European Journal of Statistics and Probability
Vol.10, No.2, pp., 39-53, 2022
Print ISSN: 2055-0154(Print),
Online ISSN 2055-0162(Online)

Application of Autoregressive Integrated Moving Average Model and Weighted Markov Chain on Forecasting Under-Five Mortality Rates in Nigeria

***Christogonus Ifeanyichukwu Ugoh[1], Osuji George Amaeze[2], Nwankwo Chike Henry[3], Nneka Chidinma Nwabueze[4], Eze Theophile Chinaza[5], Orji Gabriel Oyo[6]**
Department of Statistics, Faculty of Physical Sciences, Nnamdi Azikiwe University, Awka, Nigeria

Ugoh C.I., Osuji G.A., Nwankwo C.H., Nwabueze N.C., Eze T.C., Orji G.O. (2022) Application of Autoregressive Integrated Moving Average Model and Weighted Markov Chain on Forecasting Under-Five Mortality Rates in Nigeria, *European Journal of Statistics and Probability*, Vol.10, No.2, pp., 39-53,

ABSTRACT: *The aim of this study is to obtain an optimal model between the traditional time series model (ARIMA) and Weighted Markov Chain. The historical dataset of U5MR in Nigeria from 1980-2019 is obtained from the official website of World Bank. ARIMA modeling involved differencing of the data to attain stationarity, while WMC involved classification of the datasets into clusters using k-means cluster analysis and transition of states. Two performance measures Theil's U Statistic and MAPE are used to evaluate the two models based on in-sample and out-sample. The results shows that ARIMA(0,3,2) is a better model to forecast U5MR in Nigeria.*

KEYWORDS: U5MR, ARIMA, Weighted Markov Chain (WMC), MAPE, Theil's U Statistic, K-Mean Cluster

INTRODUCTION

Under-Five Mortality Rate is the probability of a child dying between birth and exactly 5 years of age, expressed per 1,000 live births in a given year for a particular geographical area [4,13]. In developing countries, childhood mortality rate is affected by socioeconomic, demographic, health variable, and region [9]. Nigeria and other countries in Sub Saharan Africa though experienced a decline in U5MR from 1980 to 2019, still maintain relatively and unacceptable high Mortality compared to many countries in Europe and America [2,3,8]. [1] compared the effect of ARIMA, Artificial Neural Networks, and Exponential Smoothing. [5] studied the U5MR of Malaysia by gender and developed a forecasting model for future prediction; the result showed that the U5MR for both genders decreased slowly. [9] analyzed the Under-5 mortality annual closing rate (CMACR) in Nigeria using Weighted Markov Chain and ARIMA model, the findings showed that ARIMA predicts CMACR better than WMC.

This paper attempts to establish a better model based on Autoregressive Integrated Moving Average (ARIMA) and Weighted Markov Chain (WMC) to forecast the Under-five Mortality Rate in Nigeria.

MATERIALS AND METHODS

Autoregressive Integrated Moving Average (ARIMA) Model

ARIMA is a statistical model which is used to predict future values based on past values. The 'AR' stands for Autoregressive, 'MA' stands for Moving Average, and 'I' stands for Integrated (that is the data values are replaced by difference between the data values and the previous values). ARIMA model is denoted by $ARIMA(p, d, q)$ and it is written as

$$y_t' = c + \phi_1 y_{t-1}' + \cdots + \phi_p y_{t-p}' + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{1}$$

where $\phi_1, \phi_2, \cdots, \phi_p$ are Autoregressive model's parameters; $\theta_1, \theta_2, \cdots, \theta_q$ are Moving Average model's parameters; c is a constant; ε_t is a white noise, and y_t' is the differenced series which might been differenced more than once

Autoregressive Moving Average (ARMA) Model

When the time series data is stationary and however does not require differencing, then the resultant model is an Autoregressive Moving Average (ARMA) model. ARMA model is denoted by $ARMA(p, q)$ and it is written as

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t \tag{2}$$

Autoregressive (AR) Model

AR model is the regression of the current observations against one or more past observations. That is the current observation y_t are generated by a weighted averages of past time series data going back p periods, together with a random disturbance in the current period. The AR of order p denoted by $AR(p)$ is defined as

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \varepsilon_t \tag{3}$$

Where ε_t is a white noise; $\phi_1, \phi_2, \cdots, \phi_p$ are the parameters of the AR model; y_t is the current observation, $y_{t-1}, y_{t-2}, \cdots, y_{t-p}$ are past observations.

Moving Average (MA) Model

MA is a linear combination of error terms occurring at various times in the past. MA model of order q is denoted as $MA(q)$ and it is written as

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \tag{4}$$

ARIMA Fitting

ARIMA model is fitted to the time series data (historical data) using the Box-Jenkins method. Four (4) steps are employed here, which are:

1. Check whether the historical data is stationary. This is done by observing the Autocorrelation Function (ACF) Plot. If the ACF Plot shows a rapid drop as the number of lags increases, then the historical data is stationary, but if it drops relatively slow as the number of lags increases, then the historical data is non-stationary and requires to be transformed by differencing at least once.

Differencing

This is the process of making a non-stationary time series stationary. It stabilizes the mean of time series by removing the changes in the series and eliminating or reducing trend and seasonality.

First Order Differencing

First Order Differenced series denoted as y_t' is the change between consecutive observations in the original series. It is written as

$$y_t' = y_t - y_{t-1} \tag{5}$$

If the first differenced series fails to be stationary, there is need to carry out second differencing

Second Order Differencing

Second Order Differenced series denoted as y_t'' is written as

$$y_t'' = y_t' - y_{t-1}' \tag{6}$$

where

$$y_{t-1}' = y_{t-1} - y_{t-2} \tag{7}$$

Again, if the Second Order Differenced series fails to be stationary, third differencing is carried. Using the Backshift Operator B, where the general dth order difference can be written as

$$y_t^d = (1 - B)^d y_t \tag{8}$$

Third Order Differencing

$$y_t' = (1 - B)^3 y_t \tag{9}$$

$$y_t' = y_t - 3y_{t-1} + 3y_{t-2} - y_{t-3} \tag{10}$$

2. Step Two deals with Estimation of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF)
3. Step Three deals with model identification, which involves the process of selecting the appropriate orders of AR and/or MA
4. Step Four deals with diagnostic check for model adequacy. The Akaike Information Criteria (AIC) and/or Bayesian Information Criteria (BIC) is/are used to check for model adequacy. The AIC is written as

$$AIC = nlog(\hat{\sigma}^2) + 2k \tag{11}$$

k is the number of model parameters; $\hat{\sigma}^2$ is the residual sum of squares, and n is the sample size Bayesian Information Criterion (BIC) is written as

$$BIC = nlog(\hat{\sigma}^2) + klog(n) \tag{12}$$

The ARIMA model with the lowest AIC and/or BIC are/is considered the best model among others.

Weighted Markov Chain (WMC)

Markov Chain

Markov chain is a stochastic process $X_t, t = 0,1,2,\cdots$ having the property that given the present state of the system, the past and the future are conditionally independent.
Whenever the process is in state i, there is a fixed probability P_{ij} that it will be in state j next, then the property of Markov chains is define as

$$P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \cdots, X_0 = i_0) = P_{ij} \qquad (13)$$

for all states $i_0, i_1, \cdots, i_{n-1}, i, j$ and all $n \geq 0$

where j is the future state, i is the present or current state, $i_{n-1}, i_{n-2}, \cdots i_1, i_0$ are the past states

Weighted Markov Chain

The method of Weighted Markov Chain adopted in this study for forecasting the U5MR is the one expressed by [6,7,10-12], which is categorized into seven (7) steps:

1. Set up a classification standard for the historical data, Under-five Mortality Rate (U5MR) using the K-means Cluster Analysis
2. Determine the m states, that is, the states of the historical data (U5MR) according to the classification standard
3. Obtain the Frequency Matrix (or Transition Matrix) according to step 2

$$F = \begin{pmatrix} f_{11} & f_{12} & \cdots & f_{1m} \\ f_{21} & f_{22} & \cdots & f_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mm} \end{pmatrix} \qquad (14)$$

4. Obtain the One-step transition matrix $P = (p_{ij})$ and the Marginal matrix $Q = (q_i)$ using the matrix of step 3, where

$$p_{ij} = \frac{f_{ij}}{\sum_{j=1}^{m} f_{ij}}; \quad q_i = \frac{f_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}} \qquad (15)$$

5. Test whether the Transition Probability Matrix obtained in step 4 has a Markov Property, using Chi-square test, defined as

$$\chi^2 = 2\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij} \left| ln\frac{p_{ij}}{q_i} \right| \qquad (16)$$

@ECRTD-UK: https://www.eajournals.org/
Publication of the European Centre for Research Training and Development -UK

Print ISSN: 2055-0154(Print),
Online ISSN 2055-0162(Online)

with degree of freedom $(m-1)^2$. The stochastic process (Transition Probability Matrix) has a Markov Property if χ^2 is greater than $\chi^2_{\alpha,(m-1)^2}$

6. Obtain the weight of the various steps Markov Chain, w_k transition probabilities matrices, which is defined as

$$w_k = \frac{|r_k|}{\sum_{k=1}^{K} r_k} \tag{17}$$

where r_k is the autocorrelation coefficient of the historical data with $k\epsilon\{1,2,\cdots,K\}$, and is computed as

$$r_k = \frac{\sum_{t=1}^{n-k}(y_t-\bar{y})(y_{t+k}-\bar{y})}{\sum_{t=1}^{n}(y_t-\bar{y})^2} \tag{18}$$

where y_t is the time series at time t, $\bar{y}$ is the average of time series y_t, and n is the number of time series y_t

7. Take the weighted average of various predicting probabilities of the same state as predicting probabilities of the U5MR, defined as

$$\hat{p}_{ij} = \sum_{k=1}^{K} w_k p_{ij}^{(k)} \tag{19}$$

for every $j\epsilon\{1,2,\cdots,m\}$, $\hat{p}_{ij}$ is the probability for a time series y_t to be in the state j in the future.

The forecast result is in the form of a state, state j obtained by $arg\ max\{\hat{p}_{ij}, j = 1,2,\cdots,m\}$

Measure of Forecast Accuracy

The measures of forecast accuracy adopted in this study is Theil's U Forecast Accuracy and Mean Absolute Percentage Error (MAPE).

Theil's U Forecast Accuracy

The Theil's U shows how the forecast conforms to the values of the future periods. It is written as

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t-\hat{y}_t)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}y_t^2}+\sqrt{\frac{1}{n}\sum_{t=1}^{n}\hat{y}_t^2}} \tag{20}$$

where Y_t is the actual value of a point for a given time period t, $\hat{Y}_t$ is the forecast value, n is the number of the data points.

If U falls within the range $0 \leq U < 1$, the proposed model is a good fit
If $U = 0$, the proposed model is a perfect fit
If $U \geq 1$, the proposed model is not a good fit

@ECRTD-UK: https://www.eajournals.org/
Publication of the European Centre for Research Training and Development -UK

Mean Absolute Percentage Error (MAPE)

Mean Absolute Percentage Error (MAPE) is used to measure the error of both methods (ARIMA and WMC). The model with the smallest MAPE is considered the appropriate model. It is defined as

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right| \times 100\% \qquad (21)$$

RESULTS/FINDINGS

Figure 1 shows the timeplot for Under-five Mortality Rate in Nigeria for the period of 1980 to 2019.

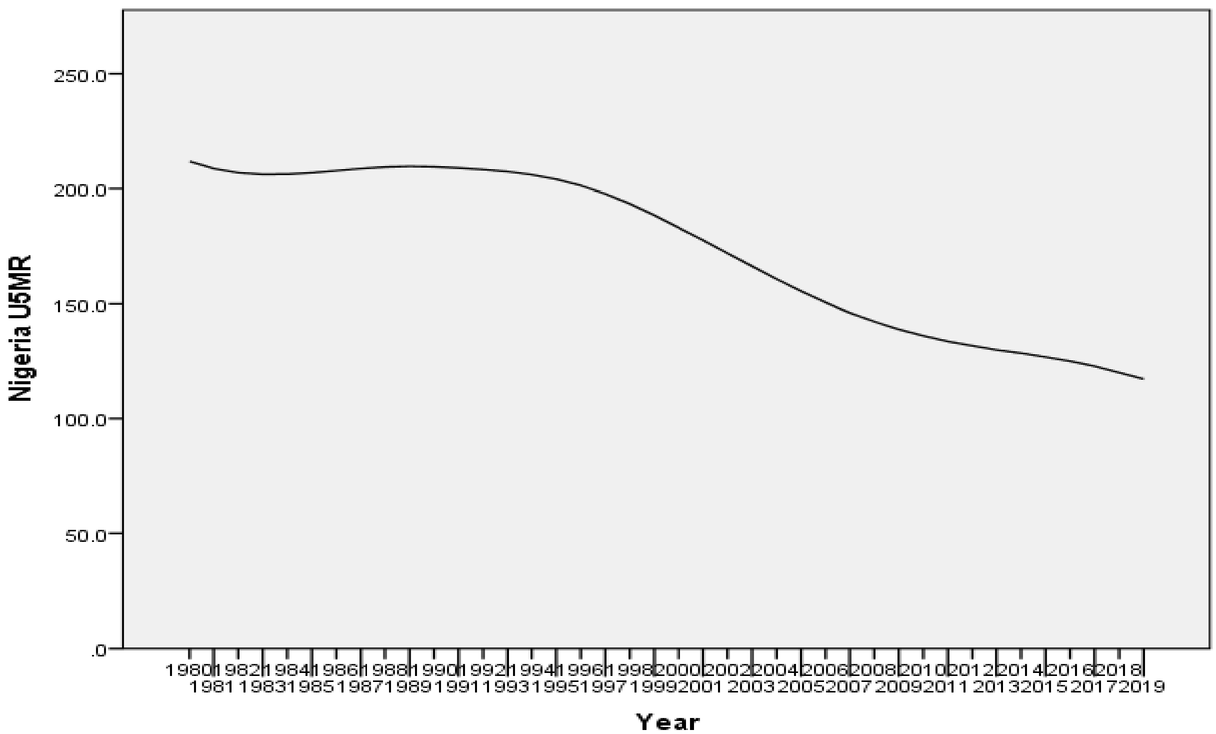
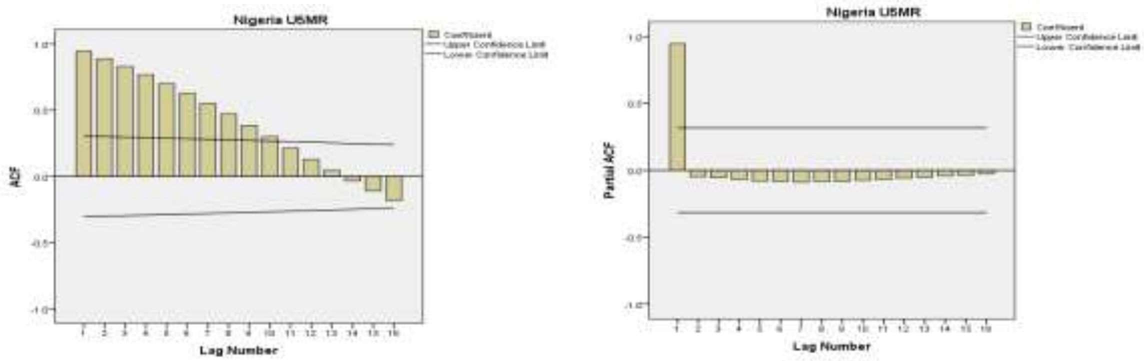


Figure 1: *Timeplot of U5MR in Nigeria from 1980-2019*

Nigeria's U5MR and Ghana's U5MR in Figure 1 are both showing a steady reduction. Where Ghana shows a deep steady fall compared to Nigeria, which implies that Nigeria recorded more Under Five Mortality than Ghana for the period review in this study.

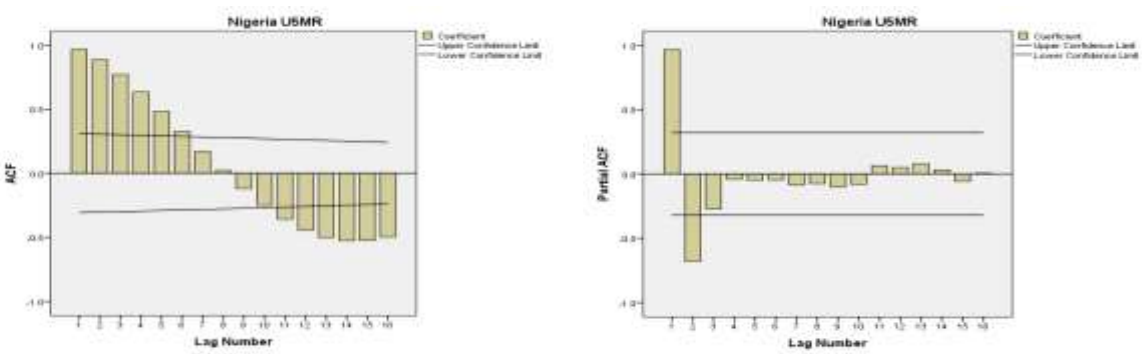
Vol.10, No.2, pp., 39-53, 2022
Print ISSN: 2055-0154(Print),
Online ISSN 2055-0162(Online)



A B

Figure 2: *(A) ACF Plot for Nigeria's U5MR (B) PACF Plot for Nigeria's U5MR*

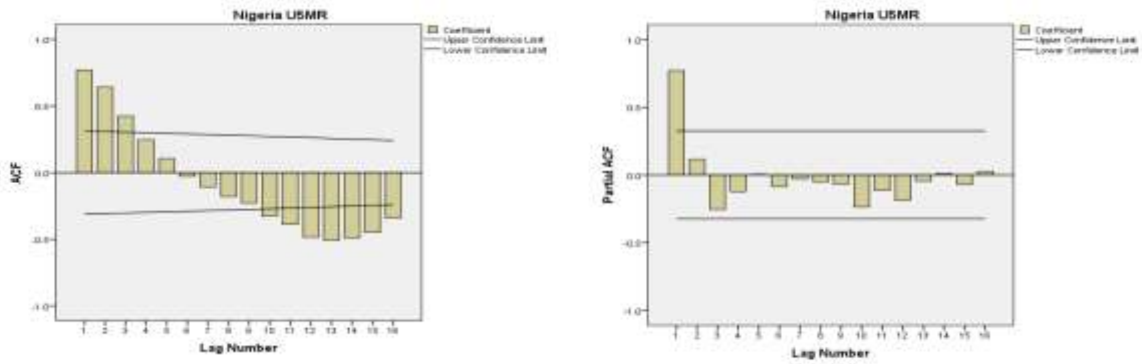
Figure 2A shows a slow fall of the lags as the lag number increases, thereby indicating a non-stationarity of Nigeria's U5MR. However, a first differenced ACF and PACF of Nigeria's U5MR is obtained as shown in Figure 3.



C D

Figure 3: *(C) First Differenced ACF Plot for Nigeria's U5MR (D) First Differenced PACF Plot for Nigeria's U5MR*

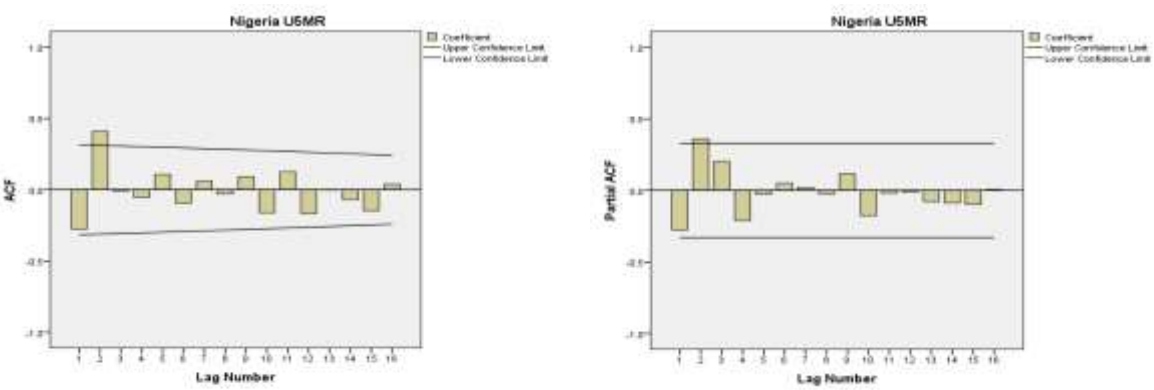
There is still a very slow fall of the lags as the lag number increases in Figure 3C which indicates that the first differenced Nigeria's U5MR is non-stationary, and will however require to be differenced the second time. Second differenced ACF and PACF of Nigeria's U5MR is obtained as shown in Figure 4



E F

Figure 4: *(E) Second Differenced ACF Plot for Nigeria's U5MR (F) Second Differenced PACF Plot for Nigeria's U5MR*

There is still a very slow fall of the lags as the lag number increases in Figure 4E which indicates that the second differenced Nigeria's U5MR is non-stationary, and will however requires to be differenced the third time. Third differenced ACF and PACF of Nigeria's U5MR is obtained as shown in Figure 5.



G H

Figure 5: *(G) Third Differenced ACF Plot for Nigeria's U5MR (H) Third Differenced PACF Plot for Nigeria's U5MR*

Figure 5G shows a quick fall at lag 1, thereby implying that the third differenced Nigeria's U5MR is now stationary. Lag 2 is significant as it cuts through the upper bound. And in Figure 5H, lag 2 is the only significant lag, which implies that the required ARIMA model is of order 2.

Print ISSN: 2055-0154(Print),
Online ISSN 2055-0162(Online)

In Table 2, ARIMA(0,3,2) has the smallest Bayesian Information Criteria (BIC) of -2.679 which indicates that the required ARIMA model that will be used to forecast Nigeria's U5MR is the ARIMA(0,3,2). The ARIMA (0,3,2) model that will be used to forecast the Nigeria's U5MR is written as $y_t' = -0.581\varepsilon_{t-1} + \varepsilon_t$

Table 1: *ARIMA Model Adequacy for Nigeria's U5MR*

ARIMA(p,d,q)	BIC
ARIMA(0,3,2)	-2.679
ARIMA(2,3,0)	-2.372
ARIMA(2,3,2)	-2.260
ARIMA(1,3,2)	-2.375
ARIMA(2,3,1)	-2.281
ARIMA(1,3,0)	-2.339
ARIMA(0,3,1)	-2.295

The Nigeria's U5MR is classified into six(6) blocks with the states in which each classified U5MR falls as shown in Table 2

Table 2: *Classification of Nigeria's U5MR*

State	Block of the Nigeria's U5MRs
1	$y \leq 129.4$
2	$129.4 < y \leq 155.9$
3	$155.9 < y \leq 180.3$
4	$180.3 < y \leq 197.6$
5	$197.6 < y \leq 207.9$
6	$y > 207.9$

The Nigeria's U5MR from 1980 to 2019 and the states in which they fall, including the state of transitions of the rates are shown in Table 3.

Table 3: *U5MR and State of Transition*

Year	U5MR	State	State Transition
1980	211.8	6	
1981	208.7	6	66
1982	206.9	5	65
⋮	⋮	⋮	⋮
2000	183.1	4	44
2001	177.7	3	43
2002	172.0	3	33
⋮	⋮	⋮	⋮
2018	120.0	1	11
2019	117.2	1	11

In Table 3, taking year 1980, it is placed in state 6 but did not transit from another state because there is no state preceding it. In the case of 1982, the U5MR transited from state 6 to state 5 in which it is being classified. And again, for 2019, the U5MR is transited from state 1 (U5MR of 2018) which comes before it.
The state transitions in Table 4 are arranged in the frequency matrix (Transition matrix) as shown below

$$F = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{pmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 8 & 1 \\ 0 & 0 & 0 & 0 & 2 & 6 \end{pmatrix} \end{array} \begin{array}{l} \\ = 5 \\ = 9 \\ = 4 \\ = 3 \\ = 10 \\ = 8 \end{array} \quad (22)$$

The one-step transition probability matrix and marginal matrix of the system in equation (25) are obtained using equation (15).

$$P = (p_{ij}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.11 & 0.89 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0.75 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.10 & 0.80 & 0.10 \\ 0 & 0 & 0 & 0 & 0.25 & 0.75 \end{pmatrix} \quad (23)$$

$$q_{ij} = \begin{pmatrix} 0.128 & 0 & 0 & 0 & 0 & 0 \\ 0.026 & 0.205 & 0 & 0 & 0 & 0 \\ 0 & 0.026 & 0.077 & 0 & 0 & 0 \\ 0 & 0 & 0.026 & 0.051 & 0 & 0 \\ 0 & 0 & 0 & 0.026 & 0.205 & 0.026 \\ 0 & 0 & 0 & 0 & 0.051 & 0.153 \end{pmatrix} \quad (24)$$

then the Marginal matrix $Q = (q_i)$ is given as

$$Q = (q_i) = \begin{pmatrix} 0.128 \\ 0.231 \\ 0.103 \\ 0.077 \\ 0.257 \\ 0.204 \end{pmatrix} \quad (25)$$

The information in the Transition Probability Matrix and Marginal Matrix is used to test whether the stochastic process has a Markov Property as shown in Table 4.

Table 4: *Chi-square Test for Markov Property Verification*

State, i	f_{ij}	p_{ij}	q_i	$ln\frac{p_{ij}}{q_i}$	$f_{ij}\left(ln\frac{p_{ij}}{q_i}\right)$
1	5	1.00	0.128	2.0557	10.2786
2	1	0.11	0.231	-0.7419	-0.7419
2	8	0.89	0.231	1.3488	10.7904
3	1	0.25	0.103	0.8867	0.8867
3	3	0.75	0.103	1.9853	5.9560
4	1	0.33	0.077	1.4553	1.4553
4	2	0.67	0.077	2.1635	4.3269
5	1	0.10	0.257	-0.9439	-0.9439
5	8	0.80	0.257	1.1355	9.0843
5	1	0.10	0.257	-0.9439	-0.9439
6	2	0.25	0.204	0.2033	0.4067
6	6	0.75	0.204	1.3020	7.8117
					48.3669

The Chi-square computed in Table 4 is$\chi^2 = 48.3669$ is greater than the Chi-square tabulated $\chi^2_{\alpha,(m-1)^2} = \chi^2_{0.05,25} = 37.652$, where $m = 6$ (number of states) and $(m-1)^2 = 25$ (degree of freedom), however, the stochastic process obtained has a Markov Property. Moreover, the weights w_k and the autocorrelation coefficients r_k for previous five-time series are computed and shown in Table 5.

Table 5: *Estimated Autocorrelation Coefficients and Weights of Markov Chain*

	k					
	1	2	3	4	5	Total
r_k	0.94610	0.89009	0.83132	0.76859	0.70121	4.13731
w_k	0.22868	0.21514	0.20093	0.18577	0.16948	

$$P^{(2)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.44 & 0.56 & 0 & 0 & 0 & 0 \\ 0.03 & 0.41 & 0.56 & 0 & 0 & 0 \\ 0 & 0.08 & 0.47 & 0.45 & 0 & 0 \\ 0 & 0 & 0.03 & 0.15 & 0.67 & 0.15 \\ 0 & 0 & 0 & 0.03 & 0.39 & 0.58 \end{pmatrix}$$

$$P^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.58 & 0.42 & 0 & 0 & 0 & 0 \\ 0.07 & 0.51 & 0.42 & 0 & 0 & 0 \\ 0 & 0.20 & 0.50 & 0.30 & 0 & 0 \\ 0 & 0 & 0.07 & 0.17 & 0.57 & 0.19 \\ 0 & 0 & 0 & 0.06 & 0.46 & 0.48 \end{pmatrix}$$

$$P^{(4)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.68 & 0.32 & 0 & 0 & 0 & 0 \\ 0.13 & 0.55 & 0.32 & 0 & 0 & 0 \\ 0.03 & 0.30 & 0.47 & 0.20 & 0 & 0 \\ 0 & 0.03 & 0.11 & 0.17 & 0.50 & 0.19 \\ 0 & 0 & 0.02 & 0.08 & 0.49 & 0.41 \end{pmatrix}$$

$$P^{(5)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.76 & 0.24 & 0 & 0 & 0 & 0 \\ 0.19 & 0.57 & 0.24 & 0 & 0 & 0 \\ 0.06 & 0.28 & 0.42 & 0.24 & 0 & 0 \\ 0 & 0.05 & 0.12 & 0.16 & 0.46 & 0.20 \\ 0 & 0 & 0.05 & 0.10 & 0.50 & 0.35 \end{pmatrix}$$

Table 6: *U5MR Forecast for 2015*

Initial Year	State	Step, k	Weight, w_k	1	2	3	4	5	6	Probability Source
2014	1	1	0.22868	1	0	0	0	0	0	$P^{(1)}$
2013	2	2	0.21514	0.44	0.56	0	0	0	0	$P^{(2)}$
2012	2	3	0.20093	0.58	0.42	0	0	0	0	$P^{(3)}$
2011	2	4	0.18577	0.68	0.32	0	0	0	0	$P^{(4)}$
2010	2	5	0.16948	0.76	0.24	0	0	0	0	$P^{(5)}$
$\hat{p}_{ij}$ (Weighted Average)				0.695	0.305	0	0	0	0	

Table 6 shows $max\{\hat{p}_{ij}\} = 0.695$, indicating that the U5MR in 2014 is in state 1 with the highest probability of 0.695, and it satisfies the interval $y \leq 129.4$. But the actual U5MR in 2015 is 126.8 and it falls within the interval. Taking the average of the interval, then, the forecasted U5MR is 64.7. Similarly, forecasting the U5MR for 2016 using 2011-2015 as initial states as give as given in **Table 7**, U5MR is in state 1 with probability 0.815, and it falls within the interval $y \leq 129.4$. The actual U5MR in 2016 which is 125 shows that the prediction is also true. The same procedure is used to forecast for the remaining years. **Table 8** shows the In-sample and Out-of-Sample forecast of U5MR for 2000-2030 using ARIMA(0,3,2) and WMC and performance measures.

Table 7: *U5MR Forecast for 2016*

Initial Year	State	Step, k	Weight, w_k	1	2	3	4	5	6	Probability Source
2015	1	1	0.22868	1	0	0	0	0	0	$P^{(1)}$
2014	1	2	0.21514	1	0	0	0	0	0	$P^{(2)}$
2013	2	3	0.20093	0.58	0.42	0	0	0	0	$P^{(3)}$
2012	2	4	0.18577	0.68	0.32	0	0	0	0	$P^{(4)}$
2011	2	5	0.16948	0.76	0.24	0	0	0	0	$P^{(5)}$
$\hat{p}_{ij}$ (Weighted Average)				0.815	0.185	0	0	0	0	

Table 8: *In-sample and Out-of-Sample Forecast of U5MR for 2000-2030 and Performance Measures*

Year	2000	2001	⋯⋯	2029	2030	MAPE	Theil's U Statistics
ARIMA(0,3,2)	182.9	177.3	⋯⋯	97.4	96.6	0.174 3 36%	0.000014
WMC	182.0	168.1	⋯⋯	64.7	64.7	17.5101 30%	0.001702

From Table 8 shows that the best forecasting performance is obtained by using ARIMA(0,3,2) model, which has the lowest MAPE of 0.174336% and a Theil's U statistic of 0.000014 which is very closed to zero (0) depicting a perfect fit compared to WMC.

Again, the out-sample forecast of Nigeria's U5MR using ARIMA(0,3,2) shows a steady decrease, where in 2030, the U5MR will drop to 96.6 deaths per 1000 live births, which shows a drop of 20.6%. Figure 6 shows the timeplot for the historical data from 1980-2019 and out-of-sample forecast of U5MR for 2020-2030.

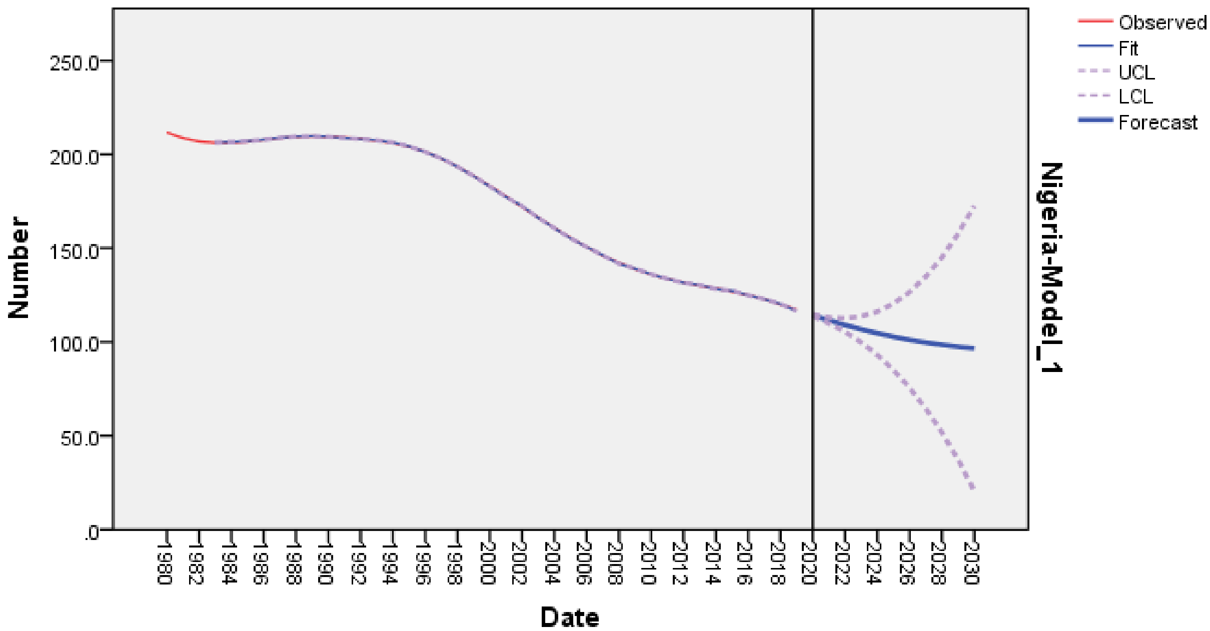


Figure 6: *Timeplot of the Actual U5MR and Out-Sample Forecast of U5MR*

CONCLUSION

The purpose of this paper is to identify the best model to forecast U5MR in Nigeria. ARIMA(0,3,2) predicts U5MR better than WMC, and based on the modeling and forecasting, the U5MR is showing an instrinsic decrease from year to year. The findings of this study can help promote health policies in order to address and to reduce U5MR in the future, as well as to

establish a basis for implementing optimal strategies that can be used to overcome U5MR in order to meet up with the target of SDGs.

References

[1] Adeyinka, D. A. and Muhajarine, N. Time series prediction of under-five mortality rates for Nigeria: Comparative analysis of artificial neural networks, Holt-Winters exponential smoothing and autoregressive integrated moving average models, BMC Medical Research Methodology, 2020, 20:292

[2] Adekanmbi, V. I., Kayode, G. O. and Uthman, O. A. Individual and contextual factors associated with childhood stunting in Nigeria: A multilevel analysis, Maternal Child Nutr, 2013, 9(2), 244-259

[3] Aremu, O., Lawoko, S. and Dalal, K. Neighbourhood Socioeconomic disadvantage, individual wealth status and patterns of delivery care utilization in Nigeria: A multilevel discrete choice analysis, International Journal of Women's Health, 2011, 3, 167-174

[4] Eke, D. O. and Ewere, F. Modeling and forecasting under-five mortality rate in Nigeria using auto-regressive integrated moving average approach, Earthline Journal of Mathematical Sciences, 2020, 4(2), 2581-8147

[5] Husin, W. Z. W., Ramli, R. Z., Muzaffar, A. N., Nasir, N. F. A. and Rahmat, S. N. E. Trend analysis and forecasting models for under-five mortality rate in Malaysia, Palarch's Journal of Archaeology of Egypt/Egyptology, 2020, 17(10), 875-889

[6] Kafi, R. A., Safitri, Y. R., Widyaningsih, Y. and Handari, B. D. Comparison of weighted Markov chain and Fuzzy time series Markov chain in forecasting stock closing price of company X, AIP Conference Proceedings 2168, 020033(2019), htpps://doi.org/10.1063/1.5132460

[7] Kordnoori, S., Mostafaei, H. and Kordnoori, S. Applied SCGM(1,1)c Model and weighted Markov chain for Exchange Rate Ratios, Hyperion Economic Journal, 2015, 3(4), 12-22

[8] Mesike, G. and Mojekwu, N. Environmental determinants of child mortality in Nigeria, Journal of Sustainable Development, 2012, 5(1), https://doi.org/10.5539/jsd.v5n1p65

[9] Obasohan, P. E. Comparing weighted Markov chain and autoregressive integrated moving average in the prediction of under-five mortality annual closing rates in Nigeria, International Journal of Statistics and Probability, 2020, 9(3), 13-22

[10] Peng, Z., Changjun Bao, Yang Zhao, Honggang Yi, Letian Xia, Hao Yu, Hongbing Shen and Feng Chen, Weighted Markov chains for forecasting and analysis in incidence of infectious diseases in Jiangsu Province, China, Journal of Biomedical Research, 2010, 24(3), 207-214

[11] Shahdoust, M., Sadeghifar, M., Poorolajal, J., Javanrooh, N. and Amini, P. Predicting hepatitis B monthly incidence rates using weighted Markov chains and time series methods, Journal of Research in Health Sciences, 2015, 15(1), 28-31

[12] Zhou, Qing-xin, Application of weighted Markov chain in stock price forecasting of China sport industry, International Journal of u- and e-Service, Science and Technology, 2015, 8(2), 219-226

[13] UNICEF. (2021). Child Survival. Retrieved from http://www.data.unicef.org/child_survival