longrightarrow \infty$$

Therefore, $\lim_{s_i \to \infty}$.[- $\frac{2^{s_i}}{ln(2)}$] = $\frac{e^{\pi I}}{ln(2)}$ = $\frac{-1}{ln(2)}$ which is obviously a constant.

Case II: S are NOT non-trivial zeta roots,

For otherwise $s \neq s_i$, where s is NOT a zeta root,

$$\lim_{s\to s_k\neq s_i} \cdot \frac{\sum ln(s-s_i)}{\sum(s-s_1)\ldots(s-s_{(i-1)})(s-s_{(i+1)})\ldots(s-s_n)} = \frac{\sum ln(s-s_i)}{\sum(s-s_1)\ldots(s-s_{(i-1)})(s-s_{(i+1)})\ldots(s-s_n)}$$

$$\frac{\sum ln(s-s_i)}{\sum(s-s_1)\ldots(s-s_{(i-1)})(s-s_{(i+1)})\ldots(s-s_n)} = \frac{ln(s^n)}{(n-1)s^{(n-1)}} = \frac{(nln(s))}{(n-1)s^{(n-1)}}$$

$$= (\frac{1}{1-(\frac{1}{n})})(\frac{(ln(s))}{s^{(n-1)}})$$

$$= 0 \qquad \text{when } n \to \infty$$

(N.B. By Ratio test, one may get $\frac{(n+1)(n-1)ln(s)s^{(n-1)}}{(s^n)n^2 ln(s)}$ = (1 - $\frac{1}{n^2}$) $\frac{1}{s} = \frac{1}{s}$ < 1 and hence converges when $n \to \infty$ and s > 1)

Hence, for any contour integral without zeta zeros inscribed as shown above with the interesting ingenious design, the limit will tends to zeros or the vice versa. Otherwise, for the above non-zero limit (= $\frac{-1}{ln(2)}$ and is a constant. Moreover, by the Residue Theorem, the limit must be a multiple of 2πI.) which does indicate that there is a zeta zero inscribed inside the contour integral such as the case in the contour (0.75+14.25*I, 0.75+14*I, 0.25+14.25*I, 0.25+14.25*I) or the vice versa.

CONCLUSION

To conclude, in the present paper, this author have presented three methods to find the zeta roots over the real value x = 0.5. These methods are:

I) Commercial Mathematics software for numerical simulation like U.S.A. Matlab & Mathematic-a, Canada's Maple Soft etc;

Initial elementary search for non-trivial Riemann Zeta zeros for a closed interval of the real-complex plane,

II) Cauchy's Residue Theorem;

Contour Integral gives two types of answer – zero or multiple of 2πI but one may need to show the uniqueness or the nonexistence of the zeta roots between assumed two conseuctive zeta root interval,

III) My well & ingenious designed method;

Contour Integral always gives you answer – zero and hence one may need to check with the limit of $\frac{ln(zeta(z))}{(zeta\prime(z))}$ at the contour interval to determine zeta zeros or not.

Actually, for the computer simulation method, the main idea is to employ double "looping" algorithm. One will be used for looping along real axis while the other will be employed for looping along the imaginary axis for the Re(z) < 0.5. For the second Residue theorem, the main concept is that for any contour integral equal to zero does NOT imply that there were NO zeta zeros laying on the closed interval between any two non-trivial zeros. Thus, this author have already proved that as in the aforementioned section that there is NO other non-trivial zeros between any non-trivial zeta zeros' interval (or the uniqueness). Finally, for the ingenious designed method, this author have shown that the limit of $\frac{ln(zeta(z))}{(zeta\prime(z))}$ will tend to ∞ for any zeta non-trivial zeros included in the range of a contour integral. On the other hand, the limit of $\frac{ln(zeta(z))}{(zeta\prime(z))}$ will tend to zeros when there is NO zeta non-trivial zeros included in the rang of a contour integral. All of these three methods work pretty well in finding the non-trivial zeta roots (and hence establish the corresponding model equation). On the other hand by a Fourier transformation to (the zeta roots model equation) and hunt for the series of our prime number (with model equation) or the vice versa. Technically, there is a duality relationship and hence the Fourier-Inverse Quantum Fourier Transform between the momentum space and the position space for our particles description in order to establish the quantum computer. May there also be similar anologically properties existing between our present heatest university depiction theories/models as this author have mentioned in [2] such that we can find the knack to travel in our deep space for practicing. However, the above suggested Fourier transform needs to be further proved in an abstracted way through the experimental data obtained from our deep universe observatory appartus together with the advances in the various mathematical methods etc. Therefore, the aforementioned proof for such kind of universe modelings' FT-IFT may be presently out of the scope in my research.

Last but NOT least, this author have combined the above three methods for searching, finnering and checking the non-trivial zeta zeros without a missing by the following algorithm:

Step 1: Use the computer software with some suitable program codes for an elementary search of feasible non-trivial zeta values among the closed real-complex plane interval – Method Matlab Simulation for searching zeta zeros;

Step 2: Substitute back the values laying in the contour interval for zeta as found in Step 1 into the limit of $\frac{ln(zeta(z))}{(zeta\prime(z))}$ in order to adjust the answer in a finer and accurate way (just like the case of Newton's method etc) with more decimal digitals – Method Ingenious Design for finer the zeta zero's values;

Step 3: Employ the Cauchy Residue Theorem for a check and hence confirm the previous found non-trivial zeta roots' uniqueness without any zeta zeros laying in between the two consecutive zeta roots – Method Cauchy's Residue for checking those already found zeta zeros.

REFERENCE

[1] Lam Kai Shun (2023) A Full and Detailed Proof for the Riemann Hypothesis & the Simple Inductive proof of Goldbach's Conjecture, International Journal of Mathematics and Statistics Studies, Vol.11, No.3, pp.1-10

[2] Analytic Continuation and the Zeta Function, Zeta-math, https://www.youtube.com/watch?v=CjSKmcWRFzE

[3] Lars V. Ahlfors, 1979, Complex Analysis – An Introduction to the Theory of Analytic Functions of One Complex Variable, McGraw-Hill Book Company.

[4] Mathematics Stack Exchange, June, 2017 https://math.stackexchange.com/questions/2243917/why-does-the-integral-for-the-average-value-of-a-complex-function-on-a-circle-no

[5] Wunsch, A. D. (1994). *Complex variables with applications* (2nd ed.). , p. 162-164, p.170, p.180-182, Addison-Wesley Pub. Co.

[6] K.S.Lam, 2021, Special issue AJMS Jul-Sep 2021 Combined 20211223 V1, Asian Journal of Mathematical Science, India. https://www.researchgate.net/publication/357321899_Special_issue_AJMS_Jul-Sep_2021_Combined_20211223_V1

[7] Complex Integration and Finding Zeros of the Zeta Function, Zeta-math, https://www.youtube.com/watch?v=uKqC5uHjE4g

[8] William, T.S. 2006, 2008, Complex Analysis with Mathematica, Cambridge University Press, p.278-p.282, England.

[9] William, F. 1995, Algebraic Topology – A First Course, Chapter 9, Springer Publication

[10] MathWorks, Help Centre, Complex Line Integral, https://www.mathworks.com/help/matlab/math/complex-line-integrals.html

[11] MathWorks, MatLab Answer, How can I create a nested loop? https://www.mathworks.com/matlabcentral/answers/393736-how-can-i-create-a-nested-loop

[12] Wolfram MathWorld, Riemann Zeta Function, https://mathworld.wolfram.com/RiemannZetaFunction.html

[13] CliffsNotes, Partial Integration, https://www.cliffsnotes.com/study-guides/differential-equations/review-and-introduction/partial-integration

[14] Dr. Trefor Bazett, The Dirichlet Integral is destroyed by Feynmann's Trick https://www.youtube.com/watch?v=ZZccxuOpb4k

[15] Mathematics Stack Exchange, Order of Integration, 2012, https://math.stackexchange.com/questions/164666/order-of-integration

[16] Mathematics Stack Exchange, Shifting the line of integration over the critical stripe of an L-function, 2011, https://math.stackexchange.com/questions/67616/shifting-the-line-of-integration-over-the-critical-stripe-of-an-l-function

[17] Selberg, Atle (1992), "Old and new conjectures and results about a class of Dirichlet series", *Proceedings of the Amalfi Conference on Analytic Number Theory (Maiori, 1989)*, Salerno: Univ. Salerno, pp. 367–385, MR 1220477, Zbl 0787.11037 Reprinted in Collected Papers, vol **2**, Springer-Verlag, Berlin (1991)

[18] Conrey, J. Brian; Ghosh, Amit (1993), "On the Selberg class of Dirichlet series: small degrees", *Duke Mathematical Journal*, **72** (3): 673–693, arXiv:math.NT/9204217, doi:10.1215/s0012-7094-93-07225-0, MR1253620, Zbl 0796.11037

[19] Murty, M. Ram (1994), "Selberg's conjectures and Artin L-functions", Bulletin of the American Mathematical Society, New Series, **31** (1): 1–14, arXiv:math/9407219, doi:10.1090/s0273-0979-1994-00479-3, MR 1242382, S2CID 265909, Zbl 0805.11062

[20] Murty, M. Ram (2008), *Problems in analytic number theory*, Graduate Texts in Mathematics, Readings in Mathematics, vol. 206 (Second ed.), Springer-Verlag, Chapter 8, doi:10.1007/978-0-387-72350-1, ISBN , MR 2376618, Zbl 1190.11001

[21] Conway, J. B. (2012). *Functions of One Complex Variable II* (Vol. 159). Springer Nature.

[22] MathOverFlow, 2018, Conformal Mappings that preserve angles and areas but not perimeters? https://mathoverflow.net/questions/311659/conformal-mappings-that-preserve-angles-and-areas-but-not-perimeters

[23] Shun L.K. (2023) The boundary between the Prime Gaps and the Interchangeable be-tween a Quantum Mechanics System & a Classical Chaos One, International Research Journal of Pure and Applied Physics, Vol.10 No.1, pp.16-23 *doi: https://doi.org/10.37745/irjpap.13vol10n1623*

[24] Lam Kai Shun (2023) A Full and Detailed Proof for the Riemann Hypothesis & the Simple Inductive proof of Goldbach's Conjecture, International Journal of Mathematics and Statistics Studies, Vol.11, No.3, pp.1-10 *doi: https://doi.org/10.37745/ijmss.13/vol11n3110*

[25] Shun L.K. (2023) The Quantized Constants with Remmen's Scattering Amplitude to Explain Riemann Zeta Zeros, International Journal of English Language Teaching, Vol.11, No.4, pp.,20-33 *doi: https://doi.org/10.37745/ijelt.13/vol11n42033*

[26] Is this limit actually Solvable? https://www.youtube.com/watch?v=y9z80Wwrog0

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