European Journal of Statistics and Probability
Vol.10, No.1, pp., 33-48, 2021
Print ISSN: 2055-0154(Print),
Online ISSN 2055-0162(Online)

# The Iwok-Nwikpe Distribution: Statistical Properties and Application

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**Citation**: Iwok, Iberedem Aniefiok and Nwikpe, Barinaadaa John (2022) The Iwok-Nwikpe Distribution: Statistical Properties and Application, *European Journal of Statistics and Probability*, Vol.10, No.1, pp., 33-48

**ABSTRACT:** *In this paper, a new continuous probability distribution named 'Iwok-Nwikpe distribution' is proposed. Some of the important statistical properties of the new distribution were derived. The shapes of the probability density function, cumulative distribution function and the survival function have been displayed for different values of the parameter. The moment generating function, the first three raw moments, the second and third moments about the mean and the distribution of order statistics were also derived. The parameters of the new distribution were estimated using maximum likelihood method. The flexibility of the Iwok-Nwikpe distribution was demonstrated using some real life data sets. The goodness of fit showed that the Iwok-Nwikpe distribution outperforms the one parameter exponential, Lindley, Shanker, Sujatha and Amarendra distributions for the data sets used in this work.*

**KEYWORDS:** Lindley distribution, Shanker distribution, Sujatha distribution, Moment generating function, Order statistics, Parameter estimation, Survival function and Goodness of fit.

## INTRODUCTION

Recent research in distribution theory shows that most classical distributions do not give satisfactory fit to real life data sets obtained from areas such as biological sciences, insurance, engineering, finance, medical science and so on due to lack of flexibility. To address this set back, a lot of efforts have been geared towards making existing probability distributions more flexible. Consequently, several methods now exist for extending an existing probability distribution. This could sometimes be enhanced by means of generalization, using the available generalized family of distributions.

## LITERATURE REVIEW

In rencent times, a mixture of classical distributions has been proven to be an efficient way of generating new distributions. Consequently, quite a lot of new probability distributions have been developed by this method. The mixture of distributions using various mixing proportions has been used by statisticians to derive new distributions in recent times.

For instance, the well-known Lindley distribution is a mixture of exponential distribution with parameter $\theta$ and gamma $(2, \theta)$ with mixing proportions $\frac{\theta}{\theta+1}$ and $\frac{1}{\theta+1}$. With this proportions, Lindley (1958) obtained a new distribution with probability density function (p.d.f) and cumulative distribution function (c.d.f):

$$f(x\,;\,\theta) = \frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}\ ;\quad x > 0,\qquad \theta > 0 \tag{2.1}$$

$$F(x\,;\,\theta) = 1 - \frac{\theta+1+\theta x}{\theta+1}(1+x)e^{-\theta x}\ ;\quad x > 0,\qquad \theta > 0 \tag{2.2}$$

Ghitany *et al* (2008) improved upon Lindley (1958) new distribution and gave detailed discussion about the properties of the Lindley distribution and the estimation of parameters with applications.

Shanker (2015) proposed the Shanker distribution by mixing exponential and gamma distributions using component mixture of the exponential distribution with parameter $\theta$ and gamma distribution having shape parameter 2 and scale parameter $\theta$. The Shanker's p.d.f and c.d.f were:

$$f(x\,;\,\theta) = \frac{\theta^2}{\theta+1}(\theta+x)e^{-\theta x}\ ;\quad x > 0,\qquad \theta > 0 \tag{2.3}$$

$$F(x\,;\,\theta) = 1 - \frac{(\theta^2+1)+\theta x}{\theta^2+1}e^{-\theta x}\ ;\quad x > 0,\qquad \theta > 0 \tag{2.4}$$

with the mixing ratios $\frac{\theta^2}{\theta^2+1}$ and $\frac{\theta^2}{\theta^2+1}$ respectively. Shanker distribution was fitted to a number of real lifetime data sets and was found to give a better fit in almost all the data sets than the exponential and Lindley distributions.

The Sujatha distribution proposed by Shanker (2016) is a three-components mixture of exponential distribution with scale parameter $\theta$, a gamma distribution with shape parameter 2 and scale parameter $\theta$ and a gamma distribution with shape parameter 3 and scale parameter $\theta$. Shanker (2016) obtained the p.d.f. and c.d.f. of the Sujatha distribution as:

$$f(x\,;\,\theta) = \frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x}\ ;\quad x > 0,\qquad \theta > 0 \tag{2.5}$$

$$F(x\,;\,\theta) = 1 - \left[1 + \frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right]e^{-\theta x}\ ;\quad x > 0,\qquad \theta > 0 \tag{2.6}$$

The Sujatha distribution was found to have a good fit with increasing hazard rate than the exponential, Lindley and Shanker distribution.

Shanker (2016) proposed the Amarendra distribution of the form:

$$f(x\,;\,\theta) = \frac{\theta^4}{\theta^3 + \theta^2 + 2\theta + 6}(1 + x + x^2 + x^3)e^{-\theta x}\ ; \quad x > 0, \theta > 0 \qquad (2.7)$$

The Amarendra distribution is a mixture of an exponential distribution and three gamma-type distributions with mixing proportions:

$$\frac{\theta^3}{\theta^3 + \theta^2 + 2\theta + 6}, \frac{\theta^2}{\theta^3 + \theta^2 + 2\theta + 6}, \frac{2\theta}{\theta^3 + \theta^2 + 2\theta + 6} \text{ and } \frac{6}{\theta^3 + \theta^2 + 2\theta + 6}.$$

Shanker (2016) found that the Amarendra distribution have monotonically increasing hazard rate for modeling lifetime data and performed better than the aforementioned distributions.

Other distributions have been proposed and extensions made to Lindley distribution using this method by some researchers including Zakerzadeh and Dolati (2009), Nadarajah *et al* (2011), Deniz and Ojeda (2011), Elbatal *et al* (2013), Singh *et al* (2014), Pararai *et al* (2015) and Abouammoh *et al* (2015). In this study, we propose a new one-parameter continuous probability distribution with a simpler structure than the Amarendra and Sujatha type; and called it 'Iwok-Nwikpe distribution'. We also look at some statistical properties and other relevant characteristics of this new distribution.

## METHODOLOGY

### *The Iwok-Nwikpe Distribution*

Let $X$ denote a continuous random variable, then $X$ is said to follow the Iwok-Nwikpe (I-N) distribution if its probability density function (p.d.f.) is given by:

$$f(x\,;\,\phi) = \frac{\phi^3}{\phi + 2}(x^2 + x)e^{-\phi x} \quad ; \quad x > 0, \quad \phi > 0 \qquad (3.1)$$

where $\phi$ is the scale parameter.

The distribution (3.1) is a two-component mixture of gamma $(2, \phi)$ and gamma $(3, \phi)$.

Generally, a continuous random variable $X$ is said to follow a gamma distribution with parameters $\alpha$ and $\lambda$ if its probability density function is given by:

$$f(x;\ \alpha,\lambda) = \frac{\lambda^{\alpha} x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} \qquad ;\ \alpha > 0, \qquad \lambda > 0\ , \qquad x > 0$$

The two-components p.d.f.s are:

$$f_1(x\ ;\ \phi) = \text{gamma}(2,\phi) = \frac{\phi^2 x}{\Gamma(2)} e^{-\phi x} \tag{3.2}$$

$$f_2(x\ ;\ \phi) = \text{gamma}(3,\phi) = \frac{\phi^3 x^2}{\Gamma(3)} e^{-\phi x} \tag{3.3}$$

with the mixing proportions:

$$p_1 = \frac{\phi}{\phi + 2} \ \text{ and } \ p_2 = \frac{2}{\phi + 2}$$

so that (3.1) can be expressed as:

$$f(x\ ;\ \phi) = f_1(x\ ;\ \phi) p_1 + f_2(x\ ;\ \phi) p_2 \tag{3.4}$$

The graphs of the p.d.f. for different values of $\phi$ are as shown below:

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Online ISSN 2055-0162(Online)

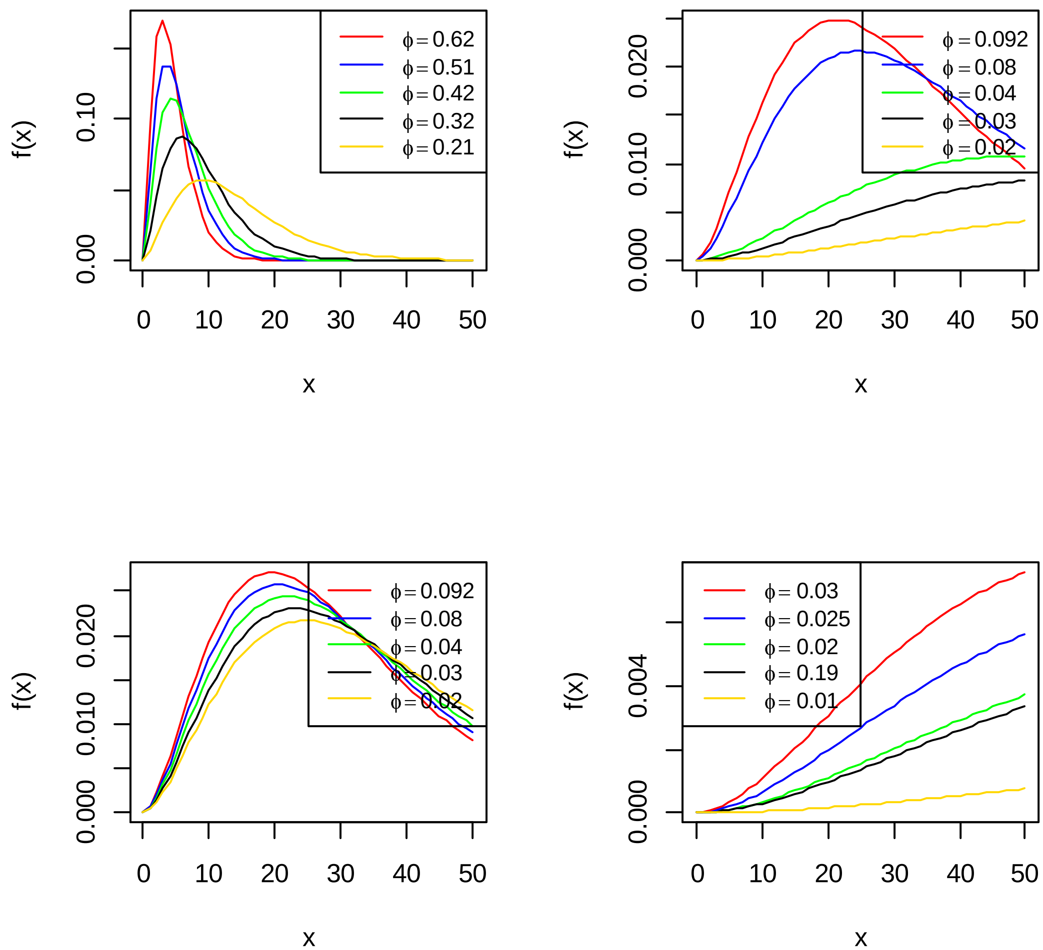


*Figure 1: Graphs of the p.d.f. of the I-N Distribution for different values of* $\phi$

***The cumulative distribution function of the Iwok-Nwikpe Distribution***

For a random variable , the cumulative distribution function (c.d.f.) is:

$$F(x) = \mathrm{P}(X \leq x), \quad x \in \mathbb{R}$$

$$\Rightarrow \mathrm{P}(X \leq x) = \int_{0}^{\mathrm{x}} f(t)dt \qquad ; \quad \text{where } \ t \sim (\mathrm{I} - \mathrm{N})$$

Hence for (I-N) distribution,

$$F(x\,;\,\phi) = \frac{\phi^3}{(\phi+2)} \int_{0}^{\mathrm{x}} (t^2 + t)e^{-\phi t} dt$$

Using Integtation by parts we have:

$$F(x\,;\,\phi) = \frac{\phi^3}{(\phi+2)} \left[ \frac{-e^{-\phi t}}{\phi}(t^2+t) - \frac{e^{-\phi t}(2t+1)}{\phi^2} - 2\frac{e^{-\phi t}}{\phi^3} \right]_0^x$$

Sustituting the limits we get:

$$F(x\,;\,\phi) = \left[ 1 - \left( 1 + \frac{\phi^2 x(x+1) + 2\phi x}{\phi+2} \right) \exp(-\phi x) \right] \qquad (3.5)$$

The graphs of the c.d.f. for different values of $\phi$ are as shown below:

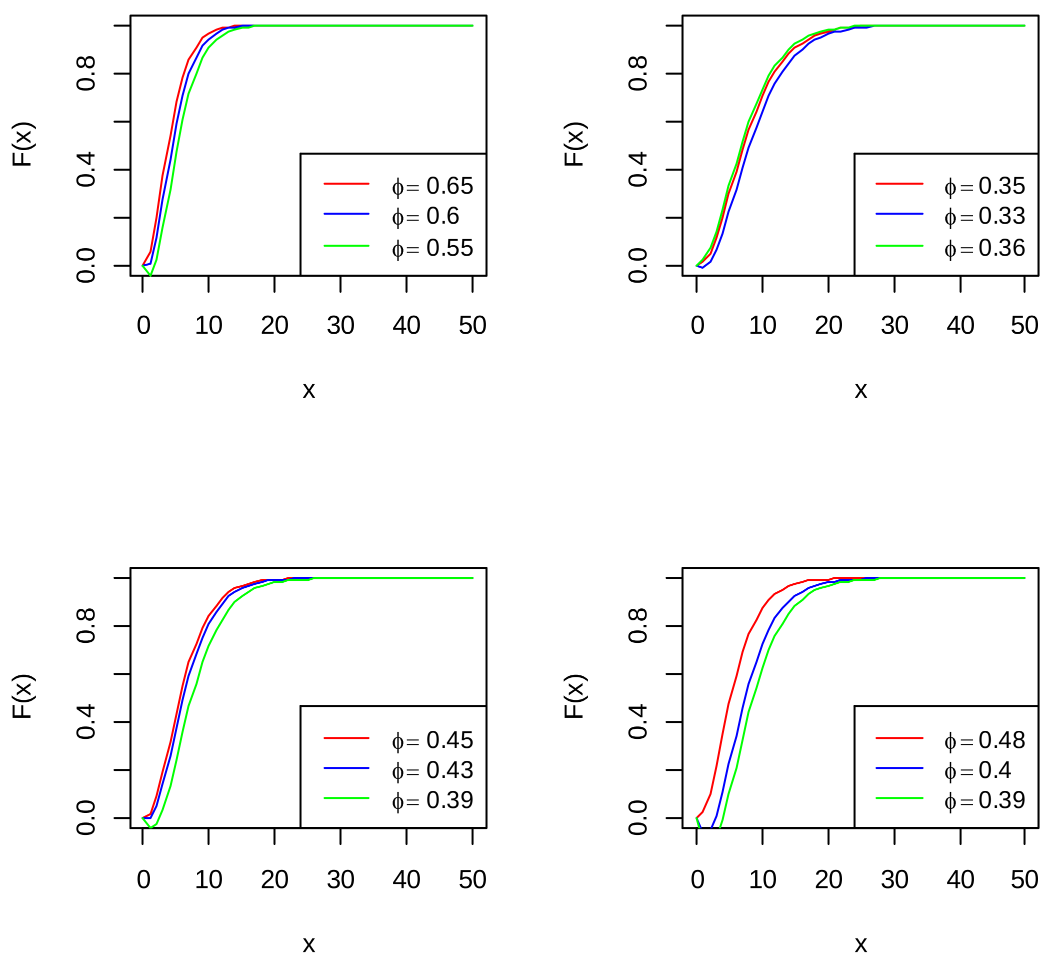


*Figure 2: Graphs of the c.d.f. of the I-N Distribution for different values of $\phi$*

***Statistical Properties***

**The Crude moments**

The $k$th raw moment of a random variable $X$ which follows the Iwok-Nwikpe (I-N) distribution is given as follows:

$$\mu_k' = E[X^k] = \int_{-\infty}^{\infty} x^k f(x)dx$$

where $f(x)$ is the p.d.f. of the I-N distribution.

$$\mu_k' = \frac{\phi^3}{\phi+2}\int_0^\infty x^k(x^2+x)e^{-\phi x}dx$$

$$= \frac{\phi^3}{\phi+2}\int_0^\infty x^{k+2}e^{-\phi x}\,dx + \int_0^\infty x^{k+1}e^{-\phi x}\,dx$$

$$= \frac{\phi^3}{\phi+2}\left(\frac{\Gamma(k+3)}{\phi^{k+3}} + \frac{\Gamma(k+2)}{\phi^{k+2}}\right)$$

$$= \frac{\phi^3}{\phi+2}\left[\frac{\Gamma(k+3)+\phi\Gamma(k+2)}{\phi^{k+3}}\right]$$

$$\Rightarrow \mu_k' = \frac{(k+2)!+\phi(k+1)!}{\phi^k(\phi+2)} \tag{3.6}$$

Thus, the first four uncorrected moments of the I-N distribution is given as follows:

$$\mu_1' = \frac{6+2\phi}{\phi(\phi\ +2)},\ \mu_2' = \frac{24+6\phi}{\phi^2(\phi\ +2)},\ \mu_3' = \frac{120+24\phi}{\phi^3(\phi\ +2)},\ \mu_4' = \frac{720+120\phi}{\phi^4(\phi\ +2)}$$

where $\frac{6+2\phi}{\phi(\phi\ +2)}$ is the mean of the distribution.

**The Second Central Moment (Variance) of the Iwok-Nwikpe Distribution**

Using the relationship between the raw and central moments we have:

$$\mu_2 = \sigma^2 = E[X-\mu]^2 = E[X^2] - \{E[X]\}^2 = \mu_2' - (\mu_1')^2$$

$$\Rightarrow \mu_2 = \frac{24+6\phi}{\phi^2(\phi\ +2)} - \left[\frac{6+2\phi}{\phi(\phi\ +2)}\right]^2 = \frac{10\phi^2+60\phi+84}{\phi^2(\phi\ +2)} \tag{3.7}$$

**The Third Central Moment of the I-N Distribution**

Recall that

$$\mu_3 = E[X-\mu]^3 \quad = \mu_3' - 3\mu\mu_2' + 2\,\mu_3'$$

$$= \frac{120 + 24\phi}{\phi^3(\phi\ + 2)} - 3\left[\frac{6 + 2\phi}{\phi(\phi\ + 2)}\right]\left[\frac{24 + 6\phi}{\phi^2(\phi\ + 2)}\right] + 2\left[\frac{120 + 24\phi}{\phi^3(\phi\ + 2)}\right]^2$$

$$\Longrightarrow \mu_3 = \frac{12[\phi^3 + 5\phi^2 + 48\phi + 22]}{\phi^3(\phi\ + 2)^3} \qquad (3.8)$$

**The Moment Generating Function of the Iwok-Nwikpe Distribution**

The Moment generating function (m.g.f.) of a random variable $X$ `which follows the I-N distribution is given by

$$M_X(t) = E[e^{Xt}] = \int_{-\infty}^{\infty} e^{xt} f(x)dx$$

where $f(x)$ is the p.d.f. of the I-N distribution.

$$M_X(t) = \frac{\phi^3}{\phi + 2}\int_0^{\infty} e^{xt}\,(x^2 + x)e^{-\phi x}dx$$

$$= \left(\frac{\phi^3}{\phi + 2}\right)\int_0^{\infty} x^2 e^{-x(\phi - t)}\,dx + \int_0^{\infty} x e^{-x(\phi - t)}\,dx$$

$$= \frac{\phi^3}{\phi + 2}\left[\frac{\Gamma(3)}{(\phi - t)^3} + \frac{\Gamma(2)}{(\phi - t)^2}\right]$$

$$= \frac{\phi^3}{\phi + 2}\left[\frac{2}{(\phi - t)^3} + \frac{1}{(\phi - t)^2}\right]$$

$$= \left(\frac{\phi^3}{\phi + 2}\right)\left\{\frac{2}{\phi^3}\sum_{k=0}^{\infty}\binom{k+2}{k}\left(\frac{t}{\phi}\right)^k + \frac{1}{\phi^2}\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\phi}\right)^k\right\}$$

$$= \left(\frac{\phi^3}{\phi + 2}\right)\sum_{k=0}^{\infty}\left[\frac{2}{\phi^3}\binom{k+2}{k} + \frac{1}{\phi^2}\binom{k+1}{k}\right]\left(\frac{t}{\phi}\right)^k$$

$$= \left(\frac{\phi^3}{\phi + 2}\right)\sum_{k=0}^{\infty}\left[\frac{(k + 2)(k + 1) + \phi(k + 1)}{\phi^3}\right]\left(\frac{t}{\phi}\right)^k$$

$$\Rightarrow M_X(t) = \sum_{k=0}^{\infty}\left[\frac{(k+2)(k+1)+\phi(k+1)}{\phi+2}\right]\left(\frac{t}{\phi}\right)^k \qquad (3.9)$$

The $r$th raw moments are the coefficients of $\frac{t^r}{r!}$ in equation (3.9) above. Hence,

$$\mu_r' = \frac{r!\,[(r+2)(r+1)+\phi(r+1)]}{\phi^r(\phi+2)}$$

**Distribution of Order Statistics**

Assuming $X_1, X_2, \dots, X_n$ are independent continuous random variables from I-N distribution, each with p.d.f. $f(x)$ and c.d.f. $F(x)$; then $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$, where $X_{(i)}$s are the $X_i$s arranged in order of increasing magnitude are the order statistics. The p.d.f. of the $k$th order statistic $[\,Y = X_{(k)}\,]$ is given by:

$$f_Y(y) = \frac{n!}{(n-k)!\,(k-1)!}\sum_{j=0}^{n-k}\binom{n-k}{j}(-1)^j F^{k+j-1}(y)\,f(y)$$

Thus, for the I-N distribution:

$$\Rightarrow f_Y(y) = \frac{n!}{(n-k)!\,(k-1)!}\sum_{j=0}^{n-k}\binom{n-k}{j}(-1)^j$$

$$\times\left\{\left[1-\left(1+\frac{\phi^2 x(x+1)+2\phi x}{\phi+2}\right)\exp(-\phi x)\right]\right\}^{k+j-1}\frac{\phi^3}{\phi+2}(x^2+x)e^{-\phi x}$$

$$= \frac{n!\,\phi^3(x^2+x)e^{-\phi x}}{(n-k)!\,(k-1)!\,(\phi+2)}\sum_{j=0}^{n-k}\sum_{q=0}^{\infty}\binom{k+j+1}{q}\binom{n-k}{j}(-1)^{j+q}$$

$$\times\left\{\left[1+\frac{\phi^2x(x+1)+2\phi x}{\phi+2}\right]\exp(-\phi x)\right\}^q$$

$$\text{Recall: } (1+x)^k=\sum_{i=0}^{k}\binom{k}{i}x^i \text{ and let } p=k+j+1$$

$$\Longrightarrow f_Y(y)=\frac{n!\,\phi^3(x^2+x)e^{-\phi x}}{(n-k)!\,(k-1)!\,(\phi+2)}\sum_{i=0}^{q}\sum_{j=0}^{n}\sum_{q=0}^{\infty}\binom{p}{q}\binom{n-k}{j}(-1)^{j+q}$$

$$\times\left[\frac{\phi^2x(x+1)+2\phi x}{\phi+2}\right]^i e^{-\phi xq}$$

$$\Longrightarrow f_Y(y)=\frac{n!\,\phi^3}{(n-k)!\,(k-1)!\,(\phi+2)^{i+1}}\sum_{i=0}^{q}\sum_{j=0}^{n}\sum_{q=0}^{\infty}\binom{p}{q}\binom{n-k}{j}(-1)^{j+q}$$

$$\times(x^2+x)e^{-\phi x(1+q)}[\phi^2x(x+1)+2\phi x]^i$$

## Parameter Estimation

### *Maximum likelihood Estimate of the parameter of the I-N Distribution*

Let $x_1, x_2, x_3, \dots, x_n$ be a $n-$ dimensional random sample from the I-N Distribution. Let the likelihood function of $n$ be $L$. By definition;

$$L=\prod_{i=1}^{n}f(x_1,x_2,\dots,x_n\,;\phi)=\prod_{i=1}^{n}f(x_i\,;\phi)$$

$$=\prod_{i=1}^{n}\frac{\phi^3}{\phi+2}(x_i^2+x_i)\,e^{-\phi x_i}$$

$$=\left[\frac{\phi^3}{\phi+2}\right]^n\prod_{i=1}^{n}(x_i^2+x_i)\,e^{-\phi\sum_{i=1}^{n}x_i}$$

Print ISSN: 2055-0154(Print),
Online ISSN 2055-0162(Online)

Taking the log of both sides

$$\log L = n\log\left(\frac{\phi^3}{\phi+2}\right) + \log\sum_{i=0}^{n}(x_i^2 + x_i) - \phi\sum_{i=1}^{n}x_i \qquad (3.10)$$

Equation (3.10) is the log of the likelihood function of the I-N distribution. We estimate the parameter, $\phi$ of the distribution by taking the partial derivative of the likelihood function with respect to $\phi$ and set it equals to 0. Thus, we have:

$$\frac{\partial \log L}{\partial \phi} = \frac{n(2\phi+6)}{\phi(\phi+2)} - \sum_{i=1}^{n}x_i = 0$$

$$\Rightarrow \frac{n(2\phi+6)}{\phi(\phi+2)} - \sum_{i=1}^{n}x_i = 0 \qquad (3.11)$$

Equation (3.11) could be solved using R programming with a given data set. The solution to equation (3.11) gives the maximum likelihood estimate of $\phi$.

**Survival Function of the New I-N Distribution**

The survival function can also be called the reliability function or survivor function.

Generally, its mathematical representation is given as:

$$s(x) = 1 - F(x)$$

For a random variable $X$ which has the I-N distribution, its survivor function is given by:

$$s(x) = 1 - \left[1 - \left(1 + \frac{\phi^2 x(x+1) + 2\phi x}{\phi+2}\right)\exp(-\phi x)\right]$$

The graphs of the survival function for different values of $\phi$ are as shown below:

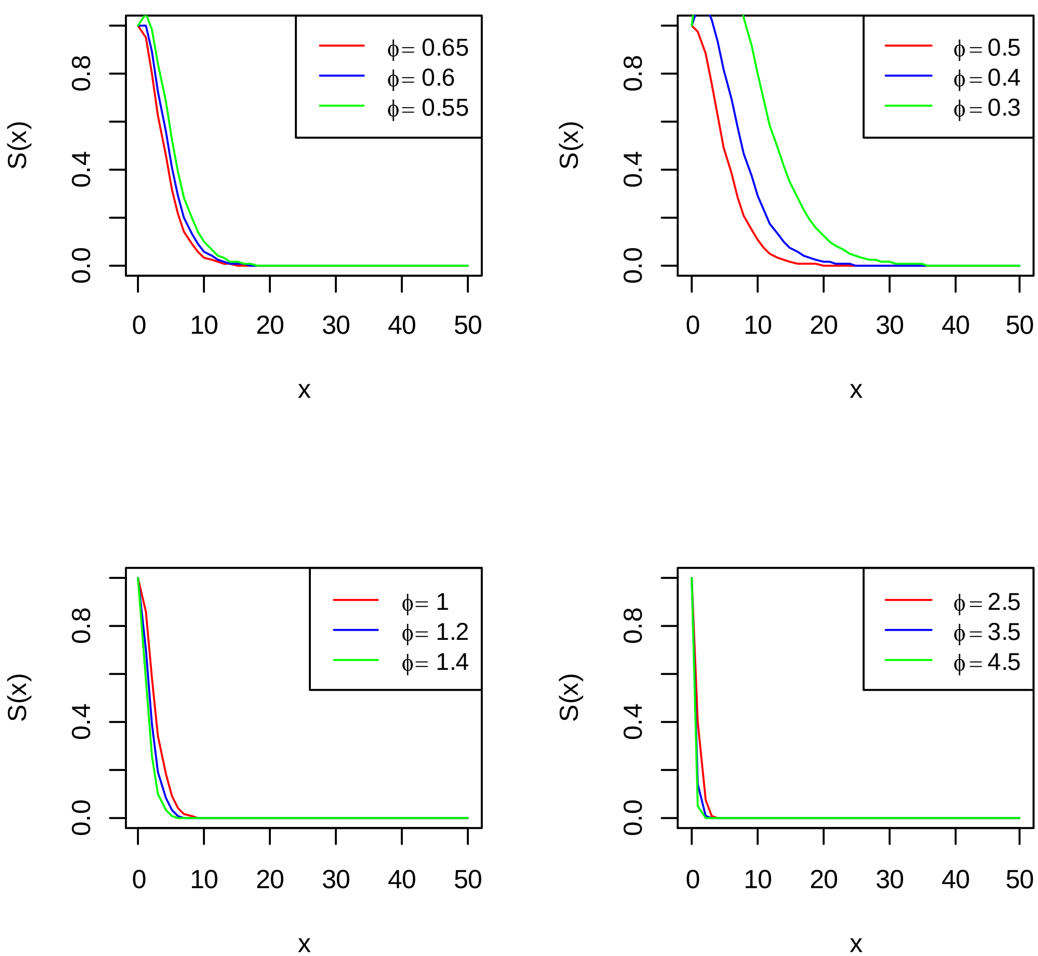


*Figure 2: Graphs of the survival function of the I-N Distribution for different values of* $\phi$

It is obvious from the graphs of $s(x)$ that the survival function is monotonically decreasing function and attend a constant rate at some higher values of $x$.

## Applications and Goodness of Fit

To apply the Iwok-Nwikpe distribution, real life data sets were used to fit the distribution. The performance of the distribution was compared with exponential, Lindley, Shanker, Sujatha and Amarendra distributions. The Akaike Information Criterion(AIC), Bayesian Information Criterion (BIC) and Akaike Information

Criterion Corrected (AICC) were used to determine the goodness of fit. The distribution that has the smallest AIC, BIC and AICC is taken to be the most flexible distribution for the data set.

The formulae for computing the different statistics are:

$$AIC = -2\ln L + 2k \ , \qquad BIC = -2\ln L + k\ln n, \ AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

where $k$ is the number of parameters and $n$ is the sample size.

***Data Sources***

The first data set used is the strength data of glass of the aircraft window reported by Fuller *et al* (1994) in Shanker (2015).

The second data set is data on tensile strength, measured in GPa, of 69 carbon fibres tested under tension at gauge lengths of 20 mm (Bader and Priest, 1982). The data was used by Shanker (2016) to fit the Sujatha distribution.

The third data is the relief times in minutes of 20 patients receiving an analgesic. The data set was given by Gross and Clark (1975, pp. 105). The data was used by Shanker (2016) to fit the Amerandra distribution.

The maximum likelihood and goodness of fit criteria were computed using the R-Software.

**First Data set :** *Strength data of glass of the aircraft window reported by Fuller et al* (*1994*)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 18.83 | 20.80 | 21.66 | 23.03 | 23.23 | 24.05 | 24.32 | 25.50 | 25.52 | 25.80 |
| 26.69 | 26.77 | 26.78 | 27.05 | 27.67 | 29.90 | 31.11 | 33.20 | 33.73 | 33.76 |
| 33.89 | 34.76 | 35.75 | 35.91 | 36.98 | 37.08 | 37.09 | 39.58 | 44.05 | 45.29 |
| 45.38 | | | | | | | | | |

**Second Data set :** *Tensile strength of 69 carbon fibres tested under tension by Bader and Priest* (*1982*)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1.312 | 1.314 | 1.479 | 1.552 | 1.700 | 1.803 | 1.861 | 1.865 | 1.944 | 1.958 |
| 1.966 | 1.997 | 2.006 | 2.021 | 2.027 | 2.055 | 2.063 | 2.098 | 2.140 | 2.179 |
| 2.224 | 2.240 | 2.253 | 2.270 | 2.272 | 2.274 | 2.301 | 2.301 | 2.359 | 2.382 |
| 2.382 | 2.426 | 2.434 | 2.435 | 2.478 | 2.490 | 2.511 | 2.514 | 2.535 | 2.554 |
| 2.566 | 2.570 | 2.586 | 2.629 | 2.633 | 2.642 | 2.648 | 2.684 | 2.697 | 2.726 |
| 2.770 | 2.773 | 2.800 | 2.809 | 2.818 | 2.821 | 2.848 | 2.880 | 2.954 | 3.012 |
| 3.067 | 3.084 | 3.090 | 3.096 | 3.128 | 3.233 | 3.433 | 3.585 | 3.585 | |

**Third Data set :** *Relief times of patients receiving analgesic by Gross and Clark* (*1975, pp. 105*)

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | 1.4 | 1.3 | 1.7 | 1.9 | 1.8 | 1.6 | 2.2 | 1.7 | 2.7 | 4.1 | 1.8 | 1.5 | 1.2 | 1.4 | 3 | 1.7 | 2.3 | 1.6 | 2 |

**Table 1 : Table of Comparison for the Five Different Distributions using Three Data Sets**

| Data Set | Model | Parameter Estimate | $-2\ln L$ | $AIC$ | $BIC$ | $AICC$ |
|---|---|---|---|---|---|---|
| First | Exponential | 0.0324 | 274.53 | 276.53 | 277.96 | 276.67 |
| | Lindley | 0.0629322 | 252.9932 | 255.9942 | 257.42331 | 256.1332 |
| | Shanker | 0.06471203 | 252.30 | 254.30 | 255.80 | 254.50 |
| | Sujatha | 0.09561026 | 241.5032 | 243.5031 | 244.9432 | 243.64301 |
| | Amarendra | 0.09706217 | 240.6818 | 242.6818 | 244.10 | 242.80 |
| | Iwok-Nwi. | 0.09588173 | 241.30 | 241.2062 | 242.51 | 241.20 |
| Second | Exponential | 0.407941 | 261.7432 | 263.7411 | 265.9655 | 263.80112 |
| | Lindley | 0.65900001 | 238.3667 | 240.3659 | 242.6134 | 240.44 |
| | Shanker | 0.6580296 | 233.0054 | 235.0054 | 237.2376 | 235.01 |
| | Sujatha | 0.9361194 | 221.6088 | 223.6088 | 225.8355 | 223.6688 |
| | Amarendra | 1.244256 | 207.947 | 209.947 | 209.7858 | 210.007 |
| | Iwok-Nwi. | 0.07670511 | 198.4607 | 200.4608 | 201.3401 | 205.023 |
| Third | Exponential | 0.5263 | 65.7 | 67.7 | 68.7 | 67.9 |
| | Lindley | 0.8161 | 60.50 | 62.50 | 63.49 | 62.72 |
| | Shanker | 0.8038668 | 59.78 | 61.78332 | 61.081 | 61.84 |
| | Sujatha | 1.1569 | 59.50 | 61.70 | 61.72 | 61.72 |
| | Amarendra | 1.4807 | 55.64 | 57.64 | 58.63 | 57.86 |
| | Iwok-Nwi. | 1.365411 | 49.7 | 51.71878 | 52.632 | 51.032 |

As stated earlier, the most flexible distribution is the one with the smallest AIC, BIC and AICC. The result in table 1 above clearly shows that the Iwok-Nwikpe distribution performs better than the exponential, Lidney, Shanker, Sujatha and Amarendra distributions.

## CONCLUSION

In this paper, we proposed a new continuous probability distribution named Iwok-Nwikpe distribution. The new distribution is a mixture of $\text{gamma}(2,\phi)$ and $\text{gamma}(3,\phi)$ distributions. Its moments, distribution of order statistics and moment generating function were derived. The probability density function and the cumulative distribution function were plotted. The method of maximum likelihood estimation was used for parameter estimation. The flexibility and supremacy of the I-N distribution was established using a real life data set. The results obtained in table 1 confirmed that the Iwok-Nwikpe distribution gave the best fit to the data sets used for this study and was considered more flexible than the exponential and other competing distributions.

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